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MULTIPLICATIVE SARIMA MODELLING OF DAILY NAIRA – EURO EXCHANGE RATES

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ABSTRACT: *The time plot of the series DNEER shows an upward secular trend from early December 2012 to early February 2013 followed by a downward trend till end of March 2013. No seasonality is observable. A seven-day differencing yields the series SDDNEER with an overall slightly negative trend. Seasonality is still not discernible. A further (non-seasonal) differencing yields the series DSDDNEER which has an overall horizontal trend. The correlogram of DSDDNEER shows a negative significant spike at lag 7 and comparable spikes at lags 6 and 8. This reveals seven-day seasonality as suspected. It also suggests the involvement of a seasonal moving average component of order one and the product of two moving average components: one seasonal and the other non-seasonal, both of order one. Hence a (0, 1, 1)x(0, 1, 1)$_7$ SARIMA model is proposed. It is fitted and shown to be adequate for the data***.**

KEYWORDS: Daily Naira – Euro Exchange Rates, SARIMA models, Nigeria

INTRODUCTION

Many economic and financial time series are known to be seasonal, this seasonality resulting from the seasonal pattern of economic and financial activities. Foreign exchange rates are amongst such series. Foreign exchange rates in respect of the Nigerian Naira have engaged the attention of many researchers in recent times. A few of the researchers who have concerned themselves with these rates are Etuk(2012), Etuk and Igbudu(2013) and Oyediran and Afieroho(2013). In this work interest is in the modelling, by seasonal autoregressive integrated moving average (SARIMA) techniques, of daily Nigerian Naira – European Euro exchange rates. Etuk(2013) has modelled the monthly exchange rates as a (0, 1, 1)x(1, 1, 1)$_{12}$ SARIMA model.

A SARIMA model was proposed by Box and Jenkins(1976) specifically for series that are seasonal in nature. A few other authors that have written extensively on such models are Madsen(2008), Priestley(1981), Surhatono(2011) and Saz(2011).

LITERATURE/THEORITICAL BACKGROUND

A stationary time series $\{X_t\}$ is said to follow an *autoregressive moving average model of orders p and q* (designated ARMA(p, q)) if

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - ... - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + ... + \beta_q \varepsilon_{t-q} \qquad (1)$$

Here the α's and the β's are constants such that the model (1) is both stationary and invertible and $\{\varepsilon_t\}$ is a white noise process. Model (1) may be put as

$$A(L)X_t = B(L)\varepsilon_t \qquad (2)$$

where $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - ... - \alpha_p L^p$ and $B(L) = 1 - \beta_1 L - \beta_2 L^2 - ... - \beta_q L^q$ and $L^k X_t = X_{t-k}$.

Many real life time series are non-stationary. For such a series Box and Jenkins(1976) proposed that differencing of sufficient degree could render it stationary. Let such a degree of differencing be d. Suppose $\nabla^d X_t$ represents the d^{th} difference of X_t, where $\nabla = 1 - L$. If $\{\nabla^d X_t\}$ follows an ARMA(p, q) then $\{X_t\}$ is said to follow an *autoregressive integrated moving average model of orders p, d and q* denoted by ARIMA(p, d, q).

Suppose the series is seasonal of period s. Box and Jenkins(1976) proposed further that such a series could be modelled by

$$A(L)\Phi(L^s)\nabla^d\nabla^D{}_s X_t = B(L)\Theta(L^s)\varepsilon_t \qquad (3)$$

where $\Phi(L)$ and $\Theta(L)$ are respectively polynomials and $\nabla_s = 1 - L^s$ and D is the minimum seasonal differencing order necessary for the series to be stationary and invertible. Then $\{X_t\}$ would be said to follow a *seasonal autoregressive moving average integrated model of orders p. d, q, P, D, Q and s* designated by $(p, d, q)x(P, D, Q)_s$ SARIMA model.

METHODOLOGY

The data for this work are daily Naira-Euro exchange rates from 8th December 2012 to 30th March 2013 published in the Nation newspaper in the website www.thenationonlineng.net .

The analysis of a realisation of a time series invariably starts with the time plot. If the series is non-stationary and seasonal, seasonal differencing is done. If the differenced series is still non-stationary, further (but non-seasonal) differencing is done. Often $d = D = 1$ and $D + d < 3$. The graph of the autocorrelation function (ACF) called the *correlogram* is used for model identification. A significant spike at the seasonal lag suggests seasonality. If this spike is negative it suggests the involvement of a seasonal moving average component but if positive it suggests the involvement of a seasonal autoregressive component. The orders P and Q might thus be estimated. The cut-off point of the ACF, if any, is indicative of the non-seasonal moving average order q, and that of the partial autocorrelation function (PACF) is indicative of the non-seasonal autoregressive order p.

In particular, a $(0, 1, 1)x(0, 1, 1)_s$ SARIMA model is suggestive if there is a negative significant spike at lag s and comparative spikes at lags s-1 and s+1 (Box and Jenkins, 1976).

After order determination estimation of the model parameters may be carried out. Because of the involvement of items of a white noise process in the model (1), parameter estimation is invariably by non-linear iterative optimization procedure like the least squares approach, the maximum likelihood approach, etc. For pure autoregressive models there are linear optimization procedures like the use of Yule-Walker equations. Similarly there are linear techniques for moving average modelling (See Oyetunji, 1985). However attempts have been made to propose linear optimization techniques for the mixed ARMA models (see for example, Etuk(1989)).

In this write-up the statistical/econometric software Eviews was used for all the analytical work of order determination and parameter estimation. This software uses the least squares approach to model estimation. An initial estimate is made. An iterative process is used, an estimate being an improvement upon its predecessor until the process converges to an optimal estimate, depending on the level of accuracy required.

A fitted model must be subjected to some residual analysis in order to ascertain its goodness-of-fit. Granted model adequacy, the residuals should be uncorrelated and should follow a Gaussian distribution.

RESULTS AND DISCUSSION

The time plot of the realisation DNEER in Figure 1 shows an upward trend from December 2012 to early February 2013. Then there is a downward trend from that time to late March. Seasonality is not so obvious. Seasonal (i.e. seven-point) differencing of DNEER yields the series SDDNEER which exhibits a generally slightly negative trend and no clear seasonality (See Figure 2). Non-seasonal differencing of SDDNEER yields the series DSDDNEER which shows a generally horizontal trend and no clear seasonality (See Figure 3). Its correlogram in Figure 4 shows a negative significant spike at lag 7 and comparable spikes at lags 6 and 8. This is an evidence of a (0, 1, 1) x$(0, 1, 1)_7$ SARIMA model. The model proposed is therefore

$$\text{DSDDNEER}_t = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_7\varepsilon_{t-7} + \beta_8\varepsilon_{t-8} \qquad (4)$$

As summarized in Table 1, this is estimated by

$$\text{DSDDNEER}_t = \varepsilon_t - \underset{(\pm 0.0712)}{0.1802}\varepsilon_{t-1} - \underset{(\pm 0.0215)}{0.9230}\varepsilon_{t-7} + \underset{(\pm 0.0746)}{0.1713}\varepsilon_{t-8} \qquad (5)$$

It is observed that all the estimated coefficients in (5) are statistically significant. Forty five percent of the variation in the data set is explained by the model. In figure 5 the model is seen to very closely agree with the data. The correlogram of the residuals in Figure 6 shows that they are uncorrelated. Finally the histogram of the residuals in Figure 7 shows by the Jarque-Bera normality test that they are normally distributed.

IMPLICATIONS TO THEORY AND PRACTICE

The negative significant spike of the ACF at lag 7 is an indication of seasonality of period 7 which implies that there exists a tendency for the rates to rise and fall on weekly basis. Moreover since time series analysis has the ultimate purpose of forecasting the SARIMA model (5) may be used as basis for forecasting daily Naira - Euro exchange rates.

CONCLUSION

It may be inferred that daily Naira-Euro exchange rates follow the (0, 1, 1)x(0, 1, 1)$_7$ SARIMA model (5). This modelled has been shown to be adequate.

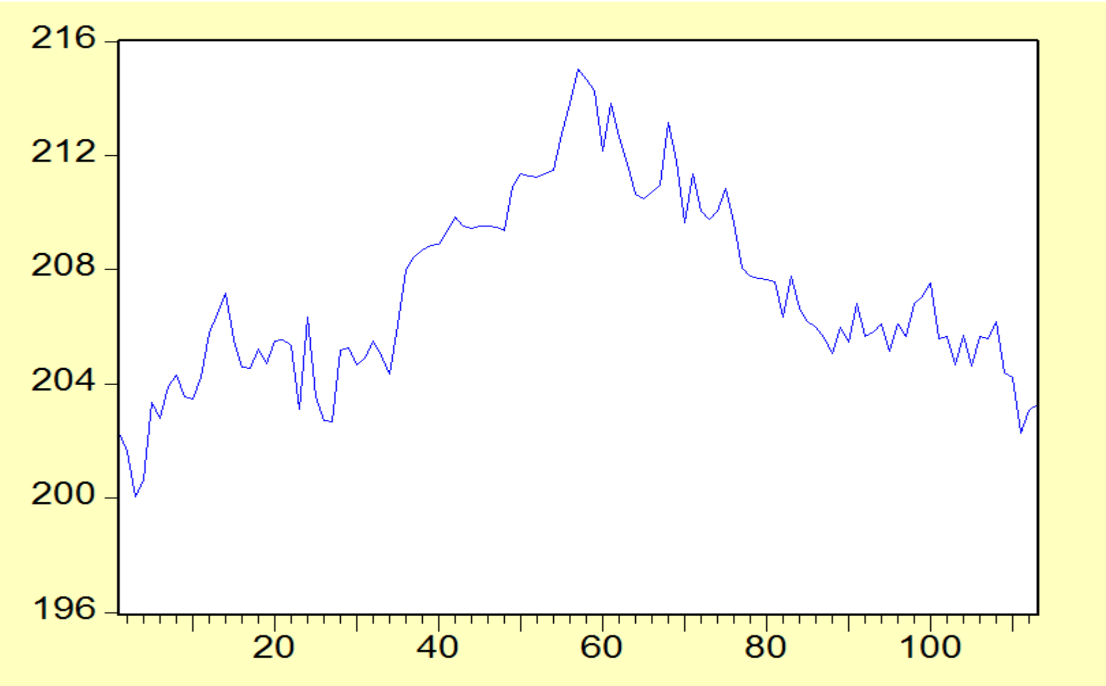


FIGURE 1: DNEER

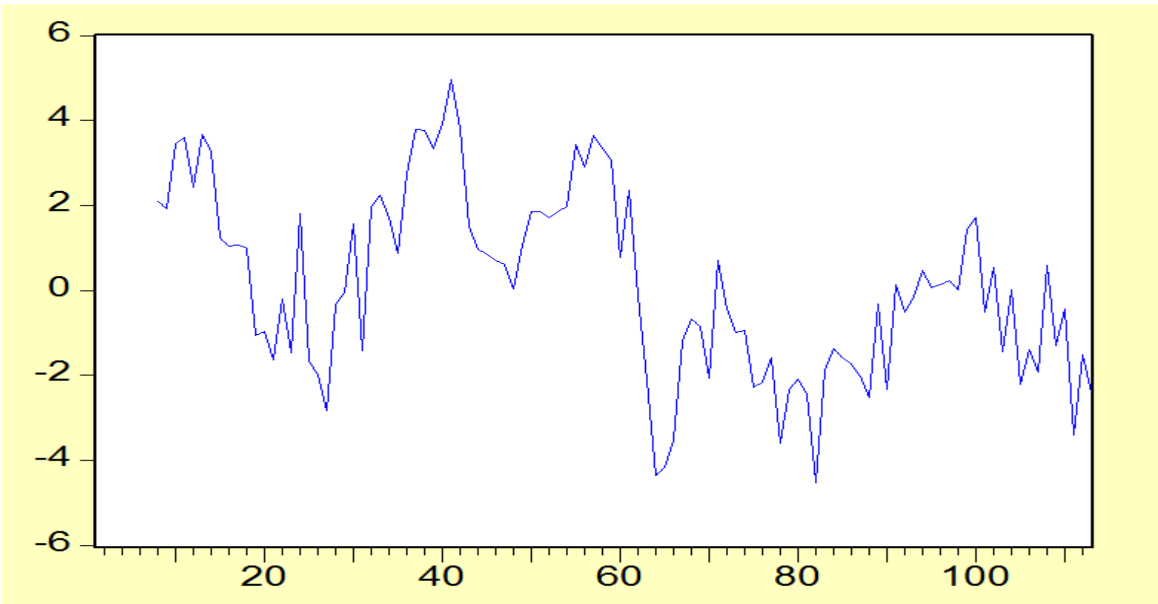
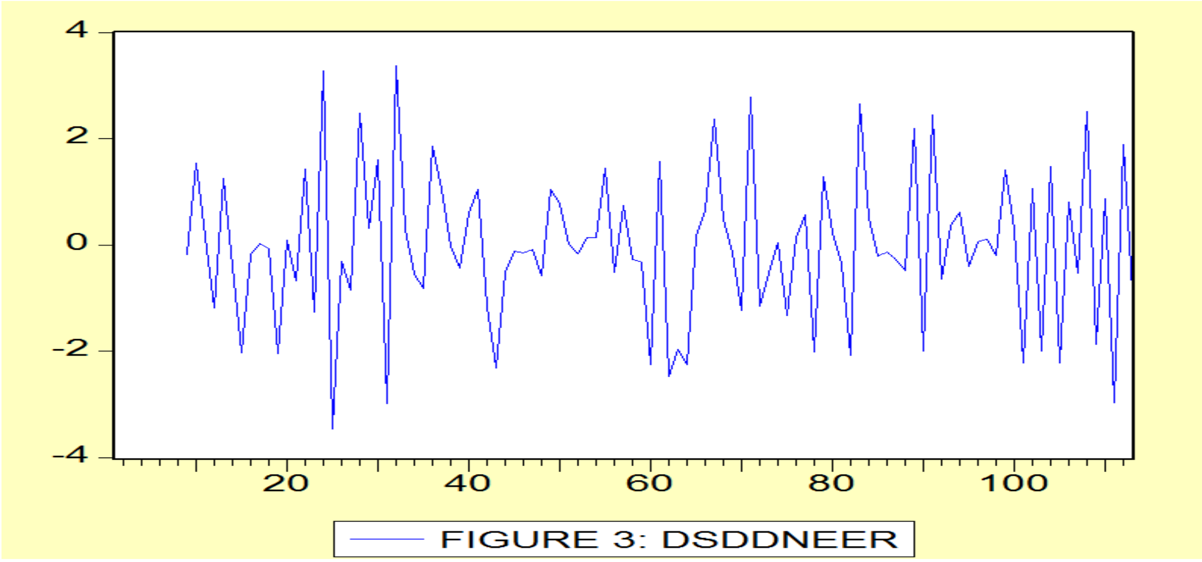
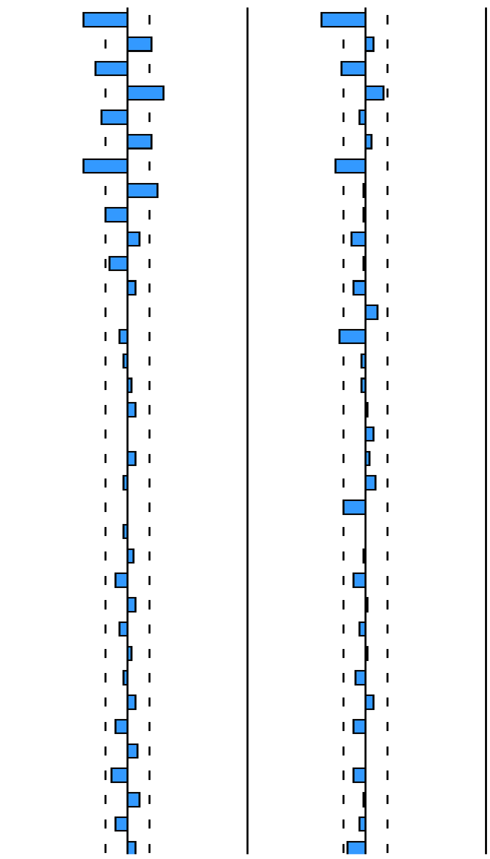


FIGURE 2: SDDNEER





	AC	PAC	Q-Stat	Prob
1	-0.386	-0.386	16.090	0.000
2	0.210	0.071	20.886	0.000
3	-0.280	-0.210	29.531	0.000
4	0.305	0.159	39.873	0.000
5	-0.232	-0.057	45.938	0.000
6	0.206	0.057	50.751	0.000
7	-0.382	-0.269	67.472	0.000
8	0.258	-0.016	75.163	0.000
9	-0.184	-0.023	79.133	0.000
10	0.102	-0.120	80.365	0.000
11	-0.156	-0.009	83.273	0.000
12	0.080	-0.112	84.051	0.000
13	-0.001	0.099	84.051	0.000
14	-0.063	-0.225	84.545	0.000
15	-0.028	-0.031	84.646	0.000
16	0.035	-0.027	84.803	0.000
17	0.058	0.008	85.231	0.000
18	0.005	0.063	85.235	0.000
19	0.064	0.039	85.776	0.000
20	-0.037	0.085	85.960	0.000
21	-0.001	-0.185	85.960	0.000
22	-0.028	0.002	86.067	0.000
23	0.053	-0.013	86.446	0.000
24	-0.111	-0.101	88.162	0.000
25	0.061	0.022	88.685	0.000
26	-0.070	-0.053	89.391	0.000
27	0.027	0.011	89.494	0.000
28	-0.026	-0.088	89.594	0.000
29	0.065	0.067	90.226	0.000
30	-0.106	-0.099	91.917	0.000
31	0.094	-0.001	93.261	0.000
32	-0.132	-0.101	95.923	0.000
33	0.106	-0.018	97.666	0.000
34	-0.111	-0.050	99.616	0.000
35	0.064	-0.162	100.26	0.000

Figure 4: Correlogram of DSDDNEER

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Table 1: Model Estimation

Dependent Variable: DSDDNEER
Method: Least Squares
Date: 04/02/13 Time: 17:54
Sample(adjusted): 9 113
Included observations: 105 after adjusting endpoints
Convergence achieved after 14 iterations
Backcast: 1 8

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	-0.180246	0.071224	-2.530702	0.0129
MA(7)	-0.922971	0.021457	-43.01532	0.0000
MA(8)	0.171280	0.074607	2.295763	0.0237

R-squared	0.452871	Mean dependent var	-0.043067
Adjusted R-squared	0.442143	S.D. dependent var	1.419439
S.E. of regression	1.060177	Akaike info criterion	2.982904
Sum squared resid	114.6455	Schwarz criterion	3.058732
Log likelihood	-153.6025	F-statistic	42.21380
Durbin-Watson stat	2.045932	Prob(F-statistic)	0.000000

Inverted MA Roots	.99	.62+.77i	.62 -.77i	.19
	-.22+.96i	-.22 -.96i	-.89+.43i	-.89 -.43i

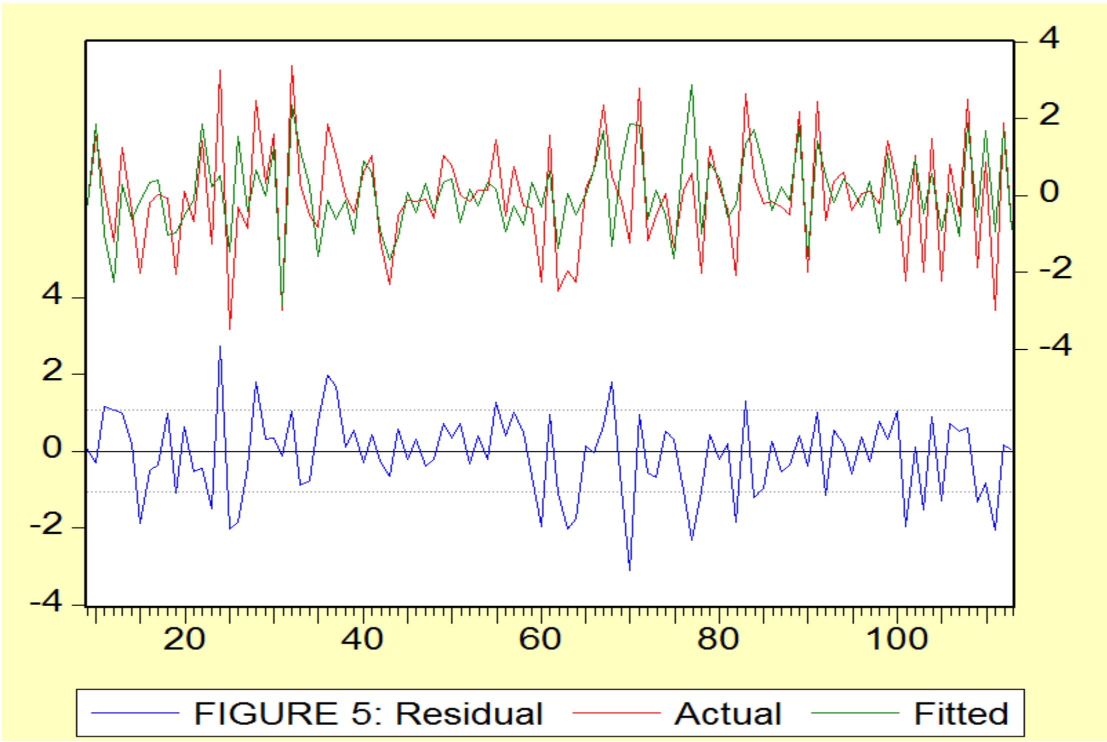

FIGURE 5: Residual Actual Fitted

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.033	-0.033	0.1166	
		2	-0.002	-0.003	0.1173	
		3	-0.150	-0.151	2.6073	
		4	0.119	0.112	4.1789	0.041
		5	-0.160	-0.161	7.0437	0.030
		6	0.099	0.080	8.1455	0.043
		7	0.084	0.119	8.9520	0.062
		8	0.173	0.126	12.404	0.030
		9	-0.142	-0.082	14.758	0.022
		10	0.068	0.065	15.306	0.032
		11	-0.063	-0.028	15.780	0.046
		12	0.021	-0.017	15.832	0.070
		13	-0.011	0.058	15.847	0.104
		14	-0.073	-0.182	16.497	0.124
		15	0.028	0.053	16.597	0.165
		16	-0.016	-0.045	16.631	0.217
		17	0.103	0.112	17.992	0.207
		18	0.067	0.093	18.571	0.234
		19	0.052	0.040	18.927	0.272
		20	-0.054	-0.008	19.308	0.311
		21	-0.043	-0.038	19.551	0.359
		22	-0.028	0.038	19.657	0.415
		23	-0.020	-0.099	19.711	0.476
		24	-0.022	-0.003	19.778	0.535
		25	0.103	0.010	21.274	0.504
		26	-0.021	-0.043	21.337	0.560
		27	-0.047	-0.033	21.655	0.600
		28	-0.075	-0.062	22.468	0.609
		29	0.002	0.008	22.469	0.663
		30	-0.060	-0.065	23.004	0.685
		31	0.000	0.039	23.004	0.733
		32	-0.007	-0.038	23.011	0.776
		33	0.054	0.050	23.459	0.796
		34	-0.101	-0.086	25.084	0.764

Figure 6: Correlogram of the Residuals

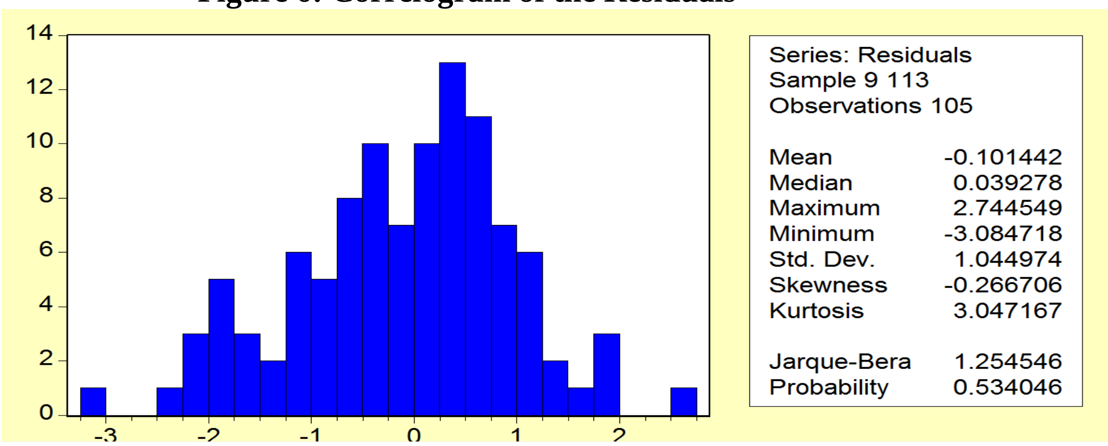

Figure 7: Histogram of the Residuals.

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