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MODELS AND QUALITY CONTROL CHARTS FOR THE PREDICTION OF COMPRESSIVE CEMENT STRENGTH

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ABSTRACT: *This paper presents quality control charts techniques usually applied in quality control of compressive cement strength. Nonlinear regression model useful for the prediction of compressive cement strength at 28 days was proposed. Combining the prediction and quality control tools, a PI (Proportional Integral) controller useful for the regulation of 28 days compressive cement strength around a target (39 Mpa) was constructed. Results of the one-year prediction of quarterly compressive cement strength aligned with the values of the historical data obtained from a leading Cement Company in Nigeria for the years, 2011-2015.*

KEYWORDS: Quality Control, Cement, Compressive Strength, Prediction, Nonlinear Regression

INTRODUCTION

Control charts methodologies are generally applied as useful tools in the assessment of different characteristics of various processes and products. It is important to note that when goods are produced in their precise specifications, not all of them will look exactly the same because of some natural causes of variations which are inevitable that occur either between the items or within the items. A production process is only said to be in a state of statistical control if it is operating in the presence of natural causes of variation and is free from assignable causes, Gupta [11].

Quality control is basically aimed at possibly reducing such assignable variations to a minimum level, so as to improve on the quality and acceptability of the products. This can be achieved if the result of the producer's quality control process falls within or closely about the general mean of the specified control chart. In the building materials sector and particularly for cement and concrete, Dimitris [7] stated that the ISO 8258:1991 standard has been adopted.

Several examples of works involving implementation of quality control techniques, exponentially weighted moving averages and PID controllers can be found in [2, 3, 4, 5, 6, 7, 8, 10, 12, 13, 15 16]. For quality control to be effective, remedial measures must be taken as soon as a deviating trend is observed.

As presented by Dimitris [7], modelling of compressive strength of cement could be based on regression or non-linear regression of existing data. These models utilize results of cement produced in industrial scale to estimate the model parameters.

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In this paper the techniques of control charts will be combined with the compressive strength predictive models techniques to predict cement strength. Thereafter, a PI controller will be designed for the regulation of the 28 days compressive strength around a predetermined target.

BUILDING OF CONTROL CHARTS

Control limits for the R – Chart:

The Central Limit (CL) = $\bar{R}$

$$\bar{R} = \frac{1}{n}\sum_{i=0}^{n} R_i$$

The Upper Control Limit

(UCL) = $\bar{R}D4$

The Lower Control Limit

(LCL) = $\bar{R}D3$

D3 and D4 are obtained from Quality Control tables.

Control limits for the X bar Chart:

The Central Limit (CL) = $\bar{\bar{X}}$ where $\bar{\bar{X}} = \frac{1}{n}\sum_{i=0}^{n} \bar{X}_i$

The Upper Control Limit (UCL) = $\bar{\bar{X}} + {}^{3\sigma}/_{\sqrt{n}}$

The Lower Control Limit (LCL) = $\bar{\bar{X}} - {}^{3\sigma}/_{\sqrt{n}}$

$$\sigma = \frac{\bar{R}}{d2}$$

The values of d_2 are obtained from statistical quality control tables.

Strength Predicting Models

The Exponentially Weighted Moving Averages (EWMA)

EWMA control charts are used to obtain the average trend values [7, 9, 14]. For a variable X, the following procedures are used to build the charts:

(i) For time i = 0 the initial moving average Y_0 is expressed as (1):

$$Y_0 = X_0 \quad (1)$$

(ii) For a parameter δ, where $0 < \delta \leq 1$, the statistic Y_i is computed using (2):

$$Y_i = \delta X_i + (1 - \delta)Y_{i-1} \quad (2)$$

(iii) If δ = 1, the moving average values are equal to the current ones. For smaller δ values, the rate of change decreases and trends of higher duration can be revealed.

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The $Y_i's$ are the Exponentially Weighted Moving Averages and i, represents the serial number of the sample in use.

Nonlinear Regression Model

Following Agnihotri and Waghmare [1], the nonlinear regression model suitable for the prediction of 28 days compressive strength is proposed as:

$$S(t) = e^{-at}\cos(bt + c) + d \qquad (3)$$

The Proportional Integral (PI) Controller

Using the combined action of control charts and the nonlinear regression as well as EWMA models predicting strength, a PI Controller regulating 28 days strength around a predefined target (39 Mpa) is constructed. The transfer functions constituting the closed loop is given in form of a block diagram in Fig. 1.

The PI controller is a special case of the PID controller. The lack of derivative action makes the system steadier in the steady state in the case of noisy data (as in this case). This is because derivative action is more sensitive to higher-frequency terms in the inputs. Without derivative action, a PI-controlled system is less responsive to real (non-noise) and relatively fast alterations in state and so the system will be slower to reach set point and slower to respond to perturbations than a well-tuned PID system may be. Here, the derivative (D) of the error was not used.

The controller output in model form is given by:

$$CO = K_p \triangle + K_i \int \triangle \, dt \qquad (4)$$

Where $\triangle$ is the error or deviation from actual measured value (PV) from the set point (SP).

$\triangle = SP - PV$

K_p = Proportional gain

K_i = integral gain.

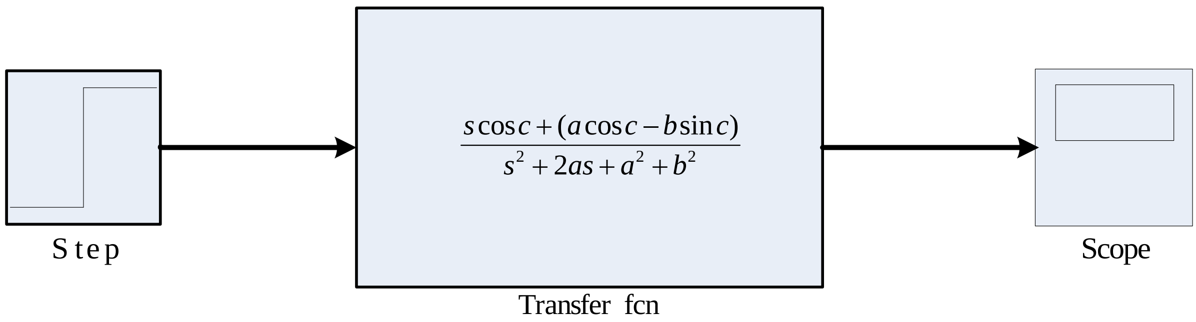

Fig.1:ClosedLoopBlockDiagram

ANALYSIS AND RESULTS

Table 1 presents the monthly mean strength of cement after twenty eight days of molding for the past years 2011 to 2015 from the Cement Company.

Table 1: Monthly Average Strength of Cement at Day 28 (2011-2015)

MONTH	YEAR				
	2011	2012	2013	2014	2015
JAN	38.12	43.48	40.05	38.03	37.57
FEB	38.41	39.18	42.28	36.54	36.63
MAR	36.20	40.07	40.72	36.11	38.18
APR	34.07	39.86	41.49	36.57	38.92
MAY	39.35	40.00	40.42	39.67	37.48
JUN	38.91	41.24	40.62	37.73	37.56
JUL	37.50	40.38	42.11	39.21	39.89
AUG	41.37	42.18	41.35	38.38	40.57
SEPT	38.24	38.9	38.05	37.71	40.91
OCT	38.56	39.23	38.71	38.27	40.17
NOV	39.67	38.53	39.23	37.22	38.42
DEC	43.26	0.00	38.36	36.91	38.80

The X bar and R Charts for the quarterly strength of cement after twenty eight days of molding is shown in Figs. 2 and 3. The charts indicate no point falling outside the two control limits and so, the process is concluded to have been in statistical process control for the years 2011 to 2015.

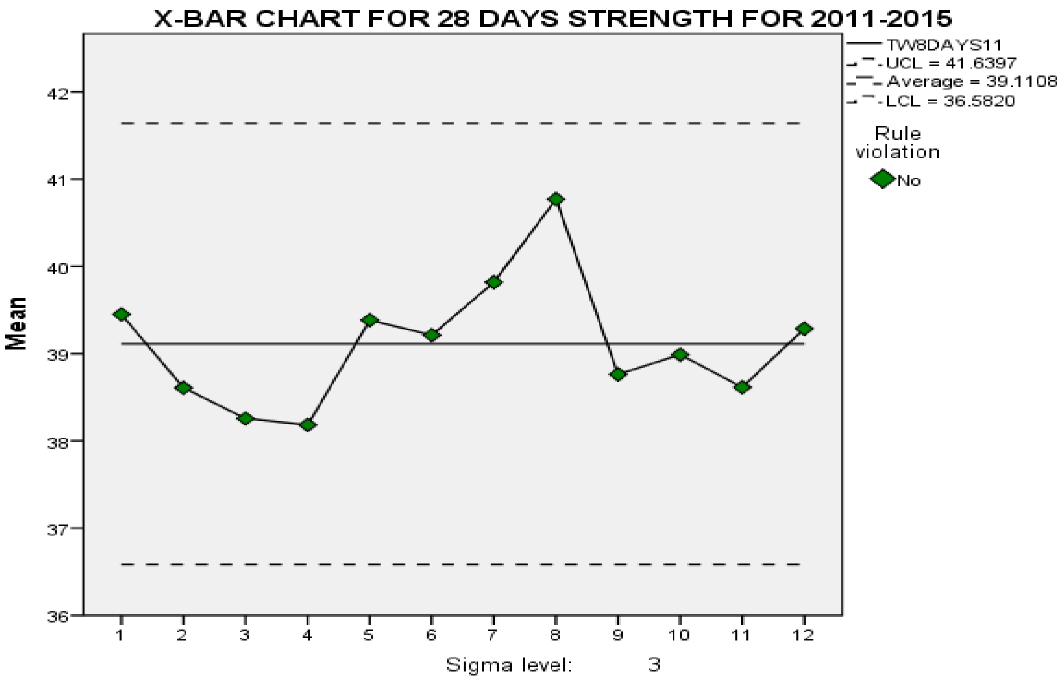

Fig. 2: X bar chart for 28days strength (2011-2015)

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Process parameters:

UCL = 41.6397

CL = 39.1108

LCL = 36.5820

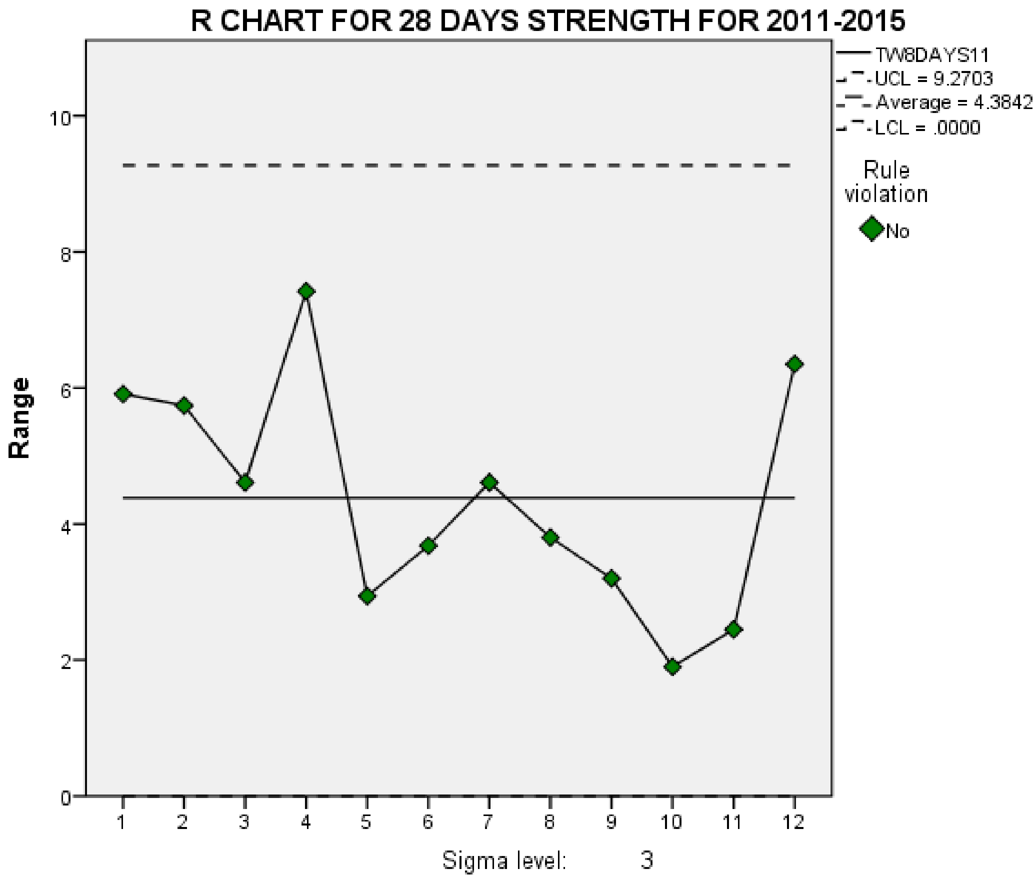

Fig. 3: R chart for 28days strength (2011-2015)

Process parameters:

UCL = 9.2703

CL = 4.3842

LCL = 0.0000

EWMA Strength Analysis

Table 2 presents the quarterly forecast of compressive strength of cement using Exponentially Weighted Moving Averages and the actual quarterly mean compressive strength of cement. The residuals show the closeness between the actual and forecast values.

Table 2: EWMA for 28 days Quarterly Strength of Cement (2011-2015)

YEARLY QUARTERS	ACTUAL MEAN STRENGHT	FORECASTED STRENGHT	SMOOTHED EWMA STRENGHT	RESIDUALS
1st QTR-2011	37.55	37.55	37.55	0.00
2nd QTR-2011	37.47	37.55	37.53	-0.08
3rd QTR-2011	39.08	37.53	37.92	1.55
4th QTR-2011	40.59	37.92	38.59	2.67
1st QTR-2012	40.75	38.59	39.13	2.17
2nd QTR-2012	40.41	39.13	39.45	1.28
3rd QTR-2012	40.55	39.45	39.72	1.10
4th QTR-2012	38.87	39.72	39.51	-0.85
1st QTR-2013	40.97	39.51	39.88	1.46
2nd QTR-2013	40.83	39.88	40.12	0.96
3rd QTR-2013	40.45	40.12	40.20	0.33
4th QTR-2013	38.76	40.20	39.84	-1.44
1st QTR-2014	36.87	39.84	39.10	-2.97
2nd QTR-2014	37.94	39.10	38.81	-1.16
3rd QTR-2014	38.36	38.81	38.69	-0.45
4th QTR-2014	37.46	38.69	38.39	-1.24
1st QTR-2015	37.43	38.39	38.15	-0.95
2nd QTR-2015	37.97	38.15	38.10	-0.18
3rd QTR-2015	40.50	38.10	38.70	2.40
4th QTR-2015	39.06	38.70	38.79	0.36

Strength Analysis Using the Proposed Models

Table 3 presents the values of the actual quarterly mean strength of cement from the raw data, the forecasted quarterly EWMA and the estimates using the proposed nonlinear regression equation model for 28 days compressive strength of cement.

Table 3: EWMA Forecasts and Nonlinear Regression Estimates

QUARTERS	PERIOD	QTRLY MEAN STRENGHT	FORECASTED EWMA	NONLINEAR REGRESSION ESTIMATION
1st QTR-2011	1	37.55	37.55	38.04895939
2nd QTR-2011	2	37.47	37.55	37.99006827
3rd QTR-2011	3	39.08	37.53	38.06363737
4th QTR-2011	4	40.59	37.92	38.25819201
1st QTR-2012	5	40.75	38.59	38.54626248
2nd QTR-2012	6	40.41	39.13	38.88824886
3rd QTR-2012	7	40.55	39.45	39.23785565
4th QTR-2012	8	38.87	39.72	39.54835128
1st QTR-2013	9	40.97	39.51	39.7788074

2nd QTR-2013	10	40.83	39.88	39.89948576
3rd QTR-2013	11	40.45	40.12	39.89566486
4th QTR-2013	12	38.76	40.20	39.76941543
1st QTR-2014	13	36.87	39.84	39.5391137
2nd QTR-2014	14	37.94	39.10	39.23678557
3rd QTR-2014	15	38.36	38.81	38.9036634
4th QTR-2014	16	37.46	38.69	38.58457035
1st QTR-2015	17	37.43	38.39	38.32189612
2nd QTR-2015	18	37.97	38.15	38.14997289
3rd QTR-2015	19	40.50	38.10	38.09059657
4th QTR-2015	20	39.06	38.70	38.15027651

Fig. 4 presents the trends of the actual quarterly mean strength as obtained from the raw data (series 1) and the EWMA (series 2). χ^2 - test for the goodness of fit using the quarterly strength data at 28 days for EWMA gave a result of 1.074. This result is less than the χ^2 table value at 5% level of significance with 19 degrees of freedom (1.729). Thus, there is no significant difference in the quarterly mean strength values and the EWMA.

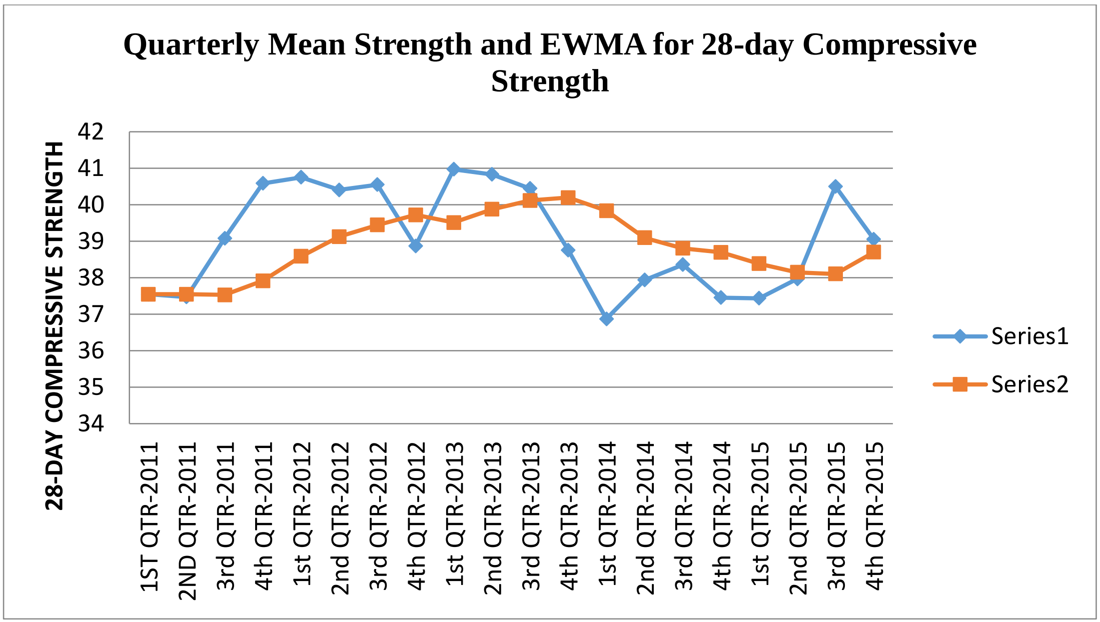

Fig. 4: Actual quarterly mean strength and EWMA for cement at 28 days

The graphs of the actual quarterly mean strength as obtained from the EWMA and the proposed nonlinear regression model are shown in Fig. 5. χ^2 - test for the goodness of fit using the EWMA and the nonlinear regression model gave a result of 0.05. This result is less than the χ^2 table value at 5% level of significance with 19 degrees of freedom (1.729). Thus, there is no significant difference in the quarterly mean strength values of cement. Also the pattern of movement of both strength values are similar.

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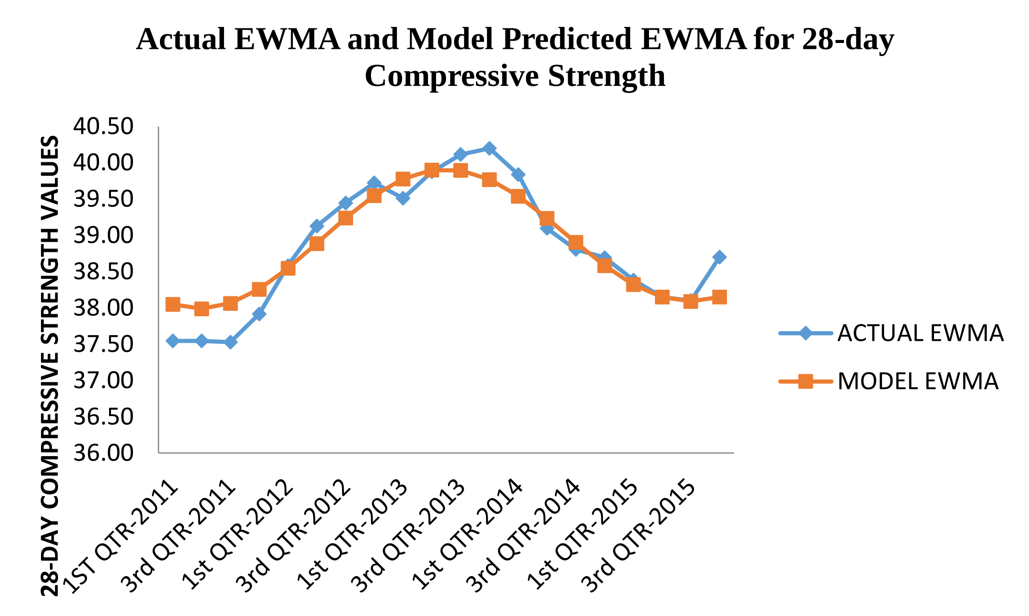

Fig. 5: Actual EWMA and nonlinear model strengths for cement at 28 days

Construction of the Proportional Integral (PI) Controller

To construct the PI controller, the proposed nonlinear equation model was implemented in Microsoft Excel Solver to obtain the parameter values. Thus,

$$S(t) = e^{-at}\cos(bt + c) + d$$

a = 0.006309

b = 0.368413

c = 2.409061

d = 38.97752

These parameter values were obtained by iteration using Microsoft Excel Solver.

Taking Laplace transform of the transient part of S(t):

$$L(S_{transient}(t)) = \frac{s\cos c + (a\cos c - b\sin c)}{s^2 + 2as + a^2 + b^2}$$

Substituting values of a, b, and c gives:

$$S_t(s) = \frac{-0.743s - 0.2692}{s^2 + 0.0126s + 0.136}$$

The above Laplace model was used as the transfer function in Fig 6.

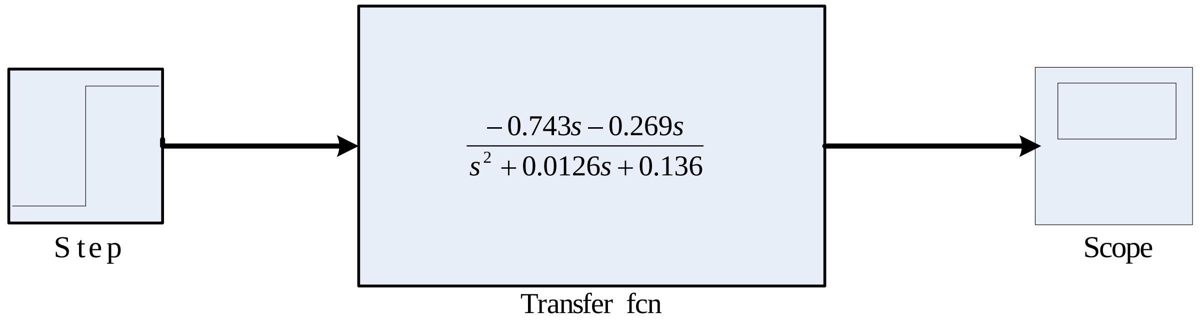

Fig. 6: The Loop implemented in MATLAB for the proposed model

The complete feedback Loop for implementing the PI controller is presented in Fig.7, where the step indicates the set point. The set point allows for an instant switch from a certain initial value (in this case zero) to a given final value (in this case the steady state strength, 39Mpa). The set-point can be set to any value, 39 was used because it was the steady-state value of the actual EWMA.

The tuning parameters and their values were set in the PID box. In this case, the Integral action (decreasing overshoot) which sums the error over time and creates a corrective action proportional to the overall error was set to -0.7, the Proportional part which controls the settling time was set to -5 while the Derivative part was set to 0, using Zeigler - Nichols PID parameter tuning. Thus,

Proportional gain = -5

Integral gain = -0.7

Derivative gain = 0

The transfer function box contains the Laplace transform of the nonlinear regression model and the parameter, s is the Laplace Transform parameter. MATLAB converts the Laplace parameter s to time domain and uses time to calculate the output.

The slider gain measures the output of the system and sends the output to the controller to compare with the set point. It was set to a gain of 1.

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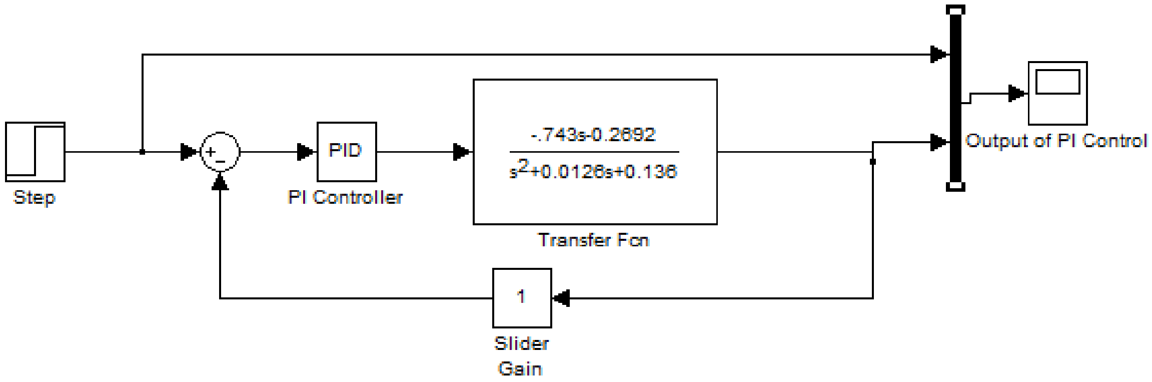

Fig. 8: The complete feedback loop for the PI controller construction

The scope of the PI controller displaying how the system begins from the step input of the controller (in this case 0) to the desired set point which is the overall mean for the strength of cement after 28 days of molding i.e. 39 Mpa (yellow line in the graph) is presented in Fig. 8. How the system overshoots from the onset of the simulation and how it settles overtime as the system continues to adjust itself is shown by the purple line with the integral part of the controller reducing the overshoot and the proportional part of the controller increasing the time it takes for the system to settle. The point where the purple line becomes straight is the point where the system stabilizes and the PI controller parameters become equal.

During the period when the system overshoots and settles, the PI controller tries to adjust the production process to the desired set point. It achieves this by reducing and increasing some of the materials responsible for the cement strength. The Cement Company is expected to experience some loss because the goods produced within this period will vary unpredictably until the system finally settles. At the point where the system stabilizes, the company's production process will be in a state of statistical control producing goods that are at most similar to each other with no or just a little variation which may be negligible.

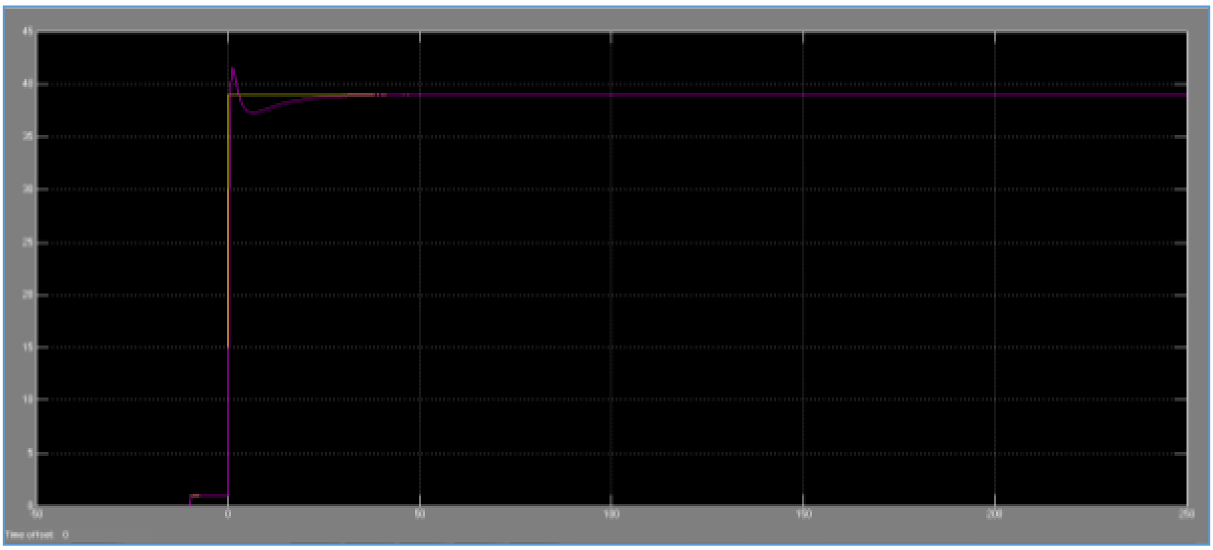

Fig. 8: Screen shot of the output of complete feedback loop implemented in MATLAB

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Predictions for 28 Days Compressive Strength of Cement

Table 4 shows forecast for the 28 days quarterly strength of cement produced by the Cement Company for the year 2016, using the proposed nonlinear regression model.

Table 4: Forecast of quarterly strength of Cement at 28 days for 2016

YEARLY QUARTERS	SERIAL NO	FORCASTED STRENGTH
1ST QTR-2016	21	38.31956
2ND QTR-2016	22	38.57449
3rd QTR-2016	23	38.87998
4th QTR-2016	24	39.19466

The forecasted mean values in Fig. 4, for the four quarters in 2016, using the proposed nonlinear regression model, follow very closely the actual quarterly mean values of cement compressive strength for 2011 – 2015.

CONCLUSION

This paper has demonstrated that quality control of cement compressive strength can be achieved through the combination of some statistical tools. The paper combined nonlinear regression model with control charts to predict the quarterly compressive strength of cement at 28 days.

The strength predictions paved the way for a PI controller, to regulate the 28 days compressive strength of cement around a target mean of 39 Mpa, to be constructed. The outcome of the implementation of the PI controller in MATLAB shows its regulatory stages. Its self-adjusting capability when the process is out of control, will save the Cement Company human and material resources and ultimately build consumer confidence in their product.

The one-year quarterly compressive strength prediction values, using the proposed nonlinear regression model, agrees adequately with the historical data for the years 2011 – 2015.

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