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NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONUSING TWO-STAGE SEMI-IMPLICIT HYBRID RUNGE-KUTTA SCHEME

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Abstract*: In this paper, family of two-stage semi-Implicit hybrid Runge-Kutta schemes were developed, analyzed and computerized to solve ordinary differential equations.Their development and analysis make use of Taylor series expansion, Dahquist stability test model equation respectively. The theoretical results show that the schemes are Consistent, Convergent and A-stable with large interval of absolute stability. (-∞, 0). The results of this scheme are compare with result of classical Runge-Kutta and Rational Runge-Kutta. The numerical results obtained confirmed that the schemes are accurate.*

Keywords: A-stable, Accurate, Semi-Implicit, Hybrid, Local-truncation Error.

1.0 Introduction

Ordinary differential equations (ODEs) arise in many different context including geometry, mechanics, astronomy, population modeling and many others.
These often lead to an ODEs of the form

$$y' = f(x, y), \; y(x_o) = y_o \tag{1.1}$$

Researchers have contributed to the development of the schemes that solves these ODEs. The consequences of these contribution leads to an introduction of the Rational One-step method of the form

$$y_{n+1} = \frac{y_n^2}{y_n - hf_n} \tag{1.2}$$

called Inverse Euler's scheme by Fatunla (1982). This prompted Hong-Yuanfu (1982) to introduced Rational Runge-Kutta Scheme of the general from

$$y_{n+1} = \frac{y_n + \sum_{i=1}^{R} W_i K_i}{1 + y_n \sum_{i=1}^{R} V_i H_i} \tag{1.3}$$

where,

$$K_1 = hf(x_n, y_n)$$

$$K_i = hf\left(x_n + C_1 h, y_n + \sum_{j=1}^{R} a_{ij} K_j \right) \qquad (1.4)$$

$$H_1 = hg(x_n, z_n)$$

$$H_i = hg\left(x_n + d_i h,\ z_n + \sum_{j=1}^{R} a_{ij} H_j \right) \qquad (1.5)$$

$$g(x_n,\ z_n) = -z_n^2 f(x_n y_n) = -1/y_n^2\ f(x_n, y_n) \qquad (1.6)$$

Although the method is suitable, accurate and stable. It is bedeviled by the difficult nature of the function evaluation of f and g, the method which we are considering in this works is of the form

$$y_{n+1} = \frac{y_n}{1 + y_n \sum_{i=1}^{R} V_i H_i} \qquad (1.7)$$

where,

$$H_i = hg\left(x_n + d_i h,\ z_n + \sum_{j=1}^{R} b_{ij} H_j \right) \qquad (1.8)$$

With

$$g(x_n\ z_n) = -z_n^2 f(x_n, y_n) \text{ and } Z_n = 1/y_n \qquad (1.9)$$

With the constraints

$$d_i = \sum_{j=1}^{R} b_{ij} \qquad (1.10)$$

and it is called Hybrid R-stage Runge-Kutta scheme. This method was classified into Explicit, Semi-Implicit and Implicit. Babatola (2010) proposed the one-stage semi-implicit scheme.

In this paper, we consider semi-implicit of two-stage for numerical solution of ODEs.

2. 0 Development of the New Schemes

Now setting R=2 in equation (1.7), then two-stage semi Implicit Hybrid R-K scheme is of the form

$$y_{n+1} = \frac{y_n}{1 + y_n (V_1 H_1 + V_2 H_2)} \tag{2.1}$$

where,

$$H_1 = hg(x_n + d_1 h,\ z_n + b_{11} H_1)$$

$$H_2 = hg(x_n + d_2 h,\ z_n + b_{21} H_1 + b_{22} H_2) \tag{2.2}$$

with

$$g(x_n\ z_n) = -z_n^2 f(x_n, y_n) \tag{2.3}$$

With constraints

$d_1 = a_{11}$

$d_2 = b_{21} + b_{22}$ (2.4)

One-step application of the scheme (2.1) will generate $T_{n+1} = y_{n+1}\left[1 + y_n \sum_{i=1}^{2} V_i H_i\right] - y_n$

$$= y_{n+1}[1 + y_n (V_1 H_1 + V_2 H_2)] - y_n \tag{2.5}$$

The parameters V_1, V_2, d_1, d_2, b_{11}, b_{21}, b_{22} are to be determine from the system of non-linear equations generated by adopting the steps below:

Step 1: Take the Taylor series expansion of y_{n+1} and H_i, 1=1, (1)2 about point (x_n, y_n)
Step 2: Insert the results of the expansion in step 1 into (2.5).
Step 3: Rearrange the final expansion about x_n in the power series of h, so that T_{n+1} can be expressed in the form

$$T_{n+1} = a_o + a_1 h y_n' + a_2 h^2 y'' + ...a_p h^p y_n^{(p)} + a_{p+1} h^{P+1} y_n^{(p+1)} + 0h^{p+2}$$

the order of accuracy of the scheme can be identified.

The scheme (2.1) is said to be or order P(Lambert 1973, 1997). If $c_p = 0$, p = 0(1)p and $C_{p+1} \neq 0$, P = 1, 2 3 … P (2.7)

Thus, to obtain the values of the above coefficient, we solve the set of non-linear system of equation (2.7) for p = 1, 2 … P with $C_{p+1} \neq 0$. So as to ensure that the values of the parameter yield computational method that have:

(i) minimum bound of local truncation error Ralston (1962)

(ii) maximum attainable order of accuracy King (1966).

(iii) Minimum computer storage space (Gill (1951)).

(iv) Large interval of absolute stability.

Expanding y_{n+1}, H_1 and H_2 in Taylor series to get a truncation error T_{n+1} specified as

$$\mathrm{y}_{n+1} = \mathrm{y}_n + hf_n + \frac{h^2}{2!}Df_n + \frac{h^3}{3!}\left(D^2 f_n + f_y Df_n\right) + \frac{h^4}{4!}\left(D^3 f_n + f_y D^2 f_n + 3Df_n Df_y\right) f_y{}^2 D f_n\Big) + 0h^5 \quad (2.8)$$

Where,

$$\mathrm{D}f_n = f_x + f_n f_y$$

$$\mathrm{D}^2 f_n = f_{xx} + 2 f_n f_{xy} + f_n^2 f_{yy}$$

$$\mathrm{D}^3 f_n + f_{xxx} + 3 f_n f_{xxy} + 3 f^2 f_{xyy} + f_n^3 f_{yyy}$$

$$D\mathrm{f}_y = f_{xy} + f_n f_y + f_y^2 \quad (2.9)$$

$$H_1 = hN_1 + h^2M_1 + h^3R_1 + h^4D_1 + 0h^5 \quad (2.10)$$

where,

$$N_1 = \frac{-f_n}{y_n^2},\ \mathrm{M}_1 = \frac{\mathrm{d}_1}{\mathrm{y}_n^2}\left(Df_n + \frac{2\mathrm{f}_n^2}{\mathrm{y}_n}\right)$$

$$R_1 = \frac{d_1^2}{y_n^2}\left(D^2 f_n - 2 f_n\left(2 f_x - f_n^2\right)\right)$$

$$D_1 = \frac{d_1^3}{y_n^2}\left(D^3 f_n - 6 f_n\left(f_{xx} + f_n^2 f_{yy}\right) - \frac{2 f_n^2}{y_n^2}\left(2 f_n f_y + 3 f_y^3\right)\right) \quad (2.11)$$

Similarly, the Taylor series expansion of H_2 is

$$H_2 = hN_2 + h^2M_2 + h^3R_2 + h^4D_2 + 0h^5 \quad (2.12)$$

where

$$N_2 = \frac{-f_n}{y_n^2},\ M_2 = \frac{d_2}{y_n^2}\left(Df_n + \frac{2f_n^2}{y_n}\right)$$

$$R_2 = \frac{d_2^2}{2y_n^2}\left(D^3 f_n - \frac{2f_n}{y_n}\left(2f_{xx} + \frac{f_n^2}{y_n}\right)\right)$$

$$D_2 = \frac{d_2^3}{y_n^2}\left(D^3 f_n - \frac{6f_n}{y_n}\left(f_{xx} + \frac{f_n f_x}{y_n}\right)\right) \quad (2.13)$$

Substitute equations (2.8), (2.10) and (2.12) into (2.5), yields

$$T_{n+1} = a_o + a_1 y_n' h + a_2 h^2 y_n'' + a_3 h^3 y_n''' + a_4 h^4 y_n'''' + 0h^5 \quad (2.14)$$

where

$a_o = 0,\ a_1 = f_n(1 - V_1 - V_2)$

$$a_2 = Df_n\left(\tfrac{1}{2} - V_1 d_1 - V_2 d_2\right) + 2f_n^2 y_n (V_1 d_1 - V_2 d_2) - \frac{f_n^2}{y_n}(V_1 + V_2)$$

$$a_3 = \frac{D^2 f_n + f_y Df_n}{6} + \left(V_1 d_1^2 + V_2 d_2^2\right)D^2 f_n + 2f_n V_1 d_1^2\left(2f_x - f_n^2\right)$$
$$+ 6V_2 d_2^2 f_n\left(f_{xx} + f_n^2 f_{yy}\right) + \left(V_1 d_1 + V_2 d_2\right)Df_n + \frac{2f_n^2}{y_n}\left(V_1 d_1 + V_2 d_2\right)$$

$$a_4 = \frac{1}{48}\left(D^3 f_n + f_y D^2 f_n + 3Df_n Df_y + f_y^2 Df_n\right) +$$
$$+ \left(V_1 d_{1\,1}^3 + V_2 d_2^3\right)D^3 f_n - 6f_n\left(f_{xx} + f_n^2 f_y\right) - 2f_n^2 f_y\left(2f_n + 3f_y^2\right) \quad (2.15)$$
$$- \frac{6f_n}{y_n}\left(f_{xx} + \frac{f_n f_x}{y_n}\right)$$

Imposing accuracy of order 3 on T_{n+1} then $a_o = a_1 = a_2 = a_3 = 0$, but $a_4 \neq 0$ the following system of non-linear equations for family of semi-implicit hybrid Runge-kutta of order three.

$V_1 + V_2 = 1$

$V_1 d_1 + V_2 d_2 = ½$

$V_1 b_{11} d_1 + V_2 (b_{21} d_1 + b_{22} d_2) = 1/6$

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$$V_1 d_1^2 + V_2 d_2^2 = \tfrac{1}{3} \qquad (2.16)$$

With the constraints

$b_{11} = d_1 \neq 0$

$$b_{21} + b_{22} = d_2 \qquad (2.17)$$

For the parameters b_{11}, b_{22}, d_1, d_2, V_1, V_2

Solving these equations we obtain

Case 1: $V_1 = 3/4$, $V_2 = 1/4$, $d_1 = b_{11} = 1/3$,

$b_{21} = b_{22} = 1/2$, $d_2 = 1$

$$y_{n+1} = \frac{y_n}{1 + \frac{y_n}{4}\left(3H_1 + H_2\right)} \qquad (2.18)$$

Where , $H_1 = hg\left(x_n + \tfrac{1}{3}h,\ z_n + \tfrac{1}{3}H_1\right)$

$$H_2 = hg\left(x_n + h,\ z_n + \tfrac{1}{2}H_1 + \tfrac{1}{2}H_2\right) \qquad (2.19)$$

Case 2:

$$V_1 = \tfrac{1}{2}, V_2 = \tfrac{1}{2},\ b_{11} = d_1 = 1/$$
$$d_2 = \tfrac{1}{3} + \frac{\sqrt{3}}{6},\ \ b_{22} = \tfrac{1}{3},\ \ b_2 = \left(1 + \sqrt{3}/6\right)$$

yields another 2-stage scheme of order 3.

$$y_{n+1} = \frac{y_n}{1 + \frac{y_n}{2}\left(H_1 + H_2\right)} \qquad (2.20)$$

Where , $H_1 = hg\left(x_n + \tfrac{1}{3}h,\ z_n + \tfrac{1}{3}H_1\right)$

$$H_2 = hg\left(x_n + (\left[\tfrac{1}{2}\ - \tfrac{\sqrt{3}}{6}h,\ z_n + \left(\frac{1-\sqrt{3}}{6}\right)H_1 + \tfrac{1}{3}H_2\right) \qquad (2.21)$$

Two stage semi-Implicit Hybrid Runge-Kutta of order four are obtained from solving system of non-linear equations (2.22) and (2.23)

$$\begin{aligned} &V_1 + V_2 = 1 \\ &V_1 d_1 + V_2 d_2 = 1/2 \\ &V_1 d_1^2 + V_2 d_2^2 = 1/3 \\ &V_1 b_{11} d_1 + V_2 (b_{21} d_1 + b_{22} d_2) = 1/6 \\ &V_1 b_{11} d_1^2 + V_2 \left(b_{21} d_1^2 + b_{22} d_2^2\right) = 1/2 \end{aligned} \qquad (2.22)$$

Subject to the following constraints

$$\begin{aligned} b_{11} &= d_1 \\ b_{21} + b_{22} &= d_2 \end{aligned} \qquad (2.23)$$

to obtain

Case 1: $V_1 = 0.\ V_2 = 1,\ d_1 = b_{11} = 1/24,\ d_2 = 1/2,\ b_{22} = b_{21} = 1/4$

Then,

$$y_{n+1} = \frac{y_n}{1 + y_n H_2} \qquad (2.24)$$

where, $H_1 = hg\left(x_n + 1/24\,h,\ z_n + 1/24\,H_1\right)$

$$H_2 = hg\left(x_n + 1/2\,h,\ z_n + 1/4\,H_1 + 1/4\,H_2\right) \qquad (2.25)$$

Case 2:

If $V_1 = 1/4,\ V_2 = 3/4,\ d_2 = 1/3,\ b_{22} = b_{21} = 1/6,\ d_1 = b_{11} = 3/2$

yields

$$y_{n+1} = \frac{y_n}{1 + \frac{y_n}{4}\left(H_1 + 3H_2\right)} \qquad (2.26)$$

where

$$\begin{aligned} H_1 &= hg\left(x_n + 3/2\,h,\ z_n + 3/2\,H_1\right) \\ H_2 &= hg\left(x_n + 1/6\,h,\ z_n + 1/12\,H_1 + 1/12\,H_2\right) \end{aligned} \qquad (2.27)$$

3.0 Analysis of the Basic Properties

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The basic properties required of a good computational method include, accuracy, consistency, convergence and stability.

3.1 Consistency

A scheme is said to be consistent if the difference equation of the computation formulas exactly approximate the differential equation it intend to solve Ademiluyi and Babatola (2001).
To prove that equation (2.1) is consistent.
Recall that

$$y_{n+1} = \frac{y_n}{1 + y_n \sum_{i=1}^{2} V_i H_i} \tag{3.1}$$

Subtract y_n from both sides of equation (3.1) and further simplification we get

$$y_{n+1} - y_n = \frac{y_n}{1 + y_n \sum_{i=1}^{2} V_i H_i} - y_n = \frac{y_n^2 \sum_{i=1}^{2} \mathrm{V_i} H_i}{1 + y_n \sum_{i=1}^{2} \mathrm{V_i} H_i} \tag{3.2}$$

$$\text{But } \mathrm{H_i} = hg\left(x_n + d_i h, \mathrm{z_n} + \sum_{j=1}^{2} b_{ij} H_j \right) \tag{3.3}$$

Hence,

$$y_{n+1} - y_n = \frac{y_n^2 h \sum \mathrm{V_i} g(x_n + d_i h, \ \mathrm{z_n} + \sum_{j=1}^{2} b_{ij} H_j)}{1 + y_n \sum_{i=1}^{2} V_i hg(x_n + d_i h, \ \mathrm{z_n} + \sum_{i=1}^{2} b_{ij} H_j)_i} \tag{3.4}$$

Dividing through by h and taking limit as h tends to zero.

$$\lim_{h \to 0} \frac{y_{n+1} - y_n}{h} = y_n^2 g(x_n, \mathrm{z_n}) \tag{3.5}$$

But $g(x_n, \mathrm{z_n}) = \frac{-1}{y_n^2} f(x_n, \mathrm{y_n})$ (3.6)

Hence,

$$\lim_{h \to 0} \frac{y_{n+1} - y_n}{h} = -y_n^2 \left(- 1/y_n^2 \ f(x_n, \mathrm{y_n}) \right) \tag{3.7}$$

$$y_n^{'} = f(x_n, \mathrm{y_n}) \tag{3.8}$$

Hence the method is consistent.

3.2 Convergence

A numerical scheme such as equation (2.1) is said to be convergent, if when applied to initial value problem (1.1), it generate a corresponding approximation y_n, which tends to the exact solution $y(x_n)$ as n approaches infinity.

To show that equation (2.1) is convergent.

Recall that

$$y_{n+1} = \frac{y_n}{1 + y_n \sum_{i=1}^{2} V_i H_i(y_n)} \tag{3.9}$$

While the exact $y(x_{n+1})$ satisfy difference equation (3.9) as

$$y(x_{n+1)} = \frac{y(x_n)}{1 + y(x_n)\sum_{i=1}^{2} V_i H_i(y(x_n)\Big)} + T_{n+1} \tag{3.10}$$

Subtracting equation (3.9) from (3.10)

$$y(x_{n+1}) - y_{n+1} = \frac{y(x_n)}{1 + y(x_n)\sum_{i=1}^{2} V_i H_i y(x_n)} - \frac{y_n}{1 + y_n \sum_{i=1}^{2} \mathrm{V_i} H_i(y_n)} + T_{n+1} \tag{3.11}$$

$$e_{n+1} = \frac{y(x_n)\left[1 + y_n \sum_{i=1}^{2} V_i H_i(y_n) - y_n\left[1 + y(x_n)\sum V_i H_i(y_n)\right]\right]}{1 + y(x_n)\sum V_i H_i(y_n) - y_n\left[1 + y(x_n)\sum V_i H_i(y_n)\right]} + T_{n+1} \tag{3.12}$$

$$e_{n+1} = \frac{\left[1 + y(x_n) y_n \sum_{i=1}^{2} V_i \frac{\partial H_i}{\partial y}\right]}{1 + y(x_n)\sum_{i=1}^{2} V_i H_i(y(x_n))\left[1 + y_n(\sum_{i=1}^{2} V_i H_i(y_n)\right]} + T_{n+1} \tag{3.13}$$

Setting

$$P_n = 1 + y(x_n)\sum_{i=1}^{2} V_i H_i(y(x_n))$$

$$Q_n = 1 + y_n \sum V_i H_i(y_n)$$

$$R_n = 1 + y_n y(x_n)\sum_{i=1}^{2} V_i \frac{\partial H_i}{\partial y}$$

Then equation (3.12) becomes

$$e_{n+1} = \frac{R_n}{P_n Q_n} + T_{n+1.} \quad , \ \mathrm{E_n} = \max_{0 \le x \le \infty} e_n \qquad (3.14)$$

Let P = $\max_{0 \le \infty}(P_n), \quad \mathrm{Q} = \max_{0 \le x \le \infty}(Q_n)$

R = $\max_{0 \le x \le \infty}(R_n), \quad \mathrm{T} = \max_{0 \le x \le \infty} T_{n+1}$

Then

$$E_{n+1} = \frac{\mathrm{R}}{\mathrm{PQ}} E_n + T \qquad (3.15)$$

Set K = $\dfrac{R}{PQ}$

Then equation (3.15) becomes

$$E_{n+1} = KE_n + T \qquad (3.16)$$

$$E_1 = KE_o + T \qquad (3.16a)$$

$$E_2 = KE_1 + T \qquad (3.16b)$$

Substitute (3.16a) into equation (3.16b)

$$E_2 \le K\,(KE_o + T) + T$$
$$K^2 E_o + KT + T$$

$$E_3 = K^3\ E_o + K^2T + \mathrm{T}$$

Therefore

$$E_{n+1} \le K^{n+1}E_o + \sum K^{1+1}T + T \qquad (3.17)$$

Since $\dfrac{R}{PQ}$ = K < 1

It is easy to see that as n → ∞, E→ 0. This shows that in this particular case, the schemes converges.

3.3 Stability properties

Since a Consistent and Convergent one-step scheme is stable, the scheme is stable. However, to ensure that the method is A-stable and P-stable and able to solve stiff initial value problem. It is adopted for solution of the A-stability model test equation

$$y' = \lambda y,\ \mathrm{y(x_o)} = y_o \qquad (3.18)$$

Applying the scheme to the initial value problem (3.18), a recurrent equation is obtained as:

$$y_{n+1} = P(z)\ \mathrm{y_n} \qquad (3.19)$$

where

$$P(z) = \frac{1 - \frac{1}{2}z - \frac{1}{2}z^2}{1 - \frac{1}{2}z + \frac{9}{2}z^2} \qquad (3.20)$$

which is (2.2) Pade's approximation to e^z since it can be expressed as

$$P(z) = 1 + z + \frac{7}{8z^2} + \frac{5}{16z^3} + \frac{5}{16z^4} + 0z^5 \qquad (3.21)$$

The scheme is A-stable since $z \in [-\infty, 0]$ and P-stable since $z \in [-\infty, \infty]$

4.1 <u>Numerical Experiment</u>

In order to confirm the applicability and suitability of the scheme for solution of ODEs, some sample problems were considered

(i) $y' = 2x + y,\quad \mathrm{y(o)} = 1$ (4.1)

with exact solution

$$y = -2(x+1) + 3e^x \qquad (4.2)$$

with h = 0.1.The results are shown in Table I.

(ii) $y' = 2xy,\quad \mathrm{y(o)} = 1$ (4.3)

with exact solution

$$y = e^{x^2} \qquad (4.4)$$

with h = 0.025.The results are shown in Table 2.

(iii) $y' = 10(y - x^3) + 3x^2 \quad y(o) = 1$ (4.5)

with the analytical solution given as $y(x) = x^3 + e^{-10x}$ (4.6)

The results of these attempts with h = 0.01 are shown in Table 3.

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TABLE 1:

NUMERICAL SOLUTION OF PROBLEM $y' = 2x + y \cdot y(0) = 1$ WITH SEMI-IMPLICIT HYBRID RUNGE-KUTTA AND SEMI IMPLICIT CLASSICAL RUNGE KUTTA SCHEME

VALUES OF X x_n	**YEXACT** $y(x_n)$	**SEMI-IMPLICIT HYBRID R-K** y_n	**SEMI IMPLICIT CLASSICAL R-K VALUE OF Y** y_n	**ERROR OF EXPLICIT HYBRID R-K** E_H	**ERROR OF CLASSICIAL R-K** E_c
0.25000000D-01	0.10259460D+01	0.10259850D+01	0.10263440D+01	0.39577480D-04	0.39839740D-03
0.50000000D-01	0.10538130D+01	0.10538790D+01	0.10542430D+01	0.65088270D-04	0.42986870D-03
0.75000000D-01	0.10836520D+01	0.10837280D+01	0.108409990D+01	0.75221060D-04	0.44620040D-03
0.10000000D+00	0.11155130D+01	0.11155820D+01	0.11159590D+01	0.69022180D-04	0.44631960D-03
0.12500000D+00	0.11494450D+01	0.11494910D+01	0.11498750D+01	0.46014790D-04	0.42998790D-03
0.15000000D+00	0.11855030D+01	0.11855080D+01	0.11858980D+01	0.50067900D-05	0.39553640D-03
0.17500000D+00	0.12237390D+01	0.12236840D+01	0.12240810D+01	0.54240230D-04	0.34272670D-03
0.20000000D+00	0.12642080D+01	0.12640760D+01	0.12644790D+01	0.13267990D-03	0.27096270D-03
0.22500000D+00	0.13069680D+01	0.13067370D+01	0.13071470D+01	0.23126600D-03	0.17893310D-03
0.27500000D+00	0.13520760D+01	0.13517260D+01	0.13521430D+01	0.34999850D-03	0.66637990D-04
0.30000000D+00	0.13995920D+01	0.13991020 D+01	0.13995250D+01	0.49018860D-03	0.67114830D-04
0.32500000D+00	0.14495770D+01	0.14489240D+01	0.14493530D+01	0.65302850D-03	0.22375580D-03
0.3500010D+00	0.15020920D+01	0.15012540D+01	0.15016900D+01	0.83768370D-03	0.40233140D-03
0.370000010D+00	0.15572030D+01	0.15561560D+01	0.15565980D+01	0.10470150D-02	0.60570240D-03
0.40000010D+00	0.16149740D+01	0.16136950D+01	0.16141420D+01	0.12794730D-02	0.83243850D-03
0.42500010D+00	0.16754740D+01	0.16739370D+01	0.16743900D+01	0.15372040D-02	0.10844470D-02

0.45000010D+00	0.17387710D+01	0.17369500D+01	0.17374090D+01	0.18210410D-02	0.13629200D-02
0.47000010D+00	0.18049370D+01	0.18028050D+01	0.18032690D+01	0.21314620D-02	0.16682150D-02
0.47500010D+00	0.18740430D+01	0.18715740D+01	0.19438020D+01	0.24693010D-02	0.20009280D-02
0.50000010D+00	0.19461640D+01	0.19433290D+01	0.20186240D+01	0.28352740D-02	0.23621320D-02
0.52500000D+00	0.20213770D+01	0.20181470D+01	0.20186240D+01	0.32300950D-02	0.27525420D-02
0.55000000D+00	0.20997590D+01	0.2096104D+01	0.20965860D+01	0.36551950D-02	0.31733510D-02
0.57500000D+00	0.21813910D+01	0.21772800D+01	0.21777660D+01	0.41110520D-02	0.36249160D-02
0.60000000D+00	0.22663560D+01	0.22617570D+01	0.22622470D+01	0.45995710D-02	0.41096210D-02
0.62499990D+00	0.23547380D+01	0.23496170D+01	0.23501110D+01	0.51205160D-02	0.46267510D-02
0.64999980D+00	0.24466220D+01	0.24409470D+01	0.24414440D+01	0.56755540D-02	0.51784520D-02
0.67499998D+00	0.25420990 +01	0.25358330D+01	0.25363340D+01	0.62651630D-02	0.57649610D-02
0.69999990D+00	0.26412580D+01	0.26343670D+01	0.26348700D+01	0.68912510D-02	0.63881870D-02
0.72499980D+00	0.27441930D+01	0.27366380D+01	0.26348700D+01	0.75547700D-02	0.70488450D-02
0.77499980D+00	0.28509990D+01	0.28427430D+01	0.27371440D+01	0.82559590D-02	0.77476500D-02
0.7999980D+00	0.29617750D+01	0.29527790D+01	0.28432890D+01	0.89967250D-02	0.84862710D-02
0.8249997D+00	0.30766220D+01	0.30668430D+01	0.29532890D+01	0.97789760D-02	0.9266150D-02
0.84999970D+00	0.31956410D+01	0.31847680D+01	0.30673550D+01	0.10873320D-01	0.10088680D-01
0.87499970D+00	0.33189390D+01	0.33069140D+01	0.3185520D+01	0.12025360D-01	0.11232140D-01
0.89999970D+00	0.34466240D+01	0.34333880D+01	0.33077070D+01	0.13236280D-01	0.12434960D-01
0.92499970D+00	0.35788080D+01	0.35642990D+01	0.34341890D+01	0.14508490D-01	13699050D-01
0.94999960D+00	0.38571270D+01	0.36997600D+01	0.35651090D+01	0.15841960D-01	0.15025140D-01
0.97499960D+00	0.40035000D+01	0.38398860D+01	0.37005770D+01	0.17240760D-01	0.16416790D-01
0.9999960D+00	0.41548430D+01	0.39847940D+01	0.38407100D+01	0.18705840D-01	0.17875190D-01
0.10250000D+01	0.43112840D+01	0.41346050D+01	0.39856250D+01	0.20238400D-01	0.19401070D-01

0.10255000D+01	0.43112840D+01	0.42894430D+01	0.42902870D+01	0.21841950D-01	0.20997520D-01

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TABLE 2:

NUMERICAL SOLUTION OF SEMI-IMPLICIT HYBRID RUNGE-KUTTA OF PROBLEM (4.3)

j	x	y_n	Yexact	Error
1	0.012500	1.0012436	1..0001563	0.001087
2	0.025000	1.0018657	1.0006253	0.001240
3	0.037500	1.0024880	1.0014073	0.001081
4	0.050000	1.0031105	1.0025032	0.000607
5	0.062500	1.0037333	1.0039139	0.000181
6	0.075000	1.0049791	1.0056409	0.001285
7	0.087500	1.0056025	1.0076857	0.002707
8	0.100000	1.0068495	1.0100502	0.004448
9	0.112500	1.0074735	1.0127367	0.006511
10	0.125000	1.0080975	1.0157477	0.008898
11	0.137500	1.0087217	1.0190861	0.11613
12	0.150000	1.0099708	1.0227550	0.014657
13	0.162500	1.0093461	1.0267580	0.018036
14	0.175000	1.0105956	1.0310987	0.021753
15	0.187500	1.0118458	1.0357815	0.025811
16	0.200000	1.0214712	1.0408108	0.030215
17	0.212500	1.0130968	1.0461913	0.034971
18	0.225100	1.0137225	1.0519284	0.030215
19	0.237500	1.0143485	1.0580274	0.045556
20	0.250000	1.0149747	1.644945	0.051398
21	0.262500	1.0156010	1.0713358	0.057613
22	0.275000	1.0162276	1.0785581	0.064210
23	0.287500	1.0168544	1.0861684	0.071194
24	0.300000	1.0174813	1.0941743	0.078573
25	0.312500	1..0191085	1.1025836	0.086356
26	0.325000	1.0187358	1.1114050	0.094551
27	0.337500	1.0168544	1.1206471	0.103166
28	0.350000	1.0181085	1.1303191	0.112211
29	0.362500	1.0187358	1.1404309	0.121695
30	0.375000	1.0193633	1.1509929	0.131630

NUMERICAL SOLUTION OF PROBLEM $y' = 10(y - x) + 3x \cdot$ WITH SEMI-IMPLICIT HYBRID RUNGE-KUTTA SCHEME AND SEMI IMPLICIT CLASSICAL RUNGE KUTTA SCHEME

VALUES OF X x_n	**YEXACT** $y(x_n)$	**SEMI-IMPLICIT HYBRID R-K** y_n	**SEMI IMPLICIT CLASSICAL R-K VALUE OF Y** y_n	**ERROR OF SEMI-IMPLICIT HYBRID R-K** E_H	**ERROR OF CLASSICIAL R-K** E_c
0.10000000D-01	0.90483840D+00	0.11041020D+01	0.10730980D+01	0.19926320D+00	0.16825960D+00
0.99999999D-01	0.36887940D+00	0.44989030D+00	0.43716390D+00	0.81010850D-01	0.68284480D-01
0.20000000D+00	0.14333530D+00	0.17312640D+00	0.16813310D+00	0.29791100D-01	0.24797830D-01
0.30000000D+00	0.76787060D-01	0.87720370D-01	0.85207890D-01	0.10933310D-01	0.84208250D-02
0.39999990D+00	0.82315610D-01	0.86300200D-01	0.84150340D-01	0.39845930D-02	0.18347280D-02
0.59999970D+00	0.21847850D+00	0.21893910D+00	0.21544030D+00	0.46066940D-03	0.30381680D-02
0.69999960D+00	0.34391130D+00	0.34400960D+00	0.33928400D+00	0.98288060D-04	0.46273470D-02
0.79999950D+00	0.51233460D+00	0.51228950D+00	0.50609780D+00	0.45120720D-04	0.62367920D-02
0.89999940D+00	0.72912200D+00	0.72901460D+00	0.72113930D+00	0.10746720D-03	0.79827900D-02
0.10099990D+01	0.10303400D+01	0.10301980D+01	0.10202290D+01	0.14233590D-03	0.10111570D-01

Conclusion

In all cases, the computed errors show that the schemes are very accurate, stable and convergent.
As can be seen from the result in tables 1 – 3, these schemes compare favourably with the existing Runge-Kutta schemes of the same order.

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