International Journal of Mathematics and Statistics Studies
Vol.10, No.2, pp.52-59, 2022
Print ISSN: 2053-2229 (Print),
Online ISSN: 2053-2210 (Online)

Newton–Cotes Formulas for Numerical Integration in Maple

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ABSTRACT: *In this paper, we started with the simplest method of numerical integration based on interpolation using the Maple package. We demonstrate some of the computational capabilities of the Maple package using the Newton -Coates formula for numerical integration. Newton - Coates formula is a very useful and simple family of generalized integral methods. The Trapezoidal method and the Simpson method can be accurately represented by the Maple package. For numerical integration and geometric analysis, the Maple is the best option. Considering a function f defined in the interval [a, b], we can draw an interpolation polynomial at some point f (x) with the Maple packet. Since it is simple to evaluate the definite integral of a polynomial, this calculation can be used to approximate the integral of* $f(x)$*. This is the Newton - Cotes approach to approximating integrals.*

KEY WORDS: Maple, Newton-Coates formula, Trapezoid, Simpson's Rule, Simpson's 3/8 Rule.

INTRODUCTION

In this study, we present an advanced family of closed Newton–Cotes numerical composite integration formulas that only employ function values on regularly spaced intervals and do not use derivative values. Computing software has advanced a lot in recent decades. Today, computing software is used not only in specialized work, but also in educational affairs, academic research and textbook writing. Founded in the early 1980s at the University of Waterloo, Canada, Maple software, in newer versions, offers many computational problems with accurate answers. This software is easy to learn because the same math symbols used in classrooms can be used to enter data. Numerical analysis is one of the most important methods that is very important for performance in many algorithms. Modern numerical analysis tries to understand the data in a shorter or more concise way. Any approximate method must converge to the correct answer [7]. Trapezoid and Simpson rules are limited to performance in a single time period. of course, since definite integrals are additive over subintervals, we can evaluate an integral by dividing the interval up into several subintervals, applying the rule separately on each one, and then totaling up. This method is called composite numerical integration strategy [5, 8]. The composite Trapezoid rule is simply the sum of Trapezoid rule approximations on adjacent subintervals, or panels. Today, using the latest applications of numerical analysis, they try to calculate the data in a shorter and more efficient way. The Maple package provides numerical methods that lead to the solution of important scientific and technical problems. Maple package also has features for input and output of image and output files. We analyzed the Newton-Coates formula in the Maple package

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considering other methods. Integration is the technique of calculating the area plotted on a graph using a function [2].

$$I = \int_a^b f(x)\, dx$$

MATERIALS AND METHODS

Because of these tasks, interpolation-based numerical integration using the following methods is accurately demonstrated by Maple.

A. Newton-Coates formula

We study Newton-Coates formulas using some Maple computational capabilities with respect to the approximate function at distances [a, b] for the integral f (x) [2].

Examples

> $\int_{1.1}^{8.0} \ln(x)\, dx$

9.630691136

> *with*(*Student*[*Calculus1*]) :
> *ApproximateInt*(ln(*x*), *x* = 1.1 ..8.0, *method* = *newtoncotes*[1]);

9.600011676

> *ApproximateInt*(ln(*x*), *x* = 1.1 ..8.0, *method* = *newtoncotes*[2]);

9.630586303

> *ApproximateInt*(ln(*x*), *x* = 1.1 ..8.0, *method* = *newtoncotes*[3]);

9.630643941

> *ApproximateInt*(ln(*x*), *x* = 1.1 ..8.0, *method* = *newtoncotes*[4]);

9.630690519

> *ApproximateInt*(ln(*x*), *x* = 1.1 ..8.0, *method* = *newtoncotes*[6]);

9.630691129

A. TRAPEZOIDAL METHODS

The trapezoidal rule is a Newton-Cotes formula for approximating the integral of a function f using linear segments. The trapezoidal rule is Newton Coates' first integration formula [5]. Trapezoidal method is a rule that evaluates curves by dividing the total area into smaller trapezoids instead of using a rectangle [6]. Let f be tabulated at points x_0 and x_1 spaced by a distance h, and write $f_n = f(x_n)$. then the trapezoidal rule states that $\int_{x_0}^{x_1} f(x)\, dx \approx h(f_0 + f_1)/2$.

Examples

> $polynomial := CurveFitting[PolynomialInterpolation]\left([x_0, x_1], [f(0), f(1)], z\right) :$

> $integrated := \int_{x_0}^{x_1} polynomial \, dz :$

> $factor(integrated)$

$$-\frac{1}{2}\left(x_0 - x_1\right)\left(f(1) + f(0)\right)$$

> $with(Student[Calculus1]) :$

> $ApproximateInt(\sin(x), x = 3..5, method = trapezoid)$

$$\frac{1}{10}\sin(3) + \frac{1}{5}\sin\left(\frac{16}{5}\right) + \frac{1}{5}\sin\left(\frac{17}{5}\right) + \frac{1}{5}\sin\left(\frac{18}{5}\right) + \frac{1}{5}\sin\left(\frac{19}{5}\right) + \frac{1}{5}\sin(4) + \frac{1}{5}\sin\left(\frac{21}{5}\right) + \frac{1}{5}\sin\left(\frac{22}{5}\right) + \frac{1}{5}\sin\left(\frac{23}{5}\right) + \frac{1}{5}\sin\left(\frac{24}{5}\right) + \frac{1}{10}\sin(5)$$

> $ApproximateInt(\cos(x), 1..100, method = trapezoid, output = animation)$

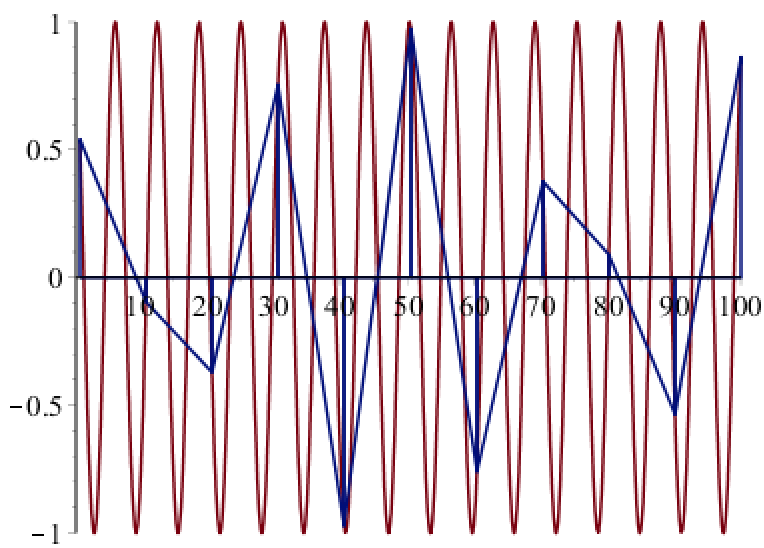

Figure 1 An animated approximation of $\int_1^{100} f(x)\,dx$ using trapezoid rule, where $f(x) = \cos(x)$ and the partition is uniform. The approximate value of the integral is 1.615815314. Number of subintervals used: 10.

THE GENERAL FORMULA OF SIMPSON'S METHODS

Simpson's rule is a numerically accurate method of approximating a definite integral using a thre e point quadrature obtained by integrating the unique quadratic that passes through these points.

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For some c in the interval $[x_0, x_1]$, provided that $f(iv)$ exists and is continuous. Concluding the derivation yields Simpson's rule [6, 3]:

$$\int_{x_0}^{x_1} f(x)dx = \frac{h}{3}(y_0 + 4y_1 + y_2) - \frac{h^5}{90} f^{(iv)}(c).$$

Where $h = x_2 - x_1 = x_2 - x_0$ and c is between x_0 and x_2.

Examples

> $polynomial := CurveFitting[PolynomialInterpolation]\left(\left[x_0, \frac{x_0 + x_1}{2}, x_1\right], \left[f(0), f\left(\frac{1}{2}\right), f(1)\right], z\right)$

> $integrated := \int_{x_0}^{x_1} polynomial\, dz :$

> $factor(integrated)$

$$-\frac{1}{6}(x_0 - x_1)\left(f(1) + 4f\left(\frac{1}{2}\right) + f(0)\right)$$

> $with(Student[Calculus1]) :$

> $ApproximateInt(\sin(x), x = 2..-3, method = simpson)$

$$-\frac{1}{6}\sin\left(\frac{5}{2}\right) - \frac{1}{3}\sin\left(\frac{11}{4}\right) - \frac{1}{12}\sin(3) - \frac{1}{12}\sin(2) - \frac{1}{3}\sin\left(\frac{9}{4}\right)$$

> $ApproximateInt(\cos(x), 1..100, method = simpson, output = animation)$

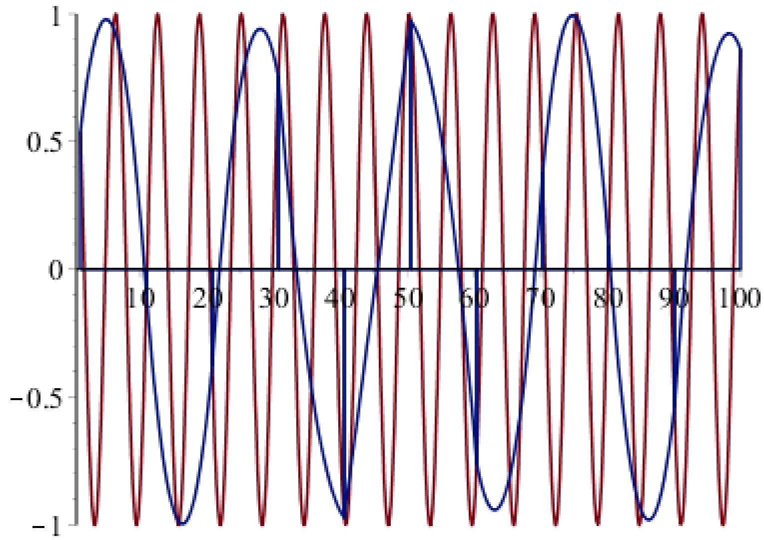

Figure 2 An animated approximation of $\int_{1}^{100} f(x)\,dx$ using Simpson's rule, where $f(x) = \cos(x)$ and the partition is uniform. The approximate value of the integral is 5.115050025. Number of subintervals used: 10.

THE GENERAL FORMULA OF SIMPSON'S 3/8 METHODS

Let the values of a function $f(x)$ be tabulated at points x_i equally spaced by $h = x_{i+1} - x_i$, so $f_1 = f(x_1)$, $f_2 = f(x_2)$, ..., $f_4 = f(x_4)$. Then Simpson's 3/8 rule approximating the integral of $f(x)$ is given by the Newton-Cotes like formula [7],

$$\int_{x_1}^{x_4} f(x)dx = \frac{3}{8}h(f_1 + 3f_3 + f_4) - \frac{3}{8}h^5(\xi).$$

This method is completely based on cubic interpolation [8]. With this method, we demonstrated some computational capabilities of the Maple package. [4].

Examples

> $polynomial := CurveFitting[PolynomialInterpolation]\left(\left[x_0, \frac{2x_0 + x_1}{3}, \frac{x_0 + 2x_1}{3}, x_1\right], \left[f(0), f\left(\frac{1}{3}\right), f\left(\frac{2}{3}\right), f(1)\right], z\right):$

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> $integrated := \int_{x_0}^{x_1} polynomial \, dz$:

> $factor(integrated)$

$$-\frac{1}{8}\left(x_0 - x_1\right)\left(f(1) + 3f\left(\frac{2}{3}\right) + 3f\left(\frac{1}{3}\right) + f(0)\right)$$

> $with(Student[Calculus1])$:

> $ApproximateInt\left(\sin(x), x = 1..6, method = simpson_{\frac{3}{8}}\right)$

$$\frac{1}{8}\sin\left(\frac{11}{2}\right) + \frac{3}{16}\sin\left(\frac{17}{3}\right) + \frac{3}{16}\sin\left(\frac{35}{6}\right) + \frac{1}{16}\sin(6) + \frac{3}{16}\sin\left(\frac{31}{6}\right) + \frac{3}{16}\sin\left(\frac{16}{3}\right) + \frac{1}{8}\sin(5) + \frac{3}{16}\sin\left(\frac{23}{6}\right) + \frac{1}{8}\sin(4) + \frac{3}{16}\sin\left(\frac{25}{6}\right) + \frac{3}{16}\sin\left(\frac{13}{3}\right) + \frac{1}{8}\sin\left(\frac{9}{2}\right) + \frac{3}{16}\sin\left(\frac{14}{3}\right) + \frac{3}{16}\sin\left(\frac{29}{6}\right) + \frac{3}{16}\sin\left(\frac{17}{6}\right) + \frac{1}{8}\sin(3) + \frac{3}{16}\sin\left(\frac{19}{6}\right) + \frac{3}{16}\sin\left(\frac{10}{3}\right) + \frac{1}{8}\sin\left(\frac{7}{2}\right) + \frac{3}{16}\sin\left(\frac{11}{3}\right) + \frac{3}{16}\sin\left(\frac{5}{3}\right) + \frac{3}{16}\sin\left(\frac{11}{6}\right) + \frac{1}{8}\sin(2) + \frac{3}{16}\sin\left(\frac{13}{6}\right) + \frac{3}{16}\sin\left(\frac{7}{3}\right) + \frac{1}{8}\sin\left(\frac{5}{2}\right) + \frac{3}{16}\sin\left(\frac{8}{3}\right) + \frac{3}{16}\sin\left(\frac{7}{6}\right) + \frac{3}{16}\sin\left(\frac{4}{3}\right) + \frac{1}{8}\sin\left(\frac{3}{2}\right) + \frac{1}{16}\sin(1)$$

> $ApproximateInt\left(\cos(x), 1..100, method = simpson_{\frac{3}{8}}, output = animation\right)$

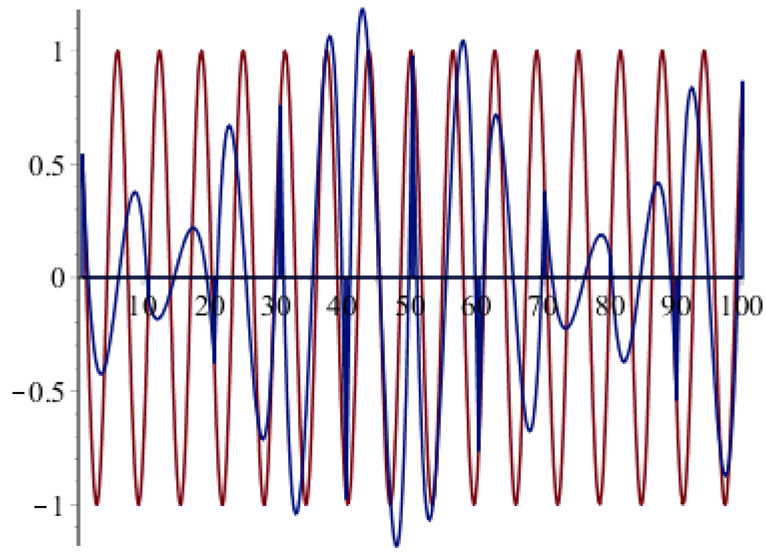

Figure 3 An animated approximation of $\int_{1}^{100} f(x)\,dx$ using Simpson's 3/8 rule, where $f(x) = \cos(x)$ and the partition is uniform. The approximate value of the integral is -0.00340010048. Number of subintervals used: 10.

CONCLUSION

The results show that the Maple package has good computational capabilities and can be very useful for analyzing generalized numerical integration methods of a software. We tested some of the computational capabilities of the Maple package using the Newton-Coates formula for numerical integration.

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