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NONPARAMETRIC TESTS FOR DETECTING MORE HNBUE-NESS OF SPECIFIC AGE

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ABSTRACT:-*In this paper, we proposed a new nonparametric test for discussing the problem of testing that two life distributions F and G are identical against the alternative that F is more harmonic new better than used in expectation (HNBUE) than G. The asymptotic normality of the proposed test is established and the asymptotic null variance is estimated from the data. Pitman asymptotic relative efficiencies (PARE) of proposed test are calculated for different distributions. Another test proposed for testing that two life distributions F and G are identical against the alternative that F is more HNBUE than G at a specific age* t_0. *The asymptotic normality of proposed test is established and the asymptotic null variance is estimated. Pitman asymptotic efficiencies (PAE) of proposed test are calculated for different distributions. A numerical example is presented for illustrative our test purpose.*

KEYWORDS: HNBUE, HNBUE-t_0,Asymptotic normal, Pitman asymptotic efficiency, U-statistic.

INTRODUCTION

Many classes of life distributions have been introduced in reliability and survival analysis. Applications of these classes can be seen in engineering and biological sciences. These classes of life distribution were discussed by many statisticians. For example Barlow and Proschan (1981), Deshpand *et al.* (1986) and Cao and Wang (1991), gave definitions of several classes of distributions, *NBU*, *NBU* (2) and *NBUE*. Rolisk (1975) introduced the HNBUE. Klefsjo (1982) and Hendi et al (1998) introduced testing versus HNBUE. In this paper, we interested in class HNBUE which defined as

Definition 1:-

The distribution F is said to be HNBUE if

$$\int_0^\infty \overline{F}(x+t)dx \le \mu e^{-\frac{t}{\mu}} \qquad \forall x,t \ge 0.$$

And a new class HNBUE-ness at specific age t_0 which introduced by Mahmoud et al. (2013) and defined as follows

Definition 1

The distribution *F* is said to be HNBUE-ness of age t_0 if

$$\int_0^\infty \overline{F}(x+t_0)dx \le \mu e^{-\frac{t_0}{\mu}} \qquad \forall x,t_0 \ge 0.$$

Mahmoud et al. (2013) introduced a test that *F* is exponential against it is HNBUE-ness of age t_0 and not exponential but in this paper, we develop this test for testing that two distributions *F* and *G* are identical against *F* is more HNBUE than *G* in section 2. The new proposed test is present for testing distributions *F* and *G* are identical against *F* is more HNBUE-ness of age t_0 than *G* in section 3. In section 4, The Pitman asymptotic relative efficiency (PARE) of first proposed test is calculate for different distributions in comparison with the test procedure due to Pandit and Gudaganavar (2009), also The Pitman asymptotic efficiency (PAE) of second proposed test is calculate for different distributions. In section 5, presents an example to illustrate the purpose of the new proposed test for testing distributions *F* and *G* are identical against *F* is more HNBUE-ness of age t_0 than *G*.

Two sample test for more HNBUE:

Let $X_1, X_2, ..., X_m$ be independent, identically distributed random variables according to a continuous life distributions *F* and $Y_1, Y_2, ..., Y_n$ be independent, identically distributed random variables according to a continuous life distributions *G*. Our goal in this section is proposed a test statistic for testing

$H_0 : F = G$ (The common distribution is unspecified), while the alternative hypothesis is

$H_1 : F$ is more HNBUE than G.

The following is the measure of departure from H_0

$$\Delta(F,G) = \Delta(F) - \Delta(G) \qquad (1)$$

Where;

$$\Delta(F) = \mu_1 - \int_0^\infty \int_0^\infty \overline{F}(x+y)dxdy \;\;, \;\mu_1 = E[X] \text{ and}$$

$$\Delta(G) = \mu_2 - \int_0^\infty \int_0^\infty \overline{G}(x+y)dxdy \;, \;\mu_2 = E[Y] \;.$$

The estimator of $\Delta(F,G)$ is obtained by using U-statistic which defined as

$$\eta_{m,n} = \eta_m - \eta_n \qquad (2)$$

Where;

$$\eta_m = \frac{1}{\overline{X}^2}(\overline{X} - \frac{1}{m(m-1)}\sum\sum_{i \neq j} X_i X_j)$$

and

$$\eta_n = \frac{1}{\overline{Y}^2}(\overline{Y} - \frac{1}{n(n-1)}\sum\sum_{i \neq j} Y_i Y_j)$$

Here $\overline{X}$ and $\overline{Y}$ the sample mean of X-sample and Y-sample respectively.

Now, we establish the asymptotic normality of $\eta_{m,n}$ in the following theorem

Theorem 1:

The asymptotic distribution of $\sqrt{N}[\eta_{m,n} - \Delta(F,G)]$ is normal with mean zero and variance given by $\sigma^2(\eta_{m,n}) = \frac{N}{m}\sigma_1^2 + \frac{N}{n}\sigma_2^2$

Where: $N=m+n$

$$\sigma_1^2 = Var(2\mu_1 - X_1^2 - \mu_1^{(2)}) \text{ and } \sigma_2^2 = Var(2\mu_2 - Y_1^2 - \mu_2^{(2)})$$

Where: $\mu_1 = E[X_1]$, $\mu_1^{(2)} = E[X_1^2]$

and $\mu_2 = E[Y_1]$, $\mu_2^{(2)} = E[Y_1^2]$

Proof: proof follows from Hoeffding (1948)

Hence, approximate α-level test rejects H_0 in favor H_1 if $\sqrt{N}\frac{\eta_{m,n}}{\sqrt{\hat{\sigma}^2(\eta_{m,n})}} > Z_\alpha$ where Z_α is the upper α-percentile point of standard normal distribution, and $\hat{\sigma}^2(\eta_{m,n})$ is a consistent estimator for $\sigma^2(\eta_{m,n})$ is obtained by replacing the empirical survival function $\overline{F}_m(x)$ by $\overline{F}(x)$, and the empirical survival function $\overline{G}_n(y)$ by $\overline{G}(y)$.

Two-Sample more HNBUE-t_0 Test.

Let $X_1, X_2, ..., X_m$ be independent, identically distributed random variables according to a continuous life distributions F and $Y_1, Y_2, ..., Y_n$ be independent, identically distributed random variables according to a continuous life distributions G. Our goal in this section is proposed a test statistic for testing

$H_0 : F = G$ (The common distribution is unspecified), while the alternative hypothesis is

$H_1 : F$ is more HNBUE-t_0 than G.

Our test statistic is motivated by considering the concept of harmonic new better than used in expectation of specified age t_0.

The following is the measure of departure from H_0

$$\xi_{t_0}(F, G) = \xi_{t_0}(F) - \xi_{t_0}(G) \qquad (3)$$

Where:

$$\xi_{t_0}(F) = \mu_1 e^{-t_0} - \int_0^\infty \overline{F}(x+t_0)dx, \quad \mu_1 = E[X_1]$$

and

$$\xi_{t_0}(G) = \mu_2 e^{-t_0} - \int_0^\infty \overline{G}(y+t_0)dy, \quad \mu_2 = E[Y_1].$$

Mahmoud et al. (2013) used $\xi_{t_0}(F)$ as a measure of departure to test F is exponential versus F belongs to *HNBUE*-t_0. The estimator of $\xi_{t_0}(F,G)$ is obtained by using U-statistic which defined as

$$U_{t_0}^{m,n} = U_{t_0}^m - U_{t_0}^n \qquad (4)$$

Where:

$$U_{t_0}^m = \frac{(e^{-t_0}-1)}{m}\sum_{i=1}^m (X_i - 1) \text{ and}$$

$$U_{t_0}^n = \frac{(e^{-t_0}-1)}{n}\sum_{i=1}^n (Y_i - 1)$$

Now, we establish the asymptotic normality of $U_{t_0}^{m,n}$ in the following theorem

Theorem 2:

The asymptotic distribution of $\sqrt{N}[U_{t_0}^{m,n} - \xi_{t_0}(F,G)]$ is normal with mean zero and variance given by $\sigma^2(U_{t_0}^{m,n}) = \frac{N}{m}\sigma^2(U_{t_0}^m) + \frac{N}{n}\sigma^2(U_{t_0}^n)$

Where: $N=m+n$

$$\sigma^2(U_{t_0}^m) = Var[(e^{t_0}-1)(X-1)]$$

and

$$\sigma^2(U_{t_0}^n) = Var[(e^{t_0}-1)(Y-1)]$$

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Proof: proof follows from Hoeffding (1948).

Hence, approximate α -level test rejects H_0 in favor H_1 if $\sqrt{N}\frac{U_{t_0}^{m,n}}{\sqrt{\sigma^2(U_{t_0}^{m,n})}} > Z_\alpha$ where Z_α is the upper α -percentile point of standard normal distribution

The Pitman Asymptotic Efficiency (PAE):

In order to compare these new proposed tests with other we used the concept of Pitman asymptotic efficiency.

(i) The Pitman asymptotic efficiency of $\eta_{m,n}$:

We compute the asymptotic efficiency of the new proposed two sample HNBUE test for two pairs of distribution $(F_{i\theta}, G)$. For this purpose we assume that G is an exponential distribution with mean one. We consider $F_{1\theta}$ as linear failure rate distribution and $F_{2\theta}$ as Weibull distribution.

The Pitman asymptotic efficiency is defined as:

$$\text{PAE}\,[\eta_{m,n}] = [\sigma_0^{-1}\frac{d\Delta(F_{i\theta}, G)}{d\theta}]^2_{\theta\to\theta_{0i}} \qquad i = 1,2.$$

Where $\theta_{01} = 0$, $\theta_{02} = 1$ and the asymptotic variance is obtained as $\sigma_0^2 = \frac{20}{\lambda(1-\lambda)}$, where $\lambda = \lim_{N\to\infty}\frac{m}{N}$.

Direct calculations of PAE [$\eta_{m,n}$**]** yield the following results: maximum efficiencies of $\eta_{m,n}$ for the above two distributions are 0.05 and 0.0252 respectively. Table (1) shows the asymptotic relative efficiency (ARE) of $\eta_{m,n}$ relative to statistic $U_{m,n}$ due to Pandit and Gudaganavar (2009).

Table (1)

Asymptotic Relative Efficiency (ARE) of $\eta_{m,n}$ W. R. T $U_{m,n}$

Linear Failure rate Distribution	Weibull Distribution
2.3702	1.119

It is evidence that the new proposed test is better than that the test statistic proposed by Pandit and Gudaganavar (2009).

(ii) The Pitman asymptotic efficiency of $U_{t_0}^{m,n}$:

We compute the asymptotic efficiency of the new proposed two sample HNBUE -t_0 test for two pairs of distribution ($F_{i\theta}, G$). For this purpose we assume that G is an exponential distribution with mean one. We consider $F_{1\theta}$ as linear failure rate distribution and $F_{2\theta}$ as

Makeham distribution.

The Pitman asymptotic efficiency is defined as:

$$\text{PAE } [U_{t_0}^{m,n}] = [\sigma_0^{-1} \frac{d\xi_{t_0}(F_{i\theta}, G)}{d\theta}]^2_{\theta \to \theta_{0i}} \qquad i = 1,2.$$

Where $\theta_{01} = \theta_{02} = 0$, and the asymptotic variance is obtained as $\sigma_0^2 = \dfrac{(e^{-t_0} - 1)^2}{\lambda(1-\lambda)}$, where $\lambda = \lim_{N \to \infty} \dfrac{m}{N}$.

Direct calculations of PAE $[U_{t_0}^{m,n}]$ yield the following results: efficiencies of $U_{t_0}^{m,n}$ for the above two distributions

$$\text{PAE [Linear Failure rate Distribution } (F_{1\theta})] = \frac{1}{\sigma_0^2}[e^{-t_0}(\frac{t_0^2}{2} + t_0)]^2$$

and

$$\text{PAE [Makeham Distribution } (F_{2\theta})] = \frac{1}{\sigma_0^2}[e^{-t_0}(2t_0^2 + e^{-t_0} - 1)]^2$$

Table (2) shows the values of maximum efficiencies of the two- sample HNBUE - t_0 for various choices t_0. Values of maximum efficiencies calculated for both functions $F_{1\theta}$ and $F_{2\theta}$. Table (2) shows that, the efficiency decreasing in t_0 as follows:

Table (2)

Maximum Efficiencies of $U_{t_0}^{m,n}$

Distribution	t_0				
	0.1	0.3	0.5	0.7	0.9
$F_{1\theta}$	0.249	0.234	0.232	0.217	0.200
$F_{2\theta}$	16.423	0.665	0.055	0.001	0.004

5- Example:

We use a subset of data considered by Proschan (1963) to illustrate our two-sample HNBUE- t_0 test. Table 3 shows the data which considered by Proschan (1963). He investigated the life distributions of the air-conditioning systems of a fleet of Boeing 720 jet airplanes. The table presents life lengths (in hours) of the air-conditioning system of planes 7909 and 8045 arranged in decreasing order. For our purpose we assume that X, s (the air-conditioning system of planes 7909) and Y, s (the air-conditioning system of planes 8045) are two random samples from two populations with distributions functions F and G, respectively. To apply our two-sample HNBUE- t_0 test

1- Compute $U_{t_0}^{29} = 27.468$, $U_{t_0}^{16} = 26.963$ and $U_{t_0}^{m,n} = 0.505$ with a corresponding

P- Value=0.377.

2- Compute the test statistic $\sqrt{N}\,\dfrac{U_{t_0}^{m,n}}{\sqrt{\sigma^2(U_{t_0}^{m,n})}} = 0.32$.

3- The decision: since the value of the test statistic is less than P –value, then the test is supports the null hypothesis that the two underlying distribution are equal, agreeing with Hollander et al (1986b) and Lim et al (2005) tests.

Table (3)

Life length of the air-conditioning systems of planes (7909) and (8045) arranged in decreasing order

X(7909)			**Y(8045)**	
310	79	44	230	54
208	76	29	209	34
208	70	26	152	32
186	62	25	134	27
156	61	24	102	14
130	60	23	67	14
118	59	20	66	
101	56	14	61	
90	49	10	59	
84	44		57	
m=29			**n=16**	

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