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NUMERICAL SOLUTION OF LAPLACE EQUATION IN CURVED BORDER USING FUZZY DATA BY FINITE DIFFERENCE

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ABSTRACT: *The fuzzification of Laplace equation in two dimensions are discussed with curve border. The interval of fuzzy interval can be determined. Finite difference method applied of two different grids using five points, first for initial values and the second to solve Laplace equation numerically.*

KEYWORDS: Fuzzy membership Function (f.m.f.), Interval of Confidence, Triangular Fuzzy Number (t.f.n.), $\alpha-$Cuts, Five Points Finite Difference, Domain with Curve Border .

INTRODUCTION

The concept of Fuzzy differential equation was first introduced by Chang Zadeh [10]. Dubois and Prade[5] has given extension principle. Raphel [8], used five points in regular domain. Here implementing five-points for finite difference method to solve Laplace equation in two variables numerically in domain with curved boundary, then fuzzified.

Definitions

A triangular Fuzzy number μ is defined by three real numbers with base as the interval $[a,c]$ and b as the vertex of triangle. The membership function are defined as follows [1,8]:

$$\mu(x)(=)\begin{Bmatrix} \frac{x-a}{b-a} & ; \ where \ \ a \le x \le b \\ \frac{x-c}{b-c} & ; \ where \ \ b \le x \le c \\ 0 & ; \ \ otherwise \end{Bmatrix} .$$

The $\alpha-$cuts are defined by $\Delta_L(\alpha)(=)a+\alpha(b-a)$ and $\Delta_R(\alpha)(=)c+\alpha(b-c)$

Finite difference using to solve Laplace equation in irregular domain

The Laplace equation in two variables is defined by

$$u_{xx}(x,y)+u_{yy}(x,y)=0 \tag{1}$$

This equation is encountered in many application, fluid mechanics, study state , electrostatics, mass transfer, and for other areas of mechanics and physics. Replacing u_{xx} and u_{yy} by the central difference formula the value of $u(x_i,x_j)$ at any mesh point is the arithmetic mean of the values at four neighboring mesh to the left, right, above and below which is called standard five points formula Fig.1 in curved boundary , use for finding the initial data respectively as follows

$$u_{i,j} = \frac{1}{\alpha_0}\left[\alpha_1 u_{i-1,j} + \alpha_2 u_{i+1,j} + \alpha_3 u_{i,j-1} + \alpha_4 u_{i,j+1}\right] \qquad (2)$$

Where $\alpha_1 = \frac{2}{h_1(h_1+h_3)}$, $\alpha_2 = \frac{2}{h_2(h_2+h_4)}$, $\alpha_3 = \frac{2}{h_3(h_1+h_3)}$, $\alpha_4 = \frac{2}{h_4(h_2+h_4)}$ and $-\alpha_0 = \sum_{i=1}^{4}\alpha_i$, where

$h_i \le h$ for $i = 1,2,3,4$ and h is the standard step length of the mesh, and if $h_i < h$, then h_i is the step length near the border [11].

Application of Fuzzy interval in Laplace Equation

From c_2 to c_{15} represents the boundary conditions of the mesh non-square with fuzzy interval as in table 1.

Table -1-

$c_1(=)[l_{1,1}; l_{1,2}; l_{1,3}]$	$c_2(=)[l_{2,1}; l_{2,2}; l_{2,3}]$	$c_3(=)[l_{3,1}; l_{3,2}; l_{3,3}]$	$c_4(=)[l_{4,1}; l_{4,2}; l_{4,3}]$
$c_5(=)[l_{5,1}; l_{5,2}; l_{5,3}]$	$c_6(=)[l_{6,1}; l_{6,2}; l_{6,3}]$	$c_7(=)[l_{7,1}; l_{7,2}; l_{7,3}]$	$c_8(=)[l_{8,1}; l_{8,2}; l_{8,3}]$
$c_9(=)[l_{9,1}; l_{9,2}; l_{9,3}]$	$c_{10}(=)[l_{10,1}; l_{10,2}; l_{10,3}]$	$c_{11}(=)[l_{11,1}; l_{11,2}; l_{11,3}]$	$c_{12}(=)[l_{12,1}; l_{12,2}; l_{12,3}]$
$c_{13}(=)[l_{13,1}; l_{13,2}; l_{13,3}]$	$c_{14}(=)[l_{14,1}; l_{14,2}; l_{14,3}]$	$c_{15}(=)[l_{15,1}; l_{15,2}; l_{15,3}]$	$c_{16}(=)[l_{16,1}; l_{16,2}; l_{16,3}]$

Fig.1

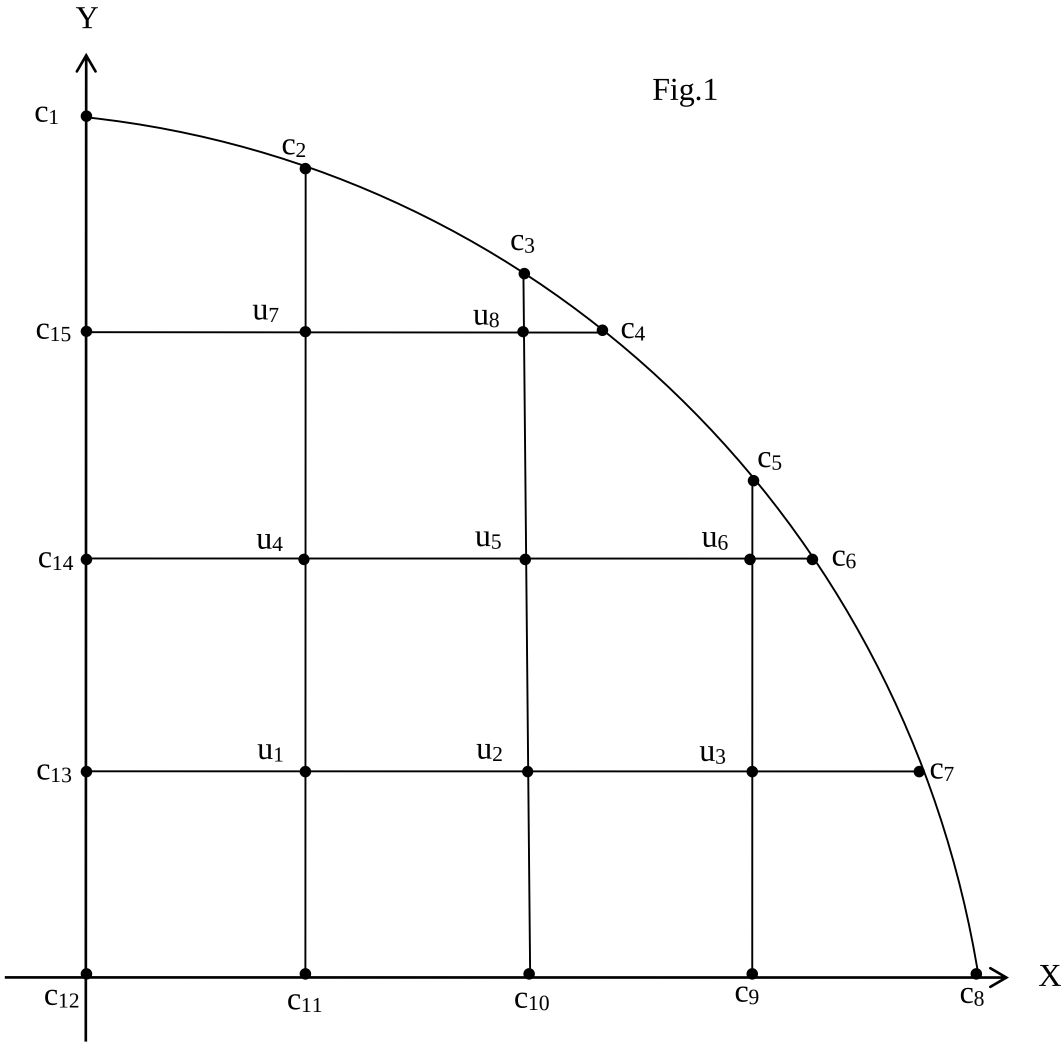
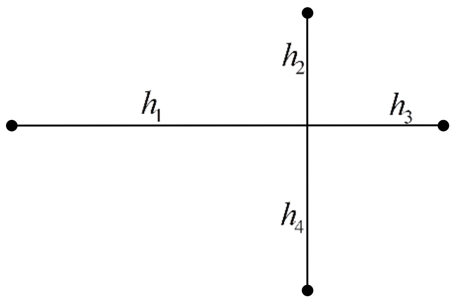


Fig.2



The interior points due to the non-square grid are u_1 to u_8.

Now to find the initial value of $u_5^{(0)}$ using standard five-points formula (2) as

$$u_5^{(0)}(=)\frac{1}{\alpha_0}\left[(\alpha_1)c_{14}(+)(\alpha_2)c_3(+)(\alpha_3)c_6(+)(\alpha_4)c_{10}\right]$$

(3)

We can write equation (9) in some details as

$$\left[u_5^{(0)}\right]^{(\alpha)}(=)\left\{\begin{array}{l} \dfrac{\alpha_1 l_{14,1}+\alpha_2 l_{3,1}+\alpha_3 l_{6,1}+\alpha_4 l_{10,1}}{\alpha_0}, \\ \dfrac{\alpha_1 l_{14,2}+\alpha_2 l_{3,2}+\alpha_3 l_{6,2}+\alpha_4 l_{10,2}}{\alpha_0}, \\ \dfrac{\alpha_1 l_{14,3}+\alpha_2 l_{3,3}+\alpha_3 l_{6,3}+\alpha_4 l_{10,3}}{\alpha_0} \end{array}\right\} \tag{4}$$

Fuzzy membership functions (f.m.f) are respective α -cuts of $c_2, c_{6,} c_{10,} and\ c_{14}$ are respectively as

Hence the $\alpha -$cuts of c_2 is $[c_2]^{(\alpha)}(=)\left[l_{2,1}+\alpha(l_{2,2}-l_{2,1}), l_{2,3}+\alpha(l_{2,2}-l_{2,3})\right]$

$$\mu_{C_6}(x)(=)\left\{\begin{array}{ll} \dfrac{x-l_{6,1}}{l_{6,2}-l_{6,1}} & ;\ where\ \ l_{6,1}\le x\le l_{6,2} \\ \dfrac{x-l_{6,3}}{l_{6,2}-l_{6,3}} & ;\ where\ \ l_{6,2}\le x\le l_{6,3} \\ 0 & ;\ \ otherwise \end{array}\right\} \tag{6}$$

Hence the $\alpha -$cuts of c_6 is $[c_6]^{(\alpha)}(=)\left[l_{6,1}+\alpha(l_{6,2}-l_{6,1}), l_{6,3}+\alpha(l_{6,2}-l_{6,3})\right]$

$$\mu_{C_{10}}(x)(=)\left\{\begin{array}{ll} \dfrac{x-l_{10,1}}{l_{10,2}-l_{10,1}} & ;\ where\ \ l_{10,1}\le x\le l_{10,2} \\ \dfrac{x-l_{2,3}}{l_{10,2}-l_{10,3}} & ;\ where\ \ l_{10,2}\le x\le l_{10,3} \\ 0 & ;\ \ otherwise \end{array}\right\} \tag{7}$$

Hence the $\alpha-$cuts of c_{10} is $[c_{10}]^{(\alpha)}(=)[l_{10,1}+\alpha(l_{10,2}-l_{10,1}), l_{10,3}+\alpha(l_{10,2}-l_{10,3})]$

$$\mu_{C_{14}}(x)(=)\begin{Bmatrix} \dfrac{x-l_{10,1}}{l_{10,2}-l_{10,1}} & ; & where \quad l_{2,1}\le x\le l_{2,2} \\ \dfrac{x-l_{14,3}}{l_{14,2}-l_{14,3}} & ; & where \quad l_{14,2}\le x\le l_{14,3} \\ 0 & ; & otherwise \end{Bmatrix} \tag{8}$$

Hence the $\alpha-$cuts of c_{14} is $[c_{14}]^{(\alpha)}(=)[l_{14,1}+\alpha(l_{14,2}-l_{14,1}), l_{14,3}+\alpha(l_{14,2}-l_{14,3})]$

Then from the equation (4), the interval of the confidence for $u_5^{(0)}$ is

$$\mu_{u_5^{(0)}}(x)(=)\begin{Bmatrix} \dfrac{\alpha_1(l_{14,2}-l_{14,1})+\alpha_2(l_{2,2}-l_{2,1})+\alpha_3(l_{6,2}-l_{6,1})+\alpha_4(l_{10,2}-l_{10,2})}{\alpha_0}\alpha \\ +\dfrac{\alpha_1 l_{14,1}+\alpha_2 l_{2,1}+\alpha_3 l_{6,1}+\alpha_4 l_{10,1}}{\alpha_0}, \\ \\ \dfrac{\alpha_1(l_{14,2}-l_{14,3})+\alpha_2(l_{2,2}-l_{2,3})+\alpha_3(l_{6,2}-l_{6,3})+\alpha_4(l_{10,2}-l_{10,3})}{\alpha_0}\alpha \\ +\dfrac{\alpha_1 l_{14,3}+\alpha_2 l_{2,3}+\alpha_3 l_{6,3}+\alpha_4 l_{10,3}}{\alpha_0} \end{Bmatrix}$$

or

$$\mu_{u_5^{(0)}}(x)(=)\begin{Bmatrix} \dfrac{(\alpha_1 l_{14,2}+\alpha_2 l_{2,2}+\alpha_3 l_{6,2}+\alpha_4 l_{10,2})-(\alpha_1 l_{14,1}+\alpha_2 l_{2,1})+\alpha_3 l_{6,1}+\alpha_4 l_{10,2})}{\alpha_0}\alpha \\ +\dfrac{\alpha_1 l_{14,1}+\alpha_2 l_{2,1}+\alpha_3 l_{6,1}+\alpha_4 l_{10,1}}{\alpha_0}, \\ \\ \dfrac{(\alpha_1 l_{14,2}+\alpha_2 l_{2,2}+\alpha_3 l_{6,2}+\alpha_4 l_{10,2})-(\alpha_1 l_{14,3}+\alpha_2 l_{2,3}+\alpha_3 l_{6,3}+\alpha_4 l_{10,3})}{\alpha_0}\alpha \\ +\dfrac{\alpha_1 l_{14,3}+\alpha_2 l_{2,3}+\alpha_3 l_{6,3}+\alpha_4 l_{10,3}}{\alpha_0} \end{Bmatrix}$$

To retain two roots with $\alpha\in[0,1]$, let

$$x_1 = \frac{(\alpha_1 l_{14,2} + \alpha_2 l_{2,2} + \alpha_3 l_{6,2} + \alpha_4 l_{10,2}) - (\alpha_1 l_{14,1} + \alpha_2 l_{2,1}) + \alpha_3 l_{6,1} + \alpha_4 l_{10,2})}{\alpha_0}\alpha + \frac{\alpha_1 l_{14,1} + \alpha_2 l_{2,1} + \alpha_3 l_{6,1} + \alpha_4 l_{10,1}}{\alpha_0},$$

and

$$x_2 = \frac{(\alpha_1 l_{14,2} + \alpha_2 l_{2,2} + \alpha_3 l_{6,2} + \alpha_4 l_{10,2}) - (\alpha_1 l_{14,3} + \alpha_2 l_{2,3} + \alpha_3 l_{6,3} + \alpha_4 l_{10,3})}{\alpha_0}\alpha + \frac{\alpha_1 l_{14,3} + \alpha_2 l_{2,3} + \alpha_3 l_{6,3} + \alpha_4 l_{10,3}}{\alpha_0}$$

Hence

$$\alpha = \frac{\alpha_0 x_1 - (\alpha_1 l_{14,1} + \alpha_2 l_{2,1} + \alpha_3 l_{6,1} + \alpha_4 l_{10,1})}{(\alpha_1 l_{14,2} + \alpha_2 l_{2,2} + \alpha_3 l_{6,2} + \alpha_4 l_{10,2}) - (\alpha_1 l_{14,1} + \alpha_2 l_{2,1}) + \alpha_3 l_{6,1} + \alpha_4 l_{10,2})}$$

and

$$\alpha = \frac{\alpha_0 x_2 - (\alpha_1 l_{14,3} + \alpha_2 l_{2,3} + \alpha_3 l_{6,3} + \alpha_4 l_{10,3})}{(\alpha_1 l_{14,2} + \alpha_2 l_{2,2} + \alpha_3 l_{6,2} + \alpha_4 l_{10,2}) - (\alpha_1 l_{14,3} + \alpha_2 l_{2,3} + \alpha_3 l_{6,3} + \alpha_4 l_{10,3})}$$

$$\mu_{u_5^0} = \begin{cases} \dfrac{\alpha_0 x_1 - (\alpha_1 l_{14,1} + \alpha_2 l_{2,1} + \alpha_3 l_{6,1} + \alpha_4 l_{10,1})}{(\alpha_1 l_{14,2} + \alpha_2 l_{2,2} + \alpha_3 l_{6,2} + \alpha_4 l_{10,2}) - (\alpha_1 l_{14,1} + \alpha_2 l_{2,1} + \alpha_3 l_{6,1} + \alpha_4 l_{10,2})} \\ where \quad \dfrac{1}{\alpha_0}(\alpha_1 l_{14,1} + \alpha_2 l_{2,1} + \alpha_3 l_{6,1} + \alpha_4 l_{10,1}) \le x \le \dfrac{1}{\alpha_0}(\alpha_1 l_{14,2} + \alpha_2 l_{2,2} + \alpha_3 l_{6,2} + \alpha_4 l_{10,2}) \\ \dfrac{\alpha_0 x_2 - (\alpha_1 l_{14,3} + \alpha_2 l_{2,3} + \alpha_3 l_{6,3} + \alpha_4 l_{10,3})}{(\alpha_1 l_{14,2} + \alpha_2 l_{2,2} + \alpha_3 l_{6,2} + \alpha_4 l_{10,2}) - (\alpha_1 l_{14,3} + \alpha_2 l_{2,3} + \alpha_3 l_{6,3} + \alpha_4 l_{10,3})} \\ where \quad \dfrac{1}{\alpha_0}(\alpha_1 l_{14,2} + \alpha_2 l_{2,2} + \alpha_3 l_{6,2} + \alpha_4 l_{10,2}) \le x \le \dfrac{1}{\alpha_0}(\alpha_1 l_{14,3} + \alpha_2 l_{2,3} + \alpha_3 l_{6,3} + \alpha_4 l_{10,3}) \\ 0 \qquad Otherwise \end{cases} \tag{9}$$

Where $\alpha \in [0,1]$.

In similar way we can find out fuzzy membership functions for $u_8^{(0)}, u_2^{(0)}, u_6^{(0)}, u_3^{(0)}, u_4^{(0)}$, $u_1^{(0)}$ and $u_7^{(0)}$ by different mesh for these initial values, then we use standard for an irregular mesh as in fig.1.

Next successive approximations with their f.m.f. as required be obtain from previous approximations and specified boundary conditions.

Numerical example

Let us consider the Laplace equation [8,9],

$$u_{xx}(+)u_{yy}(=)0 \tag{10}$$

In the domain $0 \le x \le 4, 0 \le y \le 4$ with boundary conditions $u(0,y)(=)0$, $u(x,0)(=)\frac{x^2}{2}$ and $u(x,y)(=)2x(+)2y$ in the curved boundary as in fig.1 . Leibmann's process will be applied to solve equation (10).

Solution

The boundary conditions given the numerical value of $c_1 = 0$, $c_2 = 9.745968$, $c_3 = 10.9282$, $c_4 = 11.2915$, $c_5 = 11.2915$, $c_6 = 10.9282$, $c_7 = 9.745968$, $c_8 = 8$, $c_9 = 4.5$, $c_{10} = 2.0$, $c_{11} = 0.5$, $c_{12} = 0$, $c_{13} = 0$, $c_{14} = 0$, and $c_{15} = 0$.

The initial values of $u_i = 1,2,3,...,8$, may be calculated the initial values with the help of five-points formulas to get the approximate solution. Hence interval of confidence for $u_5^{(0)}$ is

$$\left[u_5^{(0)}\right]^{(\alpha)}(=)\left[0.001\alpha + 6.389 \ , \ -0.001\alpha + 6.391\right]$$
(11)

To find f.m.f. and respective interval of confidence these eight c_i's as follows

$$\mu_{C_2}(x)(=)\begin{cases} \dfrac{x-9.744}{9.745-9.746} & ; \ where \ \ 9.744 \le x \le 9.745 \\ \dfrac{x-9.746}{9.45-9.746} & ; \ where \ \ 9.745 \le x \le 9.756 \\ 0 & ; \ otherwise \end{cases} , \quad [c_2]^{(\alpha)}(=)[0.001\alpha + 9.744, -0.001\alpha + 9.746]$$

$$\mu_{C_6}(x)(=)\begin{cases} \dfrac{x-10.927}{10.928-10.927} & ; \ where \ \ 10.927 \le x \le 10.928 \\ \dfrac{x-10.929}{10.928-10.929} & ; \ where \ \ 10.928 \le x \le 10.929 \\ 0 & ; \ otherwise \end{cases} , \quad [c_6]^{(\alpha)}(=)[0.001\alpha + 10.927, -0.001\alpha + 10.929]$$

$$\mu_{C_{10}}(x)(=)\begin{Bmatrix} \frac{x-1.999}{2-1.999} & ; & where \;\; 1.999 \le x \le 2 \\ \frac{x-2.001}{2-2.001} & ; & where \;\; 2 \le x \le 2.001 \\ 0 & ; & otherwise \end{Bmatrix}, \quad [c_{10}]^{(\alpha)}(=)[0.001\alpha+1.999, -0.001\alpha+2.001]$$

$$\mu_{C_{14}}(x)(=)\begin{Bmatrix} \frac{x+0.001}{0+0.001} & ; & where \;\; -0.001 \le x \le 0 \\ \frac{x-0.001}{0-0.001} & ; & where \;\; 0 \le x \le 0.001 \\ 0 & ; & otherwise \end{Bmatrix}, \quad [c_{14}]^{(\alpha)}(=)[0.001\alpha-0.001, -0.001\alpha+0.001]$$

Now the parameter α_1, α_2, α_3, α_4, and α_0 , for each points u_1, u_2, u_3, u_4, u_5, u_6, u_7, and u_8, can be find as in equation (2), in the following h *and* $\alpha - table$

	h (for the initial guess)				α (for the initial guess)				
	h_1	h_2	h_3	h_4	α_1	α_2	α_3	α_4	α_0
u_5	2.0	1.4641 02	1.4641 02	2.0	0.2886 75	0.39433 7	0.3943 37	0.2886 75	-1.36602
u_8	2.0	0.4641 02	0.6457 51	1.0	0.3779 65	2.94337	1.1706 2	1.3660 3	-5.85798
u_2	2.0	1.0	1.8729 84	1.0	0.2581 99	1.0	0.2757 09	1.0	-2.53391
u_6	2.0	0.6457 51	0.4641 02	2.0	1.3660 3	1.17062	2.9433 7	0.3779 65	-5.85798
u_3	1.0	1.0	0.8729 84	1.0	1.0678 1	1.0	1.2237 8	1.0	-4.29099
u_4	1.0	2.0	1.0	2.0	1.0	0.27570 9	1.0	0.2581 99	-2.53391
u_7	1.0	0.8729 84	1.0	1.0	1.0	1.22318	1.0	1.0678 1	-4.29099
u_1	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	-4.0

The following process to find the initial values of the point u_5^0 depending on the above tables with the equation (3)

$$u_5^{(0)}(=)\frac{1}{\alpha_0}\left[(\alpha_1)c_{14}(+)(\alpha_2)c_3(+)(\alpha_3)c_6(+)(\alpha_4)c_{10}\right]$$
$$(=)\{\beta_1 c_{14}(+)\beta_2 c_3(+)\beta_3 c_6(+)\beta_4 c_{10}\}, \quad where \quad \beta_i=\frac{\alpha_i}{\alpha_0}, \;\; i=1,2,3,4. \tag{12}$$

Hence we the interval of confidence for $u_5^{(0)}$, $\left[u_5^{(0)}\right]^{(\alpha)}(=)\left[0.001\alpha+6.389 \;,\; -0.001\alpha+6.391\right]$, we are to retain two roots with $\alpha\in[0,1]$. Let $0.001\alpha+6.389=x_1$ and $-0.001\alpha+6.391=x_2$, then solving for α we get

$\alpha=\frac{x_1-6.389}{0.001}$ and $\alpha=\frac{x_2-6.391}{-0.001}$, so f.m.f. for $u_5^{(0)}$ is

$$\mu_{u_5^{(0)}}(x)(=)\left\{\begin{array}{l} \frac{x-6.389}{6.390-6.389} \;\; ; \;\; where \;\; 6.389\le x\le 6.390 \\ \frac{x-6.391}{6.390-6.391} \;\; ; \;\; where \;\; 6.390\le x\le 6.391 \\ 0 \quad ; \quad otherwise \end{array}\right\}$$

In the same way we find f.m.f. for $u_8^{(0)}, u_2^{(0)}, u_6^{(0)}, u_3^{(0)}, u_4^{(0)}, u_7^{(0)}$,and $u_1^{(0)}$ are respectively.

$$\mu_{u_8^{(0)}}(x)(=)\left\{\begin{array}{l} \frac{x-9.10964}{9.11064-9.10964} \;\; ; \;\; where \;\; 9.10964\le x\le 9.11964 \\ \frac{x-9.11164}{9.11064-9.11164} \;\; ; \;\; where \;\; 9.11964\le x\le 9.11164 \\ 0 \quad ; \quad otherwise \end{array}\right\},$$

$$\mu_{u_2^{(0)}}(x)(=)\left\{\begin{array}{l} \frac{x-4.50552}{4.50652-4.50552} \;\; ; \;\; where \;\; 4.50552\le x\le 4.50652 \\ \frac{x-4.50752}{4.50652-4.50752} \;\; ; \;\; where \;\; 4.50652\le x\le 4.50752 \\ 0 \quad ; \quad otherwise \end{array}\right\},$$

$$\mu_{u_6^{(0)}}(x)(=)\left\{\begin{array}{l} \frac{x-9.60676}{9.60776-9.60676} \;\; ; \;\; where \;\; 9.60676\le x\le 9.60776 \\ \frac{x-9.60876}{9.60776-9.60876} \;\; ; \;\; where \;\; 9.60776\le x\le 9.60876 \\ 0 \quad ; \quad otherwise \end{array}\right\},$$

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$$\mu_{u_3^{(0)}}(x)(=)\begin{cases} \dfrac{x-7.18637}{7.18737-7.18637} & ; \; where \quad 5.049 \le x \le 7.18737 \\ \dfrac{x-7.18837}{7.18737-7.18837} & ; \; where \quad 7.18737 \le x \le 7.18837 \\ 0 \quad ; \quad otherwise & \end{cases},$$

$$\mu_{u_4^{(0)}}(x)(=)\begin{cases} \dfrac{x-3.71642}{3.71742-3.71642} & ; \; where \quad 3.71642 \le x \le 3.71742 \\ \dfrac{x-3.71842}{3.71742-3.71842} & ; \; where \quad 3.71742 \le x \le 3.71842 \\ 0 \quad ; \quad otherwise & \end{cases},$$

$$\mu_{u_7^{(0)}}(x)(=)\begin{cases} \dfrac{x-5.82544}{5.82644-5.82544} & ; \; where \quad 5.82544 \le x \le 5.82644 \\ \dfrac{x-5.82744}{5.82644-5.82744} & ; \; where \quad 5.82644 \le x \le 5.82744 \\ 0 \quad ; \quad otherwise & \end{cases},\text{and}$$

$$\mu_{u_1^{(0)}}(x)(=)\begin{cases} \dfrac{x-2.17999}{2.18099-2.17999} & ; \; where \quad 2.17999 \le x \le 2.18099 \\ \dfrac{x-2.18199}{2.18099-2.18199} & ; \; where \quad 2.18099 \le x \le 2.18199 \\ 0 \quad ; \quad otherwise & \end{cases}.$$

In the following there are f.m.f of the first and forth approximations respectively using five-points by the method of Lebmann's iteration process applied to equation (4) with aid of the following tables

$$\mu_{u_1^{(1)}}(x)(=)\begin{cases} \dfrac{x-2.1799834420}{2.1809834420-2.1799834420} & ; where \quad 2.1799834420 \le x \le 2.1809834420 \\ \dfrac{x-2.1819834420}{2.1809834420-2.1819834420} & ; where \quad 2.1809834420 \le x \le 2.1819834420 \\ 0 \quad ; \quad otherwise & \end{cases},$$

$$\mu_{u_2^{(1)}}(x)(=)\begin{cases} \dfrac{x-4.5240998197}{4.5250998197-4.5240998197} & ; \; where \quad 4.5240998197 \le x \le 4.5250998197 \\ \dfrac{x-4.5260998197}{4.5250998197-4.5260998197} & ; \; where \quad 4.5250998197 \le x \le 4.5260998197 \\ 0 \quad ; \quad otherwise & \end{cases},$$

$$\mu_{u_3^{(1)}}(x)(=)\begin{cases} \dfrac{x-7.1909920574}{7.1919920574-7.1909920574} & ; \ where \ \ 7.1909920574 \le x \le 7.1919920574 \\ \dfrac{x-7.192992074}{7.1919920574-7.192992074} & ; \ where \ \ 7.1919920574 \le x \le 7.192992074 \\ 0 \quad ; \quad otherwise & \end{cases},$$

$$\mu_{u_4^{(1)}}(x)(=)\begin{cases} \dfrac{x-3.6838677914}{3.6848677914-3.6838677914} & ; where \ \ 3.6838677914 \le x \le 3.6848677914 \\ \dfrac{x-3.6858677914}{3.6848677914-3.6858677914} & ; where \ \ 3.6848677914 \le x \le 3.6858677914 \\ 0 \quad ; \quad otherwise & \end{cases},$$

$$\mu_{u_5^{(1)}}(x)(=)\begin{cases} \dfrac{x-6.7825396356}{6.7835396356-6.7825396356} & ; where \ \ 6.7825396356 \le x \le 6.7835396356 \\ \dfrac{x-6.7845396356}{6.7835396356-6.7845396356} & ; where \ \ 6.7835396356 \le x \le 6.7845396356 \\ 0 \quad ; \quad otherwise & \end{cases},$$

$$\mu_{u_6^{(1)}}(x)(=)\begin{cases} \dfrac{x-9.6420649696}{9.6430649696-9.6420649696} & ; where \ \ 9.6420649696 \le x \le 9.6430649696 \\ \dfrac{x-9.6440649696}{9.6430649696-9.6440649696} & ; where \ \ 9.6430649696 \le x \le 9.6440649696 \\ 0 \quad ; \quad otherwise & \end{cases},$$

$$\mu_{u_7^{(1)}}(x)(=)\begin{cases} \dfrac{x-5.8652966744}{5.8662966744-5.8652966744} & ; where \ \ 5.8652966744 \le x \le 5.8662966744 \\ \dfrac{x-5.8672966744}{5.8662966744-5.8672966744} & ; where \ \ 5.8662966744 \le x \le 5.8672966744 \\ 0 \quad ; \quad otherwise & \end{cases}, \text{ and}$$

$$\mu_{u_8^{(1)}}(x)(=)\begin{cases} \dfrac{x-9.4245482308}{9.4255482308-9.4245482308} & ; \ where \ \ 9.4245482308 \le x \le 9.4255482308 \\ \dfrac{x-9.4265482308}{9.4255482308-9.4265482308} & ; \ where \ \ 9.4255482308 \le x \le 9.4265482308 \\ 0 \quad ; \quad otherwise & \end{cases}.$$

and the f.m.f of the fourth iterations as follows

$$\mu_{u_1^{(4)}}(x)(=)\left\{\begin{array}{ll} \dfrac{x-2.1946300042}{2.195630042-2.1946300042} & ; \ where \ \ 2.1946300042 \le x \le 2.1956300042 \\ \dfrac{x-2.1966300042}{2.1956300042-2.1966300042} & ; \ where \ \ 2.1956300042 \le x \le 2.1966300042 \\ 0 \quad ; \quad otherwise & \end{array}\right\},$$

$$\mu_{u_2^{(4)}}(x)(=)\left\{\begin{array}{ll} \dfrac{x-4.5613018816}{4.5623018816-4.5613018816} & ; \ where \ \ 4.5613018816 \le x \le 4.5623018816 \\ \dfrac{x-4.5633018816}{4.5623018816-4.5633018816} & ; \ where \ \ 4.5623018816 \le x \le 4.5633018816 \\ 0 \quad ; \quad otherwise & \end{array}\right\},$$

$$\mu_{u_3^{(4)}}(x)(=)\left\{\begin{array}{ll} \dfrac{x-7.2117460739}{7.2127460739-7.2117460739} & ; \ where \ \ 7.2117460739 \le x \le 7.2127460739 \\ \dfrac{x-7.2137460739}{7.2127460739-7.2137460739} & ; \ where \ \ 7.2127460739 \le x \le 7.2137460739 \\ 0 \quad ; \quad otherwise & \end{array}\right\},$$

$$\mu_{u_4^{(4)}}(x)(=)\left\{\begin{array}{ll} \dfrac{x-3.7352139746}{3.7362139746-3.7352139746} & ; \ where \ \ 3.7352139746 \le x \le 3.7362139746 \\ \dfrac{x-3.7372139746}{3.7362139746-3.7372139746} & ; \ where \ \ 3.7362139746 \le x \le 3.7372139746 \\ 0 \quad ; \quad otherwise & \end{array}\right\},$$

$$\mu_{u_5^{(4)}}(x)(=)\left\{\begin{array}{ll} \dfrac{x-6.8486724996}{6.8496724996-6.8486724996} & ; \ where \ \ 6.8486724996 \le x \le 6.8496724996 \\ \dfrac{x-6.8506724996}{6.8496724996-6.8506724996} & ; \ where \ \ 6.8496724996 \le x \le 6.8506724996 \\ 0 \quad ; \quad otherwise & \end{array}\right\},$$

$$\mu_{u_6^{(4)}}(x)(=)\left\{\begin{array}{ll} \dfrac{x-9.6576674056}{9.6586674056-9.6576674056} & ; \ where \ \ 9.6576674056 \le x \le 9.6586674056 \\ \dfrac{x-9.6596674056}{9.6586674056-9.6596674056} & ; \ where \ \ 9.6586674056 \le x \le 9.6596674056 \\ 0 \quad ; \quad otherwise & \end{array}\right\},$$

$$\mu_{u_7^{(4)}}(x)(=)\begin{cases} \dfrac{x-5.9075901512}{5.9085901512-5.9075901512} & ; \ where \ \ 5.9075901512 \le x \le 5.9085901512 \\ \dfrac{x-5.9095901512}{5.9085901512-5.9095901512} & ; \ where \ \ 5.9085901512 \le x \le 5.9095901512 \\ 0 \quad ; \quad otherwise & \end{cases} \text{ and}$$

$$\mu_{u_8^{(4)}}(x)(=)\begin{cases} \dfrac{x-9.4436848069}{9.4446848069-9.4436848069} & ; \ where \ \ 9.4436848069 \le x \le 9.4446848069 \\ \dfrac{x-9.4456848069}{9.4446848069-9.4456848069} & ; \ where \ \ 9.4446848069 \le x \le 9.4456848069 \\ 0 \quad ; \quad otherwise & \end{cases} .$$

CONCLUSION

For the given initial values the fourth approximations to solve the above example numerically is significant results in the domain has curved border. Using five points only, however may increased the accuracy as desired if we take more iterations.

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