LOOP RATING CURVE OF TIDAL RIVER: FLOOD AND TIDE INTERACTION

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ABSTRACT: *For a steady flow, the rating curve is unique for a non-erodible section where the flow is uniform, but it is a loop for an erodible section when the flow is non-uniform. For an unsteady flow, the rating curve does exist, but it is more complicated, Relationships among water level, flow velocity, and discharge are all affected by flood hydrograph of upstream or tide from downstream. A more general relationship between water level and flow velocity (or discharge) with flood hydrograph from upstream and tide from downstream for time-dependent is derived analytically from diffusion equation and continuity equation. The upstream and downstream boundary conditions are expressed in terms of harmonic functions rather than a step function. The analytical solutions are compared with the numerical results obtained by using finite difference model with implicit scheme based on the complete Sanit-Venant equations for unsteady flow in open channel. It is found from the study that: the peak flow times at different locations are shifted due to the kinematic wave velocity; therefore, the rating curves at different locations are spread, not complete a loop like peacock tail feather.*

The rating curves for the subcritical flows are below the line of $y^* = q^*$*, and the supercritical flow rating curves are above* $y^* = q^*$*. The dimensionless amplitude due to downstream tide is still a function of time not only function of position. The comparison between the analytical results and numerical results are in good agreement, not only for the weighting factor,* $P_t = 0.70$*, but also for* $P_t = 0.50$*. This analytical model can be used without any sophisticated computing machine; in fact, a simple desk calculator and a table of error function are sufficient in carrying out the computation based on the analytical solution.*

KEYWORDS: Diffusion Equation, Dimensionless Amplitude, Flood-Tide Interaction, Kinematic Wave Velocity, Loop Rating Curve, de Sanit-Venant Equations, and Tidal River.

INTRODUCTION

Unsteady flows are found in estuaries due to ocean tides, and rivers due to floods, in the form of translatory wave motion. Translatory waves are gravity waves that propagate in the channel and result in appreciable longitudinal displacement of the water particles. The terms "unsteady flows" and "transient flows" are used interchangeably to denote both temporal and spatial variations in depth and in the velocities of the water particles. An accurate prediction of flood wave propagation in a natural river may be a difficult task, especially for engineers to predict levels and discharges for a variety of flows. Tides are created by astronomical forces. In rivers which are directly connected with an ocean, the variation of the water surface is considered not only a direct response to such forces, but also a succession of tides created at river mouth by the ocean and progressing upstream interaction with freshwater discharge from the catchment. It will result in a very complicated flow regime in estuaries due to the combined effects of tides and river flows. The occurence of high tides downstream will create high mean water level upstream as well. If a high discharge prevails during an interval of high tides, the risk of upstream flooding will be increased.

The Chao Phraya River is the most important river in Thailand (Figures 1.1 and 1.2). There are so many people along the river, and the capital city of Thailand, Bangkok, is located on the bank of the river. It causes damage to properties of both public and private sectors in large amount of money at each flood time, such as the damage occurring in 2011 as Figure 1.3 (Daisuke Komori *et al*, 2012 snd Graham Emde, 2012). It is obvious that the two main causes of flooding are the high discharge from the upstream and high tides in the Gulf of Thailand. The high tides make the flooding more severe since the freshwater discharge to the sea is substantially retarded causing amount of water to inundate the inland area especially in the region near Bangkok.

Therefore, it is essential to investigate and evaluate quantitatively the interaction of tides and floods to get a better rating curve of depth and velocity or depth and discharge in order to make a good prediction of the flood and damage not only for the flood protection, but also for the social development and the life level arisen.The purpose of the study is to construct the rating curves for unsteady flows based on the basic continuity equation and equation of motion using tides as the downstream boundary condition, and flood wave as

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the upstream condition. Steps of Establishing the Rating Curves for Combination of Upstream and Downstream Effects are as follows:

(1) Constructing the phase of upstream effect by translating the depth y –time (t) into discharge q – time (t) relation curve to form a velocity-stage rating curve for each station.
(2) Constructing the phase plane of downstream effect by translating the water surface elevationη– t and water particle velocity in river flow direction u – t curves intoη- u rating curve each station.
(3) Combining the two phase planes together to form a useful and practical phase plane to
be used to predict the tide and flood wave effects during the flood coming periods.

Most of the results will be derived from analytical solutions which are applicable for simple geometry of river and estuary. Numerical results will be computed for some cases in order to verify the analytical solutions.

Figure 1.1 Geographical Description of the Chao Phraya River Basin

Figure 1.2 An Overview of the Gulf of Thailand

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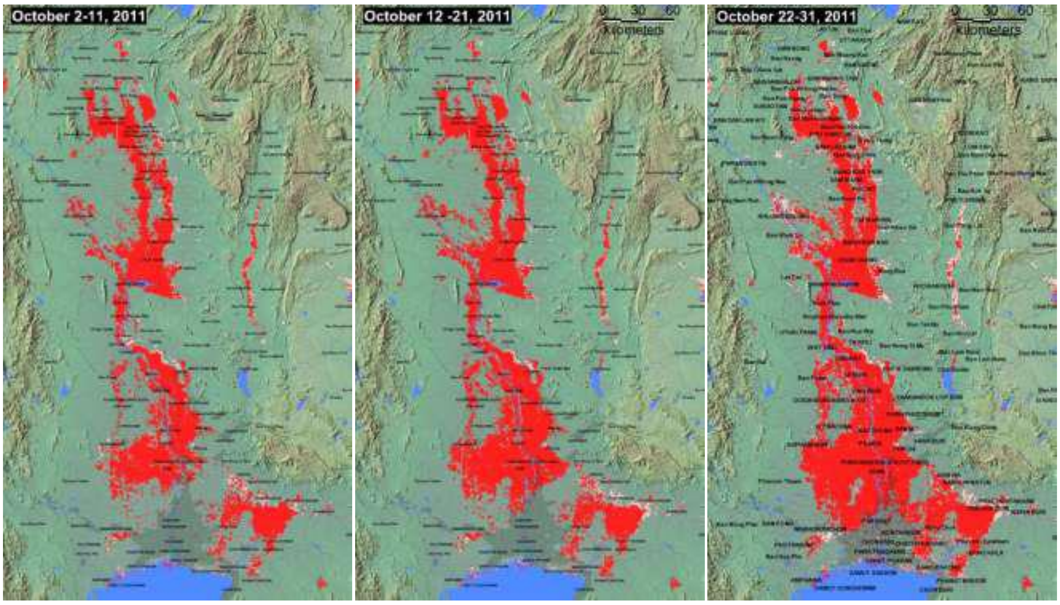

Figure 1.3 Chao Praya River Basin Flood extents 2-11, 12-21 and 22-31 October 2011 (Source: Darmouth Flood Observatory)

LITERATURE REVIEW

In many streams and rivers, flows can be accurately and economically determined using steady-flow concepts. However, hydraulic engineers frequently assume that the regulated flows occurring in some rivers and the quasiperiodic, unsteady flows prevailing in estuaries and pseudo-steady with time. While permitting simplifications which are conceptually and analytically appealing, such an assumption fails to provide a sound basis from which to fully analyze and accurately determine flows that are, in fact, transient in character. The increasing demands by modern -day society for accurate information on the unsteady flows occurring in rivers and observed in estuaries have focused attention on the need for thoroughly understanding the dynamics of unsteady flow (or transient flow) in such waterways.

Because of the difficulty encountered in attempting to determine unsteady flows in rivers and estuaries, research into the dynamics of transient open channel flows is necessary. For this purpose, the first step we need is to thoroughly explore and understand the hydrodynamics of transient flows in rivers and estuaries, and then, to develop accurate, reliable, and economically practical techniques with which to devise the systematic approach for determining or predicting such flows based on tide as the downstream

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boundary condition and flood wave as the upstream boundary condition. Up to now, even there have been several studies the relation of water depth (or elevation) and time or velocity of propagation with time, yet they were separately investigated. This study intends to make use of the availability of tide and flood wave as the boundary conditions to establish the phase plane or the unsteady rating curve.

The partial differential equations describing open-channel unsteady flows are briefly review. It is assumed that the flow in the channel is of substantially homogeneous density, that the velocity is uniform over any cross section, and the hydrostatic pressure prevails at any point in the channel. The channel is assumed to be sufficiently straight; the reach geometry to be sufficiently simple; and the channel slope to be sufficiently mild and uniform throughout the reach. The friction-resistance coefficient that is used with unsteady flow is assumed to be the same as that for steady flow, and, hence, can be approximated from the $Chezy$'s or $Manning's$ formula.

The general open-channel transient flow equations, then, can be obtained by the use of the laws of continuity and momentum by considering an element of water that is boundary by two vertical cross sections.

The equation of continuity may be obtained by considering the influx, efflux, and the accumulation of mass in the element. If the water surface elevation, z, and the discharge, Q, are used as two dependent variables, the equation of continuity can be written in the form:

Continuity Equation

$$B\frac{\partial z}{\partial t}+\frac{\partial Q}{\partial x}-q_L = 0 \qquad (2.1)$$

in which B and q_L, are the surface width and the lateral flow per unit length (positive for inflow, negative for outflow), respectively. The distance, x, in the longitudinal direction on a horizontal datum plane and the elapsed time, t, are used as two independent variables.

Equation of Motion of One-Dimension

Applying Newton's Law for momentum to one-dimension flow through the element of water, the equation of motion can be obtained as:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial z}{\partial x}+\frac{gn^2u|u|}{R^{4/3}}+\frac{q_L u}{A}=0 \qquad (2.2)$$

or

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$$\frac{\partial Q}{\partial t} + \left(\frac{Q}{A}\right)\frac{\partial Q}{\partial x} + gA\frac{\partial z}{\partial x} + \frac{gn^2 Q|Q|}{AR^{4/3}} + q_L\left(\frac{Q}{A}\right) = 0 \ldots\ldots\ldots\ldots\ldots\ldots \text{(2.3)}$$

in which g, A, and R are the acceleration of gravity, the cross sectional area, and the hydraulic radius, respectively; and n is the function of flow resistance coefficient, and the rating curve for unsteady flow without consideringω, kinematic wave velocity, andμ, dispersion parameter due to flood wave, could be presented as Figure 2.1.

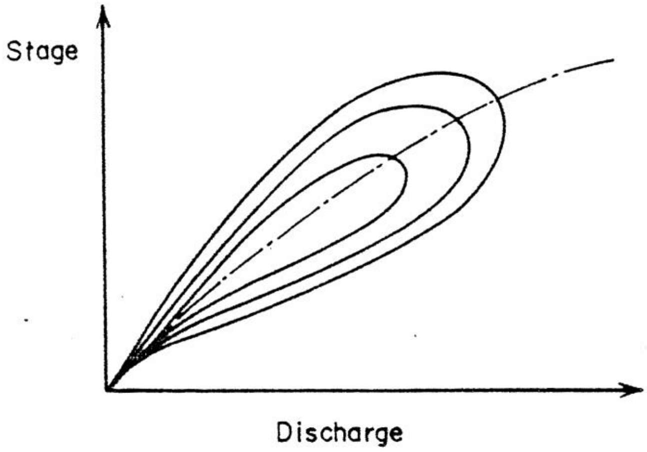

Figure. 2.1 The Loop-Rating Curve, Bird Feather Envelope, by JONES FORMULA

Characteristics of Flood Wave

First, let's consider the phenomena of flood wave moving from upstream to downstream in a river. From equation of continuity and equation of motion, by assuming the acceleration and added momentum terms to be negligible, we can obtain an equation with depth as the only dependent variable. Thus, an alternative routing procedure results from the recognition of the similarity of the flood wave profile to the transient concentration distribution curve for a mass of material diffusing in a streamflow . In its one dimensional form of the wave phenomena is described in term of flow depth by this equation:

$$\frac{\partial y}{\partial t} + \omega\frac{\partial y}{\partial x} = \mu\frac{\partial^2 y}{\partial x^2} \ldots\ldots\ldots\ldots\ldots\ldots \text{(2.4).}$$

where y is the flow depth ; x and t are the respective distance and time co-ordinate ; ω andμ are dispersion parameters.

In Eq. (2.4), the first important step we want to do is to determine the flood character and

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river basin character, ωandμ, then to solve the equation by analytical or numerical method.

HAYAMI (1951) was the first one to obtain a diffusion solution for turbulent flow in a prismatic rectangular channel with zero lateral flow. He found the disturbance on the flow caused by the channel irregularities damp away within a few kilometers and had certain limited heights and durations. He also introduced the effect of longitudinal diffusion caused by the mixing into the equation of continuity and assuming the mean flow taken over a suitable range to be steady and uniform.

HENDERSON (1966) identified ω as the velocity of kinematic flood wave and μ as the diffusion coefficient which was a function of depth and gradient of the water surface. He stated that the parameters ωandμwould be better regarded as lumped measure of the convective and diffusive characteristics of a particular river reach.

BALTZER, MOZAYENG (1969) presented the method of characteristics to solve the equations of continuity and momentum in a semi-infinite rectangular open channel to form the rating curve for unsteady flow at different locations due to sinusoidal stage change upstream. The peak stage and discharge were function of the flood amplitude, channel slope, *Manning's* friction coefficient, and the distance from the entrance section and formed as the exponential function. The linearity coefficients were identical and became constant for large distance. The results were compared with the numerical models.

SUTHERLAND and BARNETT (1972) extended the diffusion solution given by HAYAMI to apply moderately irregular channels, which vary in width and slope, with lateral inflows. The extended solution predicted that at a given time from the initiation of such a disturbance, the change in stage at a given point in the channel would bear a constant ratio to the difference between the initial and the final steady stages at that point. Channel constants derived by means of a calibration flood wave were used to successfully predict the stage-time curve for a test flood below an artificial control in a natural channel.

PRICE (1973) presented a method to calculate 00 accurately from the records of the speeds of previous flood peaks along the river. But the calculation of pi could be more

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difficult. And ω and μ could vary with the magnitude of flood significantly.

He suggested that it was preferable to define curves for ω and μ as function of discharge, which can be done by correlation values of o, u calculated for a number of recorded floods with the average peak discharge the reach in each case. Variable parameter diffusion (VPD) method was proposed by him to overcome the difficulty due to the uncertainty in the values of CO and Pl – VPD method was further stated that the linear diffusion model is as good as the VPD method except in predicting the shape of the hydrograph, where VPD method was found to be marginally more accurate.

WILFRIED BRUTSAERT (1973) said that the solution of the linearized case of de Saint Venant equation was still of interest because it provided some insight into the coexistence and the nature of kinematic and dynamic waves. Moreover, a comparison of solution of dynamic system with that of the diffusion equation gave an indication of criteria for validity of diffusion approximation and the kinematic wave approximation. The modified Bessel function and the *Green's* function were suggested useful for deriving the input and output functions or spectra.

KEEFER and McQUIVEY (1974) put forward to a multiple linearization technique which were used to overcome the limitations of single linearization models. Significant visual improvement in the timing of low flows were noticed. For most cases of practical interest the diffusion analogy worked well with multiple linearization.THOMAS N. and KEEFER A. M. (1974) obtained a multiple linearization technique to offer a useful and in expensive improvement over *Harley's* linear channel response model to a single and rapid calibrated linearization technique in one-dimensional convolution flow routing.

TINSANCHALI and MANANDHAR (1985) developed an analytical diffusion model for flood routing. The results obtained by applying the model to a hypothetical rectangular channel checked very well with those obtained by using finite difference model with implicit scheme based on the complete Saint-Venant equations for unsteady flow in open channel. The model developed provided a simple, rapid, and accurate means of tracing the course of flood waves resulting from the fluctuation in the stage at upstream, downstream ends, and the lateral discharge. Each of the effects of the upstream, the downstream, and the tributary inflow could be routed when any other two of them were under the same boundary conditions. This diffusion model also could be routed by using the daily data, and used without any sophisticated computing machine. The model was also advantageous in that the computation time required was less than that required for the finite difference model. It was applicable for the river reach where there was moderate backwater effect.

CHRISTINA W. and TSAI A. M. (2003) examined the applicability of the kinematic wave, non-inertia wave, and quasi-steady dynamic wave approximations to the full dynamic wave equations for unsteady flow routing by comparing the propagation characteristics of a sinusoidal perturbation to the steady gradually varying flow for different simplified wave models. Development of the applicability criteria provided a guideline for selecting an appropriate wave model for unsteady flow modeling, thus enabling an assessment of the capabilities and limitations of different simplified wave models.

CEVZA MELEK KAZEZYILMAZ-ALHAN and MIGUEL A. MEDINA Jr. (2007) expressed MacCormack method a particularly well suited to approximate nonlinear differential equations. The analytical solutions provided the practicing engineer with computational speed in obtaining results for overland flow problems. For large scale catchment-stream problems, the verified numerical methods provided efficient and accurate algorithms to obtain solutions. Both the analytical approaches and the MacCorcack algorithm were used to solve the same synthetic examples

Characteristics of Tide

Now, let's review the propagation of tide in canal from river mouth to upstream with linear frictional force. From continuity equation and equation of motion, we can obtain:

$$C_0^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} + gM \frac{\partial u}{\partial t} \quad \text{(2-5)}$$

or

$$C_0^2 \frac{\partial^2 \eta}{\partial x^2} = \frac{\partial^2 \eta}{\partial t^2} + gM \frac{\partial \eta}{\partial t} \quad \text{(2-6)}$$

where C is the celerity of shallow water wave, $M = 8u_{max}/3\pi C_0^2 R$, $\eta$is the water surface elevation. Ce is the Chezy's coefficient.

"Green Law" was one of the earliest models to explain the origin of many of the most

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striking distortions suffered by the tide as it propagated into shallow water. It related geometrical amplification to the conservation of wave energy flux in a non-dissipative system.

EINSTEIN and FUCHS (1954–1955) made a survey of past and present calculation methods during World War II which were used for the prediction of tidal stages and flows in canals and estuaries. Main purpose of this study was the evaluation of the methods of calculation to various practical problems and the choice of one or more preferable calculation methods for various problems. Most solutions depended on the introduction of various simplifying assumptions which might or might not be permissible in any particular application. The various calculation methods referred to later were shown to describe only partially the complicated flows such as so called " Parsons' Harmonic Theory" :

$$\frac{\partial(uH)}{\partial x} + \frac{\partial H}{\partial t} = 0 \quad \text{(2-7)}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial H}{\partial t} + fu = 0 \quad \text{(2-8)}$$

where f is the linear frictional coefficient which is in general a function of x but it 's often averaged over a given reach for convenience of calculation. Based on the fact that all the waves under cons deration are smooth and very long for their length, the result differential equations after simplification could be shown as (Figure 2-1).:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad \text{(2-9)}$$

and

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \eta}{\partial x} + \frac{gu|u|}{C_c^2 R} = 0 \quad \text{(2-10)}$$

He also considered and sought solutions of these governing equations for different channel configurations, with and without frictional effect for channel of uniform depth varying breadth and uniform depth breadth but varying depth by keeping linear frictional coefficient f constant all the cases which was not always true since f varies with depth and may not be constant throughout the channel length.

IPPEN (1966) considered tidal propagation into estuaries of rectangular sections. He derived tide phenomena for different situations of channel firstly without consideration of energy dissipation and proceeded to include frictional effects. From the basic wave

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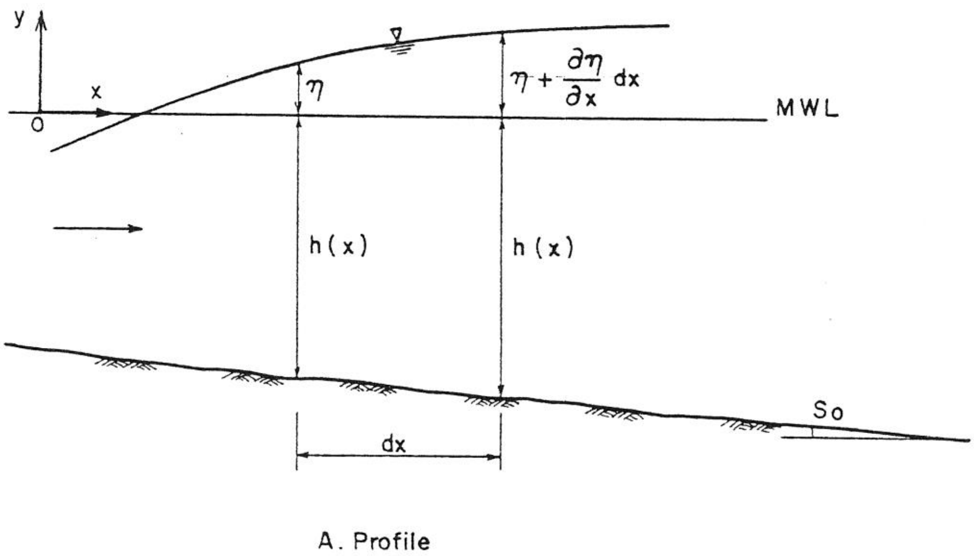
equation, be linearized them and assumed for small amplitude of the tidal waves. Hence,



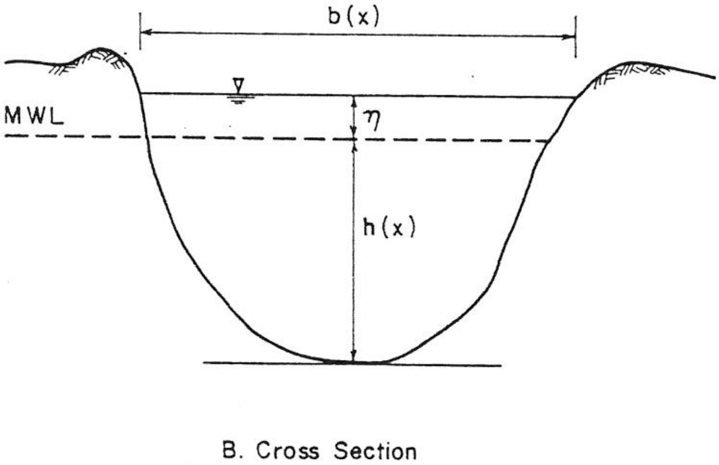


Figure. 2.1 Definition Sketch of an Element of River

The two Eqs. (2-5) and (2-6) were obtained.

The solutions were obtained in the simple mathematical forms by applying the water surface elevation, η, corresponding with the different configurations of channels. LeBLOND (1978) 2xamined the flow regime which was relevant to tidal propagation in shallow river from the scaled equations which derived from the hydrodynamic governing

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equations by introducing scaling variables. He considered for one-dimension tidal propagation with narrow rectilinear channel of uniform depth and width, took into account the downstream freshwater discharge as well. After reexamination of the momentum balance in shallow rivers with scaling appropriate to the Saint Lawrence and the Fraser in the study, he found the frictional forces exceeded accelerations over most of the tidal cycle. Consequently, tidal propagation in shallow rivers is more properly envisaged as a diffusion phenomena than as a wave propagation phenomena. The long time lags associated with low Waters which were unexplainable in terms of a simple wave propagation model, were easily accounted for by an equally simple diffusion model. This simplification was not appropriate near high-water slack when the current and hence the friction forces vanished.

GODIN (1982) studied the effect of an increased discharge on the propagation tides into a channel both on its amplitude and on its timing. He found the relation between the tide and the dis charge evidently, especially for the friction and the freshwater discharge intrinsically linked since friction is felt only when there are currents flowing. Under the theoretical considerations, using the hydrodynamic equations in one dimension form, he considered the current made up of a steady component created by freshwater discharge U_f and a time dependent component contributed by the tide, U(t).

In the downstream region where the current alternates and where the tidal is considerably larger than freshwater discharge velocity, U(t) much greater than U_f , be neglected the convective term which is relatively unimportant compared with the friction term. While for the upstream region where the current no longer alternates and U_f much greater than U (t), he linearized the equations and solved a single tidal component only. Inference from his study are, for upstream region tidal range is reduced by an increased discharge, the time of arrival of low water is accelerated while high water is retarded, the changes in range and in time can be represented by simple regression relations; for downstream region 3 Il increased discharge causes a decrease in the effective friction during flood while increase during ebb, therefore low water is retarded but high eater is accelerated. And higher frequency components of tide will propagate upstream more rapidly than the lower ones.

Studies of the Chao Phraya River

The Chao Phraya River is the major river in Thailand. During the rainy season, the flood

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superposes the tides in the lower Chao Phraya basin. The lower reach of the river is relatively flat to retain water for quite a long period. The duration of flood which will affect directly or indirectly the social activities as well as life and business and properties of the areas should neither be overlooked nor neglected. There are already several studies of this river which are reviewed as followed.TORRANIN (1969) had developed a tidal mathematical model by using the finite difference method of implicit scheme to study the flow in the Chao Phraya River. The results obtained from the study showed an acceptable agreement with the observed data.

VATCHARASINTHU (1977) had furthered these studies to include flood protection. Several flood protective schemes were studied, such as dikes along the river banks, a diversion channel at Bangsai, and a tidal barrier at km 28. A mathematical model was developed in using the finite difference approximation developed by Rossiter, J. R. and Lonnon, G. W. (1965) to simulate the flow in the Chao Phraya River which could be used as a mean to forecast water elevations, discharge along the river so as to reduce the tangible and intangible losses due to flooding in the future. At the same time, he also investigated the reach of tidal effect to present the relations among the river discharge, water depth and velocity. They were strongly affected by the river discharge. These were useful for the design or management of engineering works. The transition flows on berms at some location before it was completely inundated was also studied by using dam break techniques and the duration of flood on the flood plains. By assuming that at some locations overflow and net inflows exist, De Marchi and Houma equations had been adopted to study the effect of these variations of flow which affected the flow in reach of river under consideration. From the results of this study, we notice that simple and uniform channel geometry of the river were recommended, and Manning 's roughness n had a little effect on calculation of water surface elevation due to the building, such as houses, roads, and the grown trees increasing the value of n . The initial conditions had no significant effect on the computation of stage and discharge hydrograph so long as they started with some reasonable initial values and an adequate run-in time was allowed to elapse before output was accepted. The reasonable initial values might be the initial stages obtained by consulting the tidal gauges along the length of the estuary and by interpolating the water surface elevations at the section. The initial discharges used were those which had been assumed to have a constant discharge at every grid point. This was very important from the practical point of view because flow conditions measured simultaneously did not exist. In the calculation of reach of tidal effect, it was founded that

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in the low period of flow, tidal fluctuation could be observed as far upstream as 152 km from the river mouth. The head of the tide moved 3 downstream as the discharge increased. At a discharge of 4,000 cms no tidal influence could be observed beyond 56 km.

PHUC (1985) studied the interaction of tides and river flows in the estuary flow regime when tidal waves propagated into rivers. The interaction of each constituent with freshwater discharge was investigated, especially consideration for the lower reach of the Chao Phraya River. He applied the method of Harmonic Analysis to solve for the main constituents and analyzed the deformation of each constituent by the current. The result were then compared with the existing analytical solutions for interaction of wave and currents by Brevik and Aas (1980) . He found that the propagation and attenuation of tidal waves were affected by freshwater discharge evidently in flood season where the friction and water depth of river increase considerably and resulted in reduction of wave amplitudes associated with increasing of freshwater discharge due to more energy dissipation of tidal waves. The reduction of tidal amplitude in flood season still could not be formulated in a simple way by theories developed by Brevik and Aas (1980) due to many factors governing the phenomena. The friction of the river, geometrical changing of depth or width of the river, etc., still were not taken into account appropriately. Besides, he found that each constituent of tides was conserved and slowly attenuated when propagated upstream in dry season and had larger amplitude and propagation velocity than in flood season. While during flood, the amplitude of each component was quickly reduced. The diurnal tides were attenuated slower than the semi-diurnal tides but propagated upstream less rapidly.

ROJ ANAKAMTHORN (1986) solved the governing equations by analytical approximation method of Perturbation to investigate the interaction of tide and freshwater discharge of the Chao Phraya River. . The convective and nonlinear friction terms were also included in the derivation. Harmonic Analysis was applied to de compose the complicated interaction of the freshwater discharge with various constituents of tides into its individual interaction with each constituent. Four main constituents, Ki , O4 , M ; , S2 , were included in this study. The relations of important dimensionless parameters of the tide, especially the dimensionless damping modulus, were then determined for each solution. The partial differential equations defining the problem were solved corresponding to each power of small perturbation parameter. They were considered into

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three cases: firstly without friction and horizontal bed; secondly for linear friction; and finally for nonlinear friction. They results were obtained by straight-forward mathematical derivation. For the first case, it was shown that the solutions were diverged after a sufficient long time elapse when considered up to second order because the second order solution was a function of time. For linear friction, the solutions were expressed in Bessel function forms in which the initial amplitude of tide at river mouth was highly attenuated with time and the solutions did not show explicitly the individual interaction for each constituent of the tide with freshwater discharge. The relations of dimensionless parameters of damped tides were achieved associated with each assumed solution. It was seen from the expressions that propagation and attenuation of tides were affected evidently by freshwater discharge. The discharge caused the friction of the river and mean water level to increase considerably which resulted in more energy dissipation of tides. In other words, the amplitudes of tides were reduced following the increasing of freshwater discharge. For high discharge due to the weak point in approximation method of Perturbation, the obtained dimensionless damping modulus from analytical solutions were then modified. The results from short time duration analysis showed more accuracy in prediction of tides during high freshwater discharge fluctuation.

Other Review

GERFOV (1971) presented his studies which was about the determination of the loop discharge ration curve for flood wave propagation. The purpose of this article was to suggest a new method of determination of the loop discharge-rating curve by direct computation. An unsteady flow was usually studied as a process of flood wave propagation and its modification along rivers, but the studies differed from this by treating the changes of the hydraulic characteristics of the unsteady flow at a given point. There were some assumptions:

(1) The water movement is two dimensional.

(2) The hydraulic characteristics gradually variable with continuous derivatives along the river's reach.

(3) The resistance forces are similar to those in a steady flow.

(4) The morphometric characteristics of the river reach are almost permanent.

The practical application of the result formula was by graphical differentiation of the flood wave hodograph in order to obtain the values of the term $\Delta h / \Delta t$. It was not very

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difficult to plot the loop discharge-rating curve for a flood wave with a high degree of accuracy by means of the proposed method. It also resulted in a reduction in the amount of hydrometric discharge measurement required, especially during flood wave period. At the same time, the correct form of the upper part of the curve was determined. An article about the propagation of dynamic waves in open channel flow. The analysis that followed endeavors to apply the theory of linear stability to the sets of equations governing the motion in open channel flow. The conclusions relate to the celerity and attenuation functions of dynamic waves, expressed in terms of the Froude number of the steady uniform flow $F = u/\sqrt{gh}$, and a dimensionless wave number of the unsteady component of the motion. From equation of motion,

$$S_f = S_0 - \frac{1}{g}\frac{\partial u}{\partial t} - \frac{u}{g}\frac{\partial u}{\partial x} - \frac{\partial h}{\partial x} \qquad (2\text{-}11)$$

the wave number spectrum was divided into three bands :(1) a gravity band corresponding to large wave number, where the wave celerity was the gravity wave celerity; (2) a kinematic band corresponding to a small wave number, where the wave was the kinematic wave celerity, $S_f = S_0$. ; (3) a dynamic band corresponding to mid-spectrum values of the wave number, where the wave celerity fell between the gravity and kinematic celerity values,

$$\frac{1}{g}\frac{\partial u}{\partial t} + \frac{u}{g}\frac{\partial u}{\partial x} + \frac{\partial h}{\partial x} + \left(S_f - S_0\right) = 0 \qquad (2\text{-}12)$$

A significant conclusion regarding to dynamic wave propagation could be obtained from the following: for primary waves, F = 2 was the threshold dividing the attenuation F less than 2 and amplitude F great than 2 tendencies. For secondary wave, however, F = 1 was the threshold dividing the propagation upstream F less than 1 or downstream F great than 1 for gravity waves. Thus, F =2 was verified to be as important a threshold value as F= 1 in describing the dynamics of the unsteady flow phenomena.

MATHEMATICAL DERIVATION AND COMPUTATION PROCEDURE

Complete and Simplified Form of Diffusion Equation

The differential equations governing flow in a wide rectangular channel with no lateral inflow may be written as:

$$\frac{\partial q}{\partial x} + \frac{\partial y}{\partial t} = 0 \qquad (3\text{-}1)$$

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$$(gy^3 - q^2)\frac{\partial y}{\partial x} + 2yq\frac{\partial q}{\partial x} + y^2\frac{\partial q}{\partial t} = gy^3\left(S_0 - \frac{q^2}{C_c^2 y^3}\right) \quad (3\text{-}2)$$

in which:

X=distance along the channel, t=time, g=the acceleration of gravity, y=depth, q=discharge per unit width, S_0=the channel bed slope, Cc=the Chezy's coefficient.

By perturbation, $q= q_0 +q'$ and $y= y_0 +y'$ substituting into Eqs. (3-1) and (3-2):

$$\left(\frac{\partial q_0}{\partial x} + \frac{\partial y_0}{\partial t}\right) + \left(\frac{\partial q'}{\partial x} + \frac{\partial y'}{\partial t}\right) = 0 \quad (3\text{-}3)$$

$$\{g(y_0 + y')^3 - (q' + q_0)^2\}\cdot\left(\frac{\partial y'}{\partial x} + \frac{\partial y_0}{\partial x}\right) + 2(y_0 + y')(q_0 + q')\left(\frac{\partial q_0}{\partial x} + \frac{\partial q'}{\partial x}\right) +$$

$$(y_0 + y')^2\left(\frac{\partial q_0}{\partial t} + \frac{\partial q'}{\partial t}\right) = q(y_0 + y')^3\left\{S_0 - \frac{(q_0+q')^2}{C_c^2(y_0+y')^3}\right\} \quad (3\text{-}4)$$

(a): for *0th order:*

$$\frac{\partial q_0}{\partial x} + \frac{\partial y_0}{\partial t} = 0 \quad (3\text{-}5)$$

$$(gy_0^3 - q_0^2)\frac{\partial y_0}{\partial x} + 2y_0 q_0\frac{\partial q_0}{\partial x} + y_0^2\frac{\partial q_0}{\partial t} = gy_0^3 S_0 - \frac{gq_0^2}{C_c^2} \quad (3\text{-}6)$$

(b): for *1th order:*

$$\frac{\partial q'}{\partial x} + \frac{\partial y'}{\partial t} = 0 \quad (3\text{-}7)$$

$$(3gy_0^2 y' - 2q_0 q')\frac{\partial y_0}{\partial x} + (gy_0^3 - q_0^2)\frac{\partial y'}{\partial x} + 2(y_0 q' + y' q_0)\frac{\partial q_0}{\partial x} + 2y_0 q_0\frac{\partial q'}{\partial x} +$$

$$2y_0 y'\frac{\partial q_0}{\partial t} + y_0^2\frac{\partial q'}{\partial t} = 3gy_0^3 y' S_0 + \frac{2g}{C_c^2} q_0 q' \quad (3\text{-}8)$$

from the zeroth order, we can solve for q_0 and y_0, then for the first order:

$$\frac{\partial y_0}{\partial x} = 0;\ \frac{\partial q_0}{\partial x} = 0\ \ and\ \ \frac{\partial q_0}{\partial t} = 0.$$

Eq.(3-8) becomes:

$$(gy_0^3 - q_0^2)\frac{\partial y'}{\partial x} + 2y_0 q_0\frac{\partial q'}{\partial x} + y_0^2\frac{\partial q'}{\partial t} = 3gy_0^2 y' S_0 + \frac{2g}{C_c^2} q_0 q' \quad (3\text{-}9)$$

$$By\ \ (gy_0^3 - q_0^2)\frac{\partial}{\partial x}\big(\mathrm{Eq}(3-7)\big) - \frac{\partial}{\partial t}\big(Eq(3-9)\big):$$

$$(gy_0^3 - q_0^2)\frac{\partial^2 q'}{\partial x^2} - 2y_0 q_0\frac{\partial^2 q'}{\partial x \partial t} - y_0^2\frac{\partial^2 q'}{\partial t^2} = 3gy_0^2 S_0\frac{\partial y'}{\partial t} + \frac{2g}{C_c^2} q_0\frac{\partial q'}{\partial t} \quad (3\text{-}10)$$

divided by y_0^2:

$$(gy_0 - u_0^2)\frac{\partial^2 q'}{\partial x^2} - 2u_0\frac{\partial^2 q'}{\partial x \partial t} - \frac{\partial^2 q'}{\partial t^2} = 3gS_0\frac{\partial y'}{\partial t} + \frac{2gS_0}{u_0}q_0\frac{\partial q'}{\partial t} \qquad (3\text{-}11)$$

let $u_0^2 = C_c^2 y_0 S_0 \ or \ \frac{C_c^2}{q_0} = \frac{u_0}{S_0}y_0^2$

it is then:

$$\left(\frac{u_0}{2gS_0}\right)\left\{gy_0(1-F_0^2)\frac{\partial^2 q'}{\partial x^2} - 2u_0\frac{\partial^2 q'}{\partial x \partial t} - \frac{\partial^2 q'}{\partial t}\right\} = \frac{3}{2}u_0\frac{\partial q'}{\partial x} + \frac{\partial q'}{\partial t}$$

$$or \ \left(\frac{q_0}{2S_0}\right)(1-F_0^2)\frac{\partial^2 q'}{\partial x^2} - \frac{y_0 F_0^2}{S_0}\frac{\partial^2 q'}{\partial x \partial t} - \frac{u_0}{2gS_0}\frac{\partial^2 q'}{\partial t^2} = \frac{3}{2}u_0\frac{\partial q'}{\partial x} + \frac{\partial q'}{\partial t} \qquad (3\text{-}12)$$

If we neglect the second and third terms of left-hand side, then:

$$\left(\frac{q_0}{2S_0}\right)(1-F_0^2)\frac{\partial^2 q'}{\partial x^2} = \frac{3}{2}u_0\frac{\partial q'}{\partial x} + \frac{\partial q'}{\partial t}$$

$$\frac{\partial q'}{\partial t} = \mu\frac{\partial^2 q'}{\partial x^2} - C_0\frac{\partial q'}{\partial x} \qquad (3\text{-}13)$$

μ*:* a wave dispersion coefficient $=\frac{q_0}{2S_0}(1-F_0^2)$

C_0 *:* the wave celerity, or kinematic wave velocity $=\frac{3}{2}u_0$

and Eq (3-12) is "complete form of diffusion equation for discharge", Eq (3-13) is "simplified form of diffusion equation for discharge".

The same for constructing the diffusion for "y", by ∂/∂t (Eq(3-7))-∂/∂x (Eq(3-9)), We have :

$$\left(\frac{q_0}{2S_0}\right)(1-F_0^2)\frac{\partial^2 y'}{\partial x^2} - \frac{y_0 F_0^2}{S_0}\frac{\partial^2 y'}{\partial x \partial t} - \frac{u_0}{2gS_0}\frac{\partial^2 y'}{\partial t^2} = \frac{3}{2}u_0\frac{\partial y'}{\partial x} + \frac{\partial y'}{\partial t} \qquad (3\text{-}14)$$

$$or \ \frac{\partial y'}{\partial t} = \mu\frac{\partial^2 y'}{\partial x^2} - C_0\frac{\partial y'}{\partial x} \qquad (3\text{-}15)$$

Egs.(3-14) and (3-15) are "diffusion equation for depth".

Solution of Input Unit Step function from the Simplified Diffusion Equation

Solving for water depth, y, we follow Hayami, the flood wave diffusion equation incorporation effects of channel irregularities and lateral flow can be expressed as:

$$\frac{\partial y}{\partial t} + \frac{3}{2}u\frac{\partial y}{\partial x} = \left[k + \frac{yu}{2\left(s_0 - \frac{\partial y}{\partial x}\right)}\right]\frac{\partial^2 y}{\partial x^2} + Q_L(x,t) \qquad (3\text{-}16)$$

In which:

k: diffusivity due to channel irregularities; y: flow depth; C_c: Chezy roughness

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coefficient; u: velocity of flow $= C_c[y(S_0 - \partial y/\partial x)]^{1/2}$; S_0: bed slope; x: distance from upstream; t: time, and $Q_L(x,t)$: lateral discharge per unit width of the tributary and per unit width of the main channel. The initial condition, i.e. y(x,o)=H_0. The boundary conditions for t>0, are taken as y(o,t)= H_0+U(t), and y(l,t)= H_0+D(t), in which U(t) and D(t) are the water level variations above the initial depth, H_0, at the upstream and downstream ends; and l is the length of the channel reach. The preceding nonlinear diffusion equation is linearized to obtain and analytical solution by perturbation series:

$$y(x,t) = y_0 + \in \emptyset_1 + \in^2 \emptyset_2 + \cdots = (H_0 + h_0)\left(1 + \frac{\emptyset_0}{(H_0+h_0)} + \frac{\emptyset_0{}'}{(H_0+h_0)^2} + \cdots\right) \quad \text{.... (3-17)}$$

in which h_0 is the average height of the water level above H_0.

Let $\emptyset_0(x,t) = \emptyset_1(x,t) + \emptyset_3(x,t)$ and we only take the first approximation solution:

$$y(x,t) = (H_0 + h_0) + \emptyset_1(x,t) + \emptyset_3(x,t) \quad \text{..... (3-18)}$$

in which $\emptyset_1(x,t)$ is the solution of

$$\frac{\partial \emptyset_1}{\partial t} + \omega \frac{\partial \emptyset_1}{\partial x} = \mu \frac{\partial^2 \emptyset_1}{\partial x^2} \quad \text{..... (3-19)}$$

with the boundary conditions and initial condition:

$\emptyset_1(x,t) = -h_0$, x≧0

$\emptyset_1(x,t) = U(t) - h_0$, t≧0

$\emptyset_1(l,t) = D(t) - h_0$, t≧0..... (3-20)

And $\emptyset_3(x,t)$ is the solution of the equation:

$$\frac{\partial \emptyset_3}{\partial t} + \omega \frac{\partial \emptyset_3}{\partial x} = \mu \frac{\partial^2 \emptyset_3}{\partial x^2} + Q_L(x,t) \quad \text{..... (3-21)}$$

with the boundary conditions and initial condition:

$\emptyset_3(x,t) = 0$, x≧0

$\emptyset_3(x,t) = 0$, t≧0

$\emptyset_3(l,t) = 0$, t≧0..... (3-22)

and

$$\mu = K + (H_0 + h_0)\frac{u_0}{2S_0}$$

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$$\omega = \frac{3}{2}u_0$$

$$u_0 = C_c[(H_0 + h_0)S_0]^{1/2}$$

Let $\emptyset_3(x,t) = 0$ with no lateral flow effect, and substituting the following equations into Eq.(3-18):

$$\emptyset_1(x,t) = \tau(x,t)exp\left(\frac{\omega x}{2\mu} - \frac{\omega^2 t}{4\mu}\right)$$

$$\frac{\partial \emptyset_1}{\partial t} = \left[\frac{\tau\omega}{2\mu} + \frac{\partial \tau}{\partial x}\right] exp\left(\frac{\omega x}{2\mu} - \frac{\omega^2 t}{4\mu}\right)$$

$$\frac{\partial^2 \emptyset_1}{\partial x^2} = \left[\frac{\omega^2\tau}{4\mu^2} + \frac{\omega}{\mu}\frac{\partial \tau}{\partial x} + \frac{\partial^2 \tau}{\partial x^2}\right] exp\left(\frac{\omega x}{2\mu} - \frac{\omega^2 t}{4\mu}\right)$$

$$\frac{\partial \emptyset_1}{\partial t} = \left[-\frac{\tau\omega}{2\mu} + \frac{\partial \tau}{\partial x}\right] exp\left(\frac{\omega x}{2\mu} - \frac{\omega^2 t}{4\mu}\right)$$

we yield

$$\frac{\partial \tau}{\partial x} = \mu\frac{\partial^2 \tau}{\partial x^2} \quad \text{(3-23)}$$

And the boundary conditions are transformed into:

$$\tau(O,t) = [U(t) - h_0]exp\left(\frac{\omega^2 t}{4\mu}\right)$$

$$\tau(l,t) = [D(t) - h_0]exp\left(\frac{\omega^2 t}{4\mu} - \frac{\omega l}{2\mu}\right)$$

$$\tau(x,O) = -h_0 exp\left(-\frac{\omega x}{2\mu}\right) \quad \text{(3-24)}$$

By using Laplace Transformation, if $L\big(\tau(O,t)\big) = \theta(x,s)$ and Eq. (3-23):

$$L\left\{\frac{\partial \tau}{\partial x}\right\} = L\left\{\frac{\partial^2 \tau}{\partial x^2}\right\}$$

$$S\theta - \tau(x,O) = \mu\frac{d^2\theta}{dx^2}$$

$$\frac{d^2\theta}{dx^2} - \frac{s}{\mu}\theta = \frac{h_0}{\mu}e^{-\frac{\omega x}{2\mu}} \quad \text{(3-25)}$$

The characteristic equation of corresponding homogeneous equation is

$$m^2 - \frac{s}{\mu} = 0$$

$$m = \pm\sqrt{\frac{s}{\mu}}$$

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$$\theta_h(x,s) = C_1 e^{\sqrt{\frac{s}{\mu}}x} + C_2 e^{-\sqrt{\frac{s}{\mu}}x} \quad \ldots\ldots (3\text{-}26)$$

For the particular solution, we assume

$$\theta_p = \beta e^{-\frac{\omega x}{2\mu}} \quad \ldots\ldots (3\text{-}27)$$

From Eq.(3-25), we get

$$\beta = \frac{h_0}{\left(\frac{\omega^2}{4\mu} - S\right)}$$

$$\theta(x,s) = \theta_h(x,s) + \theta_p(x,s) = C_1 e^{\sqrt{\frac{s}{\mu}}x} + C_2 e^{-\sqrt{\frac{s}{\mu}}x} + \frac{h_0 e^{-\frac{\omega x}{2\mu}}}{\left(\frac{\omega^2}{4\mu} - S\right)} \quad \ldots\ldots (3\text{-}28)$$

And the boundary conditions are also transformed into:

$$\theta(O,s) = L\left\{[U(t) - h_0]e^{\frac{\omega^2 t}{4\mu}}\right\} = f\left(S - \frac{\omega^2}{4\mu}\right) - \frac{h_0}{S - \frac{\omega^2}{4\mu}}$$

$$\theta(l,s) = L\left\{[D(t) - h_0]e^{\frac{\omega^2 t}{4\mu}}\right\} = \left[g\left(S - \frac{\omega^2}{4\mu}\right) - \frac{h_0}{\left(S - \frac{\omega^2}{4\mu}\right)}\right] e^{-\frac{\omega x}{2\mu}}$$

From Eq.(3-28):

$$\theta(O,s) = C_1 + C_2 - \frac{h_0}{S - \frac{\omega^2}{4\mu}} = f\left(S - \frac{\omega^2}{4\mu}\right) - \frac{h_0}{S - \frac{\omega^2}{4\mu}}$$

$$C_1 + C_2 = f\left(S - \frac{\omega^2}{4\mu}\right) \quad \ldots\ldots (3\text{-}29)$$

$$\theta(l,s) = C_1 e^{\sqrt{\frac{S}{\mu}}l} + C_2 e^{-\sqrt{\frac{S}{\mu}}l} - h_0 \frac{e^{-\frac{\omega t}{2\mu}}}{S - \frac{\omega^2}{4\mu}} = \left[g\left(S - \frac{\omega^2}{4\mu}\right) - \frac{h_0}{S - \frac{\omega^2}{4\mu}}\right] e^{-\frac{\omega l}{2\mu}}$$

$$C_1 e^{\sqrt{\frac{S}{\mu}}l} + C_2 e^{-\sqrt{\frac{S}{\mu}}l} = g\left(S - \frac{\omega^2}{4\mu}\right) e^{-\frac{\omega t}{2\mu}} \quad \ldots\ldots (3\text{-}30)$$

From Eqs.(3-29) and (3-30), we can obtain C_1 and C_2:

$$C_2 = \frac{f\left(S - \frac{\omega^2}{4\mu}\right) - exp\left(-\frac{\omega l}{2\mu} - \sqrt{\frac{S}{\mu}}l\right) g\left(S - \frac{\omega^2}{4\mu}\right)}{1 - exp\left(-2\sqrt{\frac{S}{\mu}}l\right)}$$

$$C_1 = \frac{g\left(S - \frac{\omega^2}{4\mu}\right) exp\left(-\frac{\omega l}{4\mu} - \sqrt{\frac{S}{\mu}}l\right) - f\left(S - \frac{\omega^2}{4\mu}\right) exp\left(-2\sqrt{\frac{S}{\mu}}l\right)}{1 - exp\left(-2\sqrt{\frac{S}{\mu}}l\right)}$$

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$\theta(x,s) =$

$$f\left(S-\frac{\omega^2}{4\mu}\right)\frac{exp\left(-\sqrt{\frac{s}{\mu}}l\right)-exp[-(2l-x)]\sqrt{\frac{s}{\mu}}}{1-exp\left(-2\sqrt{\frac{s}{\mu}}l\right)}+g\left(S-\frac{\omega^2}{4\mu}\right)\frac{exp\left[-(l-x)\sqrt{\frac{s}{\mu}}\right]-exp\left[-(2l-x)\sqrt{\frac{s}{\mu}}\right]}{1-exp\left(-2\sqrt{\frac{s}{\mu}}l\right)}e^{-\frac{\omega t}{2\mu}}-$$

$$\frac{h_0 e^{-\frac{\omega x}{2\mu}}}{\left(s-\frac{\omega^2}{4\mu}\right)} \quad \text{...} (3\text{-}31)$$

By use inversion of Laplace Transformation to find the original solution forms:

$L^{-1}\{\bar{f}(s)\} = F(t),$

$$L^{-1}\left\{\frac{e^{-(A-Bx)\sqrt{\frac{s}{\mu}}}}{1-e^{-2\sqrt{\frac{s}{\mu}}l}}\right\} = L^{-1}\left[e^{-(A-Bx)\sqrt{\frac{s}{\mu}}}\left\{1+e^{-2\sqrt{\frac{s}{\mu}}l}+e^{-4\sqrt{\frac{s}{\mu}}l}+\cdots\right\}\right] =$$

$$L^{-1}\left\{e^{-(A-Bx)\sqrt{\frac{s}{\mu}}}+e^{-(A-Bx-2l)\sqrt{\frac{s}{\mu}}}+e^{-(A-Bx+4l)\sqrt{\frac{s}{\mu}}}+\cdots\right\} = L^{-1}\left\{\sum_{n=0}^{\infty} e^{-(A-Bx-2nl)\sqrt{\frac{s}{\mu}}}\right\}$$

$$= \sum_{n=0}^{\infty} L^{-1}\left\{e^{-(A-Bx-2nl)\sqrt{\frac{s}{\mu}}}\right\} = \sum_{n=0}^{\infty}\left\{\frac{A-Bx-2nl}{2\sqrt{\pi\mu t^3}}exp\left[-\frac{(A-Bx-2nl)^2}{4\mu t}\right]\right\}$$

$$= \frac{1}{2\sqrt{\pi\mu t^3}}\sum_{n=0}^{\infty}\left\{(A-Bx-2nl)exp\left[\frac{-(A-Bx-2nl)^2}{4\mu t}\right]\right\} \quad \text{.......................................} (3\text{-}32)$$

$$L^{-1}\left\{f\left(S-\frac{\omega^2}{4\mu}\right)\right\} = u(t)e^{\frac{\omega^2 t}{4\mu}} \quad \text{..} (3\text{-}33)$$

$$L^{-1}\left\{\frac{h_0}{S-\frac{\omega^2}{4\mu}}\right\} = h_0 e^{\frac{\omega^2 t}{4\mu}} \quad \text{...} (3\text{-}34)$$

$$L^{-1}\{\bar{f}(s)\bar{g}(s)\} = \int_0^t F(t-\tau)\,G(\tau)d\tau \quad \text{..} (3\text{-}35)$$

By using Eqs. (3-32),(3-33),(3-34) and (3-35) in the inversion of Eq. (3-31), then

$$\tau(x,t) = \frac{1}{2\sqrt{\pi\mu}}\int_0^t U(t-\lambda)\frac{e^{\frac{\omega^2(t-\lambda)}{4\mu}}}{\lambda^{3/2}}\sum_{n=0}^{\infty}\left[(x+2nl)e^{-\frac{(x+2nl)^2(t-\lambda)}{2\mu\lambda}}-[2(n+1)l-\right.$$

$$\left.x]e^{-\frac{(2(n+1)l-x)^2}{4\mu\lambda}}\right]d\lambda+\frac{e^{-\frac{\omega t}{2\mu}}}{2\sqrt{\pi\mu}}\int_0^t D(t-\lambda)\frac{e^{\frac{\omega^2(t-\lambda)}{4\mu}}}{\lambda^{3/2}}\sum_{n=0}^{\infty}\left[[2(n+1)l-x]e^{-\frac{(2(n+1)l-x)^2}{4\mu\lambda}}-\right.$$

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$$[2(n+1)l-x]e^{-\frac{(2(n+1)l-x)^2}{4\mu\lambda}}\Bigg]d\lambda-h_0e^{-\left(\frac{\omega x}{2\mu}-\frac{\omega^2 t}{4\mu}\right)} \text{ .... (3-36)}$$

If we denote *(l-x)* by *x'*, and substitute into Eq.(3-19) by all the above expression:

$$\tau(x,t)=\frac{e^{\frac{\omega t}{2\mu}}}{2\sqrt{\pi\mu}}\int_0^t U(t-\lambda)\frac{e^{-\frac{\omega^2 l}{4\mu}}}{\lambda^{3/2}}\sum_{n=0}^{\infty}\Bigg[[2nl+x]e^{-\frac{(2nl+x)^2}{4\mu\lambda}}-[2(n+1)l-$$

$$x]e^{-\frac{(2n+1)l+x^2}{4\mu\lambda}}\Bigg]d\lambda+\frac{e^{-\frac{\omega x}{2\mu}}}{2\sqrt{\pi\mu}}\int_0^t D(t-\lambda)\frac{e^{\frac{\omega^2 l}{4\mu}}}{\lambda^{3/2}}\sum_{n=0}^{\infty}\Bigg[[2nl+x']e^{-\frac{(2nl+x')^2}{4\mu\lambda}}-[2(n+1)l-$$

$$x']e^{-\frac{(2n+1)(-x')^2}{4\mu\lambda}}\Bigg]d\lambda \text{ .... (3-37)}$$

Substitute $\emptyset_1(x,t)$ into Eq. (3-2), and let $\emptyset_3(x,t)=0$*:*

$$y(x,t)-H_0=\frac{e^{\frac{\omega x}{2\mu}}}{2\sqrt{\pi\mu}}\int_0^t U(t-\lambda)\frac{e^{-\frac{\omega^2\lambda}{4\mu}}}{\lambda^{3/2}}\sum_{n=0}^{\infty}\Bigg[[2nl+x]e^{-\frac{(2nl+x)^2}{4\mu\lambda}}-[2(n+1)l-$$

$$x]e^{-\frac{[(2nl+1)l-x]^2}{4\mu\lambda}}\Bigg]d\lambda+\frac{e^{-\frac{\omega x\prime}{2\mu}}}{2\sqrt{\pi\mu}}\int_0^t D(t-\lambda)\frac{e^{-\frac{\omega^2\lambda}{4\mu}}}{\lambda^{3/2}}\sum_{n=0}^{\infty}\Bigg[[2nl+x']e^{-\frac{(2nl+x')^2}{4\mu\lambda}}-$$

$$[2(n+1)l-x']e^{-\frac{\left((2n+1)l-x'\right)^2}{4\mu\lambda}}\Bigg]d\lambda=I_1(x,t)+I_2(x',t)=\int_0^t U(t-\lambda)x(\lambda,x)\,dx+$$

$$\int_0^t D(t-\lambda)x'(\lambda,x')\,d\lambda \text{ .... (3-38)}$$

Now, let's discuss about *U(t)* and *D (t)*, and if let *u(t)* is a discrete step function, as

$U(t)=f_1 \quad 0<t\leq 1$

$U(t)=f_2 \quad 1<t\leq 2$

.

.

$U(t)=f_m \quad m-1<t\leq m$

$$I_1(x,t)=\int_0^t U(t-\lambda)x(\lambda,x)\,dx$$

$=$

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$\int_{t-1}^{t} f_1 X(\lambda, x)\, d\lambda + \int_{t-2}^{t-1} f_2 X(x, \lambda)\, d\lambda + \cdots + \int_{t-m+1}^{t-m+2} f_{m-1} X(\lambda, x)\, d\lambda +$

$\int_{0}^{t-m+1} f_m X(\lambda, x)\, d\lambda$

$=$

$f_1 \int_0^t X(\lambda, x)\, d\lambda + (f_2 - f_1) \int_0^{t-1} X(x, \lambda)\, d\lambda + \cdots +$

$(f_{m-1} - f_{m-2}) \int_0^{t-m+2} X(\lambda, x)\, d\lambda + (f_m - f_{m-1}) + \int_0^{t-m+1} f_m X(\lambda, x)\, d\lambda$ ....

(3-39)

If we define $\int_0^t X(\lambda, x)\, d\lambda = R_1(x, t)$, then:

$I_1(x, t) = f_1 R_1(x, t) + \sum_{l=1}^{m_1-1} R_1(x, t-j)[f_{l+1} - f_l]$ or
$I_1(x, t) = f_1 R_1(1) + \sum_{l=m_1-1}^{1} f_1 [R_1(x, t-j+1) - R(x, t-j)]$ .... (3-40)
If $R_1(x, t)$ tends to a constant value as t increases, i.e. the higher value of t, the effect of f_1 will be cancelled by that of f_2 and so on. If let K is the time after which $R_1(x, t)$ remains constant, so:
$R_1(K) = R_1(K+1) = R_1(K+2) = \cdots = R_1(t-1) = R_1(t)$; k<t
Then we can rewrite Eq.(3-40) as:

$I_1(x, t) = f_t R_1(x, t) + \sum_{l=m_1-1}^{t-k_1+1} f_1 [R_1(x, t-j+1) - R(x, t-j)]$ .... (3-41)

$I_1(x, t) = f_1 R_1(x, t) + \sum_{l=t-k_1+1}^{m_1-1} R_1(x, t-j)[f_{l+1} - f_l]$ .... (3-42)

where:

m_1=t or flood period whichever is minimum.

k_1=t or k defined above, whichever is minimum.

The same result:

$I_2(x', t) = g_1 R_2(x', t) + \sum_{l=t-k_2+1}^{m_2-1} R_2(x', t-j)[g_{l+1} - g_l]$ .... (3-43)

$R_1(x, t) =$

$$e^{\frac{\omega x}{2\mu}} \int_0^t \frac{-\frac{\omega^2 \lambda}{4\mu}}{2\sqrt{\pi\mu}\lambda^{3/2}} \sum_{n=0}^{\infty} \left[[2nl + x] e^{-\frac{(2nl+x)^2}{4\mu\lambda}} - [2(n+1)l - x] e^{-\frac{(2(n+1)l-x)^2}{4\mu\lambda}} \right] d\lambda$$

$R_2(x', t) =$

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$$e^{-\frac{\omega x'}{2\mu}} \int_0^t \frac{e^{-\frac{\omega^2\lambda}{4\mu}}}{2\sqrt{\pi\mu}\lambda^{3/2}} \sum_{n=0}^{\infty} \left[[2nl + x'] e^{-\frac{(2nl+x')^2}{4\mu\lambda}} - [2(n+1)l - x'] e^{-\frac{(2(n+1)l-x')^2}{4\mu\lambda}} \right] d\lambda \quad \text{.......... (3-44)}$$

If we substitute

$$\lambda = \frac{x^2}{4\mu\xi^2}$$

$$R_1(x,t) = \frac{2}{\sqrt{\pi}} e^{\frac{\omega x}{2\mu}} \int_{\frac{x}{2\sqrt{\mu t}}}^{\infty} e^{-\frac{\omega^2 x^2}{16\mu^2\xi^2}} \sum_{n=0}^{\infty} \left\{ \left(\frac{2nl}{x} + 1\right) e^{-\left(\frac{2nl}{x}+1\right)^2 \xi^2} - \left[\frac{2(n+l)l}{x} - 1\right] e^{-\left(\frac{2(n+1)l}{x}-1\right)^2 \xi} \right\} d\xi \quad \text{......... (3-45)}$$

$$\int_0^{\infty} e^{-x^2 - \frac{a^2}{x^2}} dx = \frac{e^{-2a\sqrt{x}}}{2} \quad for\ \alpha \geq 0 \quad \text{.......... (3-46)}$$

Therefore

$$\int_0^{\infty} e^{-b^2\xi^2 - \frac{c^2}{\xi^2}} dx/b = \frac{e^{-2bc\sqrt{\pi}}}{2b} \quad \text{.......... (3-47)}$$

$$\int_{\frac{x}{2\sqrt{\mu t}}}^{\infty} e^{-b^2\xi^2 - \frac{c^2}{\xi^2}} d\xi = \frac{e^{-2bc\sqrt{\pi}}}{2b} - \int_{\infty}^{\frac{x}{2\sqrt{\mu t}}} e^{-b^2\xi^2 - \frac{c^2}{\xi^2}} d\xi \quad \text{.......... (3-48)}$$

And Eq. (3-45)

$$R_1(x,t) = e^{\frac{\omega x}{2\mu}} \sum_{n=0}^{\infty} \left[\left\{ e^{-\left(\frac{\omega x}{2\mu}\right)\left[\frac{2nl}{x}+1\right]} - \frac{2}{\sqrt{\pi}} \left[\frac{2nl}{x} + 1\right] \int_{\infty}^{\frac{x}{2\sqrt{\mu t}}} e^{-\left[\frac{2nl}{x}+1\right]^2 \xi^2 - \frac{\left[\frac{\omega x}{4\mu}\right]^2}{\xi^2}} d\xi \right\} - \left\{ e^{-\frac{\omega x}{2\mu}\left[\frac{2(n+1)l}{x}-1\right]} - \frac{2}{\sqrt{\pi}} \left[\frac{2(n+1)l}{x} - 1\right] \int_{\infty}^{\frac{x}{2\sqrt{\mu t}}} e^{-\left[\frac{2nl}{x}+1\right]^2 \xi^2 - \frac{\left[\frac{\omega x}{4\mu}\right]^2}{\xi^2}} d\xi \right\} \right] \quad \text{......... (3-49)}$$

$$R_2(x',t) =$$

$$e^{-\frac{\omega x'}{2\mu}} \sum_{n=0}^{\infty} \left[\left\{ e^{-\left(\frac{\omega x'}{2\mu}\right)\left[\frac{2nl}{x}+1\right]} - \frac{2}{\sqrt{\pi}} \left[\frac{2nl}{x'} + 1\right] \int_{\infty}^{\frac{x}{2\sqrt{\mu t}}} e^{-\left[\frac{2(n+1)l}{x'}-1\right]^2 \xi^2 - \frac{\left[\frac{\omega x}{4\mu}\right]^2}{\xi^2}} d\xi \right\} - \right.$$

$$\left\{ e^{-\frac{\omega x'}{2\mu}\left[\frac{2(n+1)l}{x'}-1\right]} - \frac{2}{\sqrt{\pi}} \left[\frac{2(n+1)l}{x'} - \right. \right.$$

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$$1\Big] \int_{\infty}^{\frac{x}{2\sqrt{\mu t}}} e^{-\left[\frac{2nl}{x'}+1\right]^2 \xi^2 - \frac{\left[\frac{\omega x'}{4\mu}\right]^2}{\xi^2}} d\xi \Bigg\}\Bigg]\Bigg] \text{ .... (3-50)}$$

$$R_1(x,t) = e^{\frac{\omega x}{2\mu}} B(x,t) \text{ .... (3-51)}$$

$$R_2(x',t) = e^{-\frac{\omega x\prime}{2\mu}} B(,t) \text{ .... (3-52)}$$

$$\beta(p,t) = \sum_{n=0}^{\infty}\left[s\left(p,\frac{2nl}{p}+1,t\right) - s\left(p,\frac{2(n+1)l}{p}-1,t\right)\right] \text{ .... (3-53)}$$

$$S(p,b.t)=e^{-\frac{\omega p}{2\mu}} - \frac{2b}{\sqrt{\pi}}\int_{0}^{\frac{p}{2\sqrt{\mu t}}} e^{-b^2\xi^2-\frac{\left[\frac{\omega p}{4\mu}\right]^2}{\xi^2}} d\xi \text{ .... (3-54)}$$

f_1=fluctuation in depth at *u/s* end at time $t=i$ over the initial uniform depth, H_0
g_1=fluctuation in depth at downstream end at time $t=i$ over the initial uniform depth, H_n
p and b are dummy variable which take the values x and x' and *(2nl/p +1)* or *[2(n+1)1/p-1]*, respectively.
Solving for R1(x,t) and R2(x',t), we based on an assumed rectangular channel, for which the diffusivity due to channel irregular is K=o, and
$\phi_l(x,t) = \alpha_{t.l}R_t(x,t) + \sum_{l=1}^{m_t-1} R_t(x,t-j)(\alpha_{t.l+1} - \alpha_{t.l})$
in which the subscript $i=1.2$ corresponds to upstream, and downstream effects, respectively with $\alpha_{1,J} = u_j(upstream), \alpha_{2,J} = d_j(\text{downstream})$ *and j=1,2,3,…,m-1.*
It's also found that it's already sufficient to take into account only the terms with $n=0$ in the converging series terms of Eqs. *B (p,t)* for the 1mm accuracy of the resulting water depth.

$$\int_0^{\infty} e^{s\left(2-\frac{\delta}{2\lambda\sqrt{s}}\right)^2} d\lambda = \frac{\sqrt{\pi}}{2\sqrt{s}} \quad \text{and } \tau = \frac{b}{\lambda} \text{ with } d\lambda = -\frac{b}{\tau^2}d\tau,\ b = \frac{q}{2\sqrt{s}}\ ,$$

$$\int_0^{\infty} e^{-s\left(2-\frac{b}{2}\right)^2} d\lambda = \int_0^{\infty} e^{-s\left(\frac{b}{\tau}-\tau\right)^2}\left(-\frac{b}{\tau^2}\right)d\,\tau = \int_{\infty}^{0}\left(\frac{b}{\tau^2}\right) - e^{-s\left(\frac{b}{\tau}-\tau\right)^2} d\tau =$$

$$-\int_{\infty}^{0}\left(\frac{b}{\tau^2}\right)e^{-\left(\frac{b}{\tau}-\tau\right)^2} d\tau \text{ .... (3-55)}$$

That is: $2\int_0^{\infty} e^{-s\left(\lambda-\frac{b}{\lambda}\right)^2} d\lambda = \int_0^{\infty}\left(1+\frac{b}{\tau^2}\right)e^{-\left(\tau-\frac{b}{\tau}\right)^2} d\tau$

Let $x = \sqrt{s}\left(\tau - \frac{b}{\tau}\right) dx = \sqrt{s}\left(1+\frac{b}{\tau^2}\right)d\tau$

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$$2\int_0^\infty e^{-s\left(\lambda-\frac{b}{\lambda}\right)^2} d\lambda = \int_{-\infty}^{\infty}\left(1+\frac{b}{\tau^2}\right)e^{-x^2}\frac{1}{\sqrt{s}\left(1+\frac{b}{\tau^2}\right)}dx$$

$$= \frac{\sqrt{\pi}}{\sqrt{s}}\left(\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-x^2}\,dx\right) = \frac{\sqrt{\pi}}{\sqrt{s}} erf(\infty)$$

$$\int_0^\infty e^{-s\left(\lambda-\frac{b}{\lambda}\right)^2} d\lambda = \frac{\sqrt{\pi}}{2\sqrt{s}} = \frac{1}{2}\frac{\sqrt{\pi}}{\sqrt{s}} erf(\infty)$$, then

$$\int_0^\infty e^{-x^2-\frac{\pi^2}{x^2}}dx = \int_0^\infty e^{-\left(x-\frac{\pi}{\mu}\right)^2-2\alpha}dx = e^{-2\alpha}\int_0^\infty e^{-\left(x-\frac{\pi}{x}\right)^2}dx = e^{-2\alpha}\frac{\sqrt{\pi}}{2} \quad (3\text{-}56)$$

and

$$\int_0^\infty e^{-b^2\xi^2-\frac{c^2}{\xi^2}}d\xi = \frac{e^{-2bc}}{b}\int_0^\infty e^{-\left(x-\frac{bc}{x}\right)^2}dx = \left(\frac{e^{-2bc}}{b}\frac{\sqrt{\pi}}{2}\right)\left(\frac{2}{\sqrt{\pi}}\int_0^\infty e^{-\left(x-\frac{bc}{\mu}\right)^2}dx\right)$$

$$= \left(\frac{\sqrt{\pi}}{2b}e^{-2bc}\right) \quad \text{.....................} \quad (3\text{-}57)$$

$$if \int_0^{\alpha'} e^{-b^2\xi^2-\frac{c^2}{\xi^2}}d\xi = \int_0^{\alpha'} e^{-\left(x-\frac{bc}{x}\right)^2-2bc}\frac{1}{b}dx$$

$$=\left(\frac{1}{b}e^{-2bc}\right)\int_0^{\alpha'} e^{-\left(x-\frac{bc}{x}\right)^2}dx = \left(\frac{e^{-2bc}}{b}\right)\left(\frac{\sqrt{\pi}}{2}\right)\left(\frac{2}{\sqrt{\pi}}\int_0^{\alpha'} e^{-\left(x-\frac{bc}{x}\right)^2}dx\right)$$

$$=\left(\frac{\sqrt{\pi}e^{-2bc}}{2b}\right)\left(erf(\alpha')\right) \quad \text{.....................} \quad (3\text{-}58)$$

$$R_1(x,t) = e^{\frac{\omega x}{2\mu}}\sum_{n=0}^{\infty}\left[\left\{e^{-\left(\frac{\omega x}{2\mu}\right)\left[\frac{2nl}{x}+1\right]} - \frac{2}{\sqrt{\pi}}\left[\frac{2nl}{x}+1\right]\int_{\infty}^{\frac{x}{2\sqrt{\mu t}}} e^{-\left[\frac{2nl}{x}+1\right]^2\xi^2-\frac{\left[\frac{\omega x}{4\mu}\right]^2}{\xi^2}}d\xi\right\} -\right.$$

$$\left. e^{-\frac{\omega x}{2\mu}\left[\frac{2(n+1)l}{x}-1\right]} - \frac{2}{\sqrt{\pi}}\left[\frac{2(n+1)l}{x}-1\right]\int_{\infty}^{\frac{x}{2\sqrt{\mu t}}} e^{-\left[\frac{2nl}{x}+1\right]^2\xi^2-\frac{\left[\frac{\omega x}{4\mu}\right]^2}{\xi^2}}d\xi\right]$$

$$R_2(x',t) =$$

$$e^{-\frac{\omega x}{2\mu}}\sum_{n=0}^{\infty}\left[\left\{e^{-\left(\frac{\omega x'}{2\mu}\right)\left[\frac{2nl}{x}+1\right]} -\right.\right.$$

$$\frac{2}{\sqrt{\pi}}\left[\frac{2nl}{x'}+1\right]\int_{\infty}^{\frac{x}{2\sqrt{\mu t}}} e^{-\left[\frac{2(n+1)l}{x'}-1\right]^2\xi^2-\frac{\left[\frac{\omega x}{4\mu}\right]^2}{\xi^2}}d\xi\left[e^{-\frac{\omega x'}{2\mu}\left[\frac{2(n+1)l}{x'}-1\right]} -\right.$$

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$$\frac{2}{\sqrt{\pi}}\left[\frac{2(n+1)l}{x'}-1\right]\int_0^{\frac{x}{2\sqrt{\mu t}}} e^{-\left[\frac{2nl}{x'}+1\right]^2\xi^2-\frac{\left[\frac{\omega x'}{4\mu}\right]^2}{\xi^2}}\,d\xi\left.\left.\left.\right]\right\}\right]$$

If n=0, then

$R_1(x,t) =$

$$e^{\frac{\omega x}{2\mu}}\left[\left\{e^{-\frac{\omega x}{2\mu}}-\frac{2}{\sqrt{\pi}}\sqrt{\pi}\frac{e^{-\frac{\omega x}{2\mu}}}{2}erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right\}-\right.$$

$$\left.\left\{e^{-\omega\frac{(2l-x)}{2\mu}}-\frac{2}{\sqrt{\pi}}\left(\frac{2l}{x}-1\right)\sqrt{\pi}\frac{e^{-\omega\frac{(2l-x)}{2\mu}}}{2\left(\frac{2l}{x}-1\right)}erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right\}\right]=\left[\left\{1-erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right\}-\left\{e^{\frac{\omega(x-l)}{\mu}}-\right.\right.$$

$$\left.\left.e^{\frac{\omega(x-l)}{\mu}}erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right\}\right]=\left\{1-erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right\}-e^{-\frac{\omega x'}{\mu}}\left\{1-erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right\}=$$

$$\left[1-erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right]\left[1-e^{-\frac{\omega x'}{\mu}}\right], \quad (3\text{-}59)$$

$x' =$

$$R_2(x',t)=e^{-\frac{\omega x'}{2\mu}}\left[\left\{e^{-\frac{\omega x'}{2\mu}}-\frac{2}{\sqrt{\pi}}\frac{\sqrt{\pi}}{2}e^{-\frac{\omega x'}{2\mu}}erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right\}-\left\{e^{-\frac{\omega(2l-x')}{2\mu}}-\frac{2}{\sqrt{\pi}}\left(\frac{2l}{x'}-\right.\right.\right.$$

$$\left.\left.\left.1\right)\frac{\sqrt{\pi}}{2\left(\frac{2l}{x'}-1\right)}e^{-\omega\frac{(2l-x')}{2\mu}}erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right\}\right]=$$

$$\left[\left\{\left(e^{-\frac{\omega x'}{\mu}}-e^{-\frac{\omega x'}{\mu}}erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right)-\left(e^{-\frac{\omega l}{\mu}}-e^{-\frac{\omega l}{\mu}}erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right)\right\}\right]=\left(e^{-\frac{\omega x'}{\mu}}\right)\left(1-\right.$$

$$\left.erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right)-\left(e^{-\frac{\omega l}{\mu}}\right)\left(1-erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right)=\left(e^{-\frac{\omega x'}{\mu}}-e^{-\frac{\omega l}{\mu}}\right)\left[1-\right.$$

$$\left.erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3\text{-}60)$$

the maximum values of $R_1(x,t)$ *and* $R_2(x,t)$ are:

$$\left(1-e^{-\frac{\omega x'}{\mu}}\right) \text{ and } \left(e^{-\frac{x'\omega}{\mu}}-e^{-\frac{\omega l}{\mu}}\right)$$

$$\left(1-erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right) = 1$$

Therefore, when t increases, $R_1(x,t)$ and $R_2(x',t)$ approach to constants. $R_1(x,t-j)$ and $R_2(x',t-j)$ are constants after the time increases to K_1 and K_2.

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$\sum_{l=t-k_1+l}^{m_1-l} R_1(x,t-j)(f_{l+1}-f_l) = R_r(x,t-j)\sum_{l=t-k_2+l}^{m_1-l}[f_{l+1}-f_l] = R_1(x,t-j)(f_{ml}-f_l)$ ………………………………………………………… (3-61).

$\sum_{l=t-k_2+l}^{m_2-l} R_2(x',t-j)[g_{l+1}-g_l] = R_2(x',t-j)(g_{m2}-g_l)$ ……………… (3-62)

$(f_{ml}-f_l)$ and $(g_{m2}-g_l)$ is very small, when it is symmetric hydrograph, the values of $(f_{ml}-f_l)$ and $(g_{m2}-g_l)$ are zero. So, sometimes, we can neglect the two terms Eqs. (3-61) and (3-62).

Solving for velocity *u.(x,t)* from

$$\frac{\partial y}{\partial t} = \left[f_l\left(1-e^{-\frac{\omega x'}{\mu}}\right)\frac{\partial}{\partial t}\left\{1-erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right\} + g_l\left(e^{-\frac{x'\omega}{\mu}}-e^{-\frac{\omega l}{\mu}}\right)\frac{\partial}{\partial t}\left\{1-erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right\}\right] erf\left(\frac{x}{2\sqrt{\mu t}}\right) = \frac{2}{\sqrt{\pi}}\int_0^{\frac{x}{2\sqrt{\mu t}}} e^{-u^2}\,du = \frac{2}{\sqrt{\pi}}\left[u-\frac{1}{3}u^3+\frac{u^5}{5x2!}-\frac{u^7}{7x3!}\ldots\right] l_{u=\frac{x}{2\sqrt{\mu t}}} = \frac{2}{\sqrt{\pi}}\left[\frac{x}{2\sqrt{\mu t}}-\frac{x^3}{6\mu t\sqrt{\mu t}}+\frac{x^5}{20\mu^2t^2\sqrt{\mu t}}-\frac{x^7}{84\mu^3t^3\sqrt{\mu t}}\ldots\right]$$ ……………………………………… (3-63)

and

$$\frac{\partial}{\partial t}\left[erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right] = \frac{2}{\sqrt{\pi}}\left[-\frac{x}{4\mu^{1/2}t^{3/2}}\right]\left\{1-\frac{x^2}{1!\mu t}+\frac{x^5}{2!(\mu t)^2}-\frac{x^6}{3!(\mu t)^3}\ldots\right\}$$

$$= \frac{2}{\sqrt{\pi}}\left[-\frac{x}{4\mu^{1/2}t^{3/2}}\right]\left\{\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{x^2}{\mu t}\right)^n\right\}$$

$$e^{-u} = 1-u+\frac{u^2}{2!}-\frac{u^3}{3!}+\frac{u^4}{4!}-\cdots+\frac{(-1)^n}{n!}+\cdots$$

that is

$$\frac{\partial}{\partial t}\left[erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right] = \frac{2}{\sqrt{\pi}}\left[-\frac{x'}{4\mu^{1/2}t^{3/2}}\right]e^{-\frac{x'^2}{\mu t}}$$ ……………………………………………………… (3-64)

The same of

$$\frac{\partial}{\partial t}\left[erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right] = \frac{2}{\sqrt{\pi}}\left[-\frac{x'}{4\mu^{1/2}t^{3/2}}\right]e^{-\frac{x'^2}{\mu t}}$$ ………………………………………………………. (3-65)

$$\frac{\partial y}{\partial t} = f_1\left(1-e^{-\frac{\omega x'}{\mu}}\right)\frac{2}{\sqrt{\pi}}\left[\frac{x}{4\mu^{1/2}t^{3/2}}\right]e^{-\frac{x^2}{\mu t}} + g_1\left(e^{-\frac{\omega x'}{\mu}}-e^{-\frac{\omega l}{\mu}}\right)\frac{2}{\sqrt{\pi}}\left(\frac{x'}{4\mu^{1/2}t^{3/2}}\right)e^{-\frac{x'^2}{\mu t}}$$

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$$\int\left(\frac{\partial y}{\partial t}\right)dx=\frac{2}{\sqrt{\pi}}f_1\left(\frac{1}{4\mu^{1/2}t^{3/2}}\right)\int\left(1-e^{-\frac{\omega x'}{\mu}}\right)\left(xe^{-\frac{x^2}{\mu}}\right)dx-g_1\frac{2}{\sqrt{\pi}}\left(\frac{1}{4\mu^{1/2}t^{3/2}}\right)\left(e^{-\frac{\omega x'}{\mu}}-e^{-\frac{\omega l}{\mu}}\right)x'e^{-\frac{x'^2}{\mu t}}dx'$$

$$\int xe^{-\frac{x^2}{\mu t}}dx=\left(-\frac{1}{2}\right)ut\int e^{-\frac{x^2}{\mu t}}d\left(-\frac{x^2}{\mu t}\right)$$

$$=\left(-\frac{\mu t}{2}\right)e^{-\frac{x^2}{\mu t}}\int\left(e^{-\frac{\omega x'}{\mu}}\right)\left(xe^{-\frac{x^2}{\mu t}}\right)dx=$$

$$\left(-\frac{\mu t}{2}\right)e^{-\frac{x^2}{\mu t}}e^{-\frac{\omega x'}{\mu}}+\frac{\omega t}{2}\sqrt{\mu t}e^{\frac{4\omega x-\omega^2t^2}{4\mu t}}\sqrt{\frac{\pi}{2}}\left\{\frac{2}{\sqrt{\pi}}\int_{u_1'}^{u_2'}e^{-u^2}du\right\}=\left(-\frac{\mu t}{2}\right)\left(e^{-\frac{x^2+\omega x't}{\mu t}}\right)+\frac{\omega t\sqrt{\pi\mu t}}{4}e^{-\frac{4\omega x-\omega^2t^2}{4\mu t}}\left\{erf\left(\frac{2l-x}{2\sqrt{\mu t}}\right)+erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right\}$$

Where:

$$t^*=\frac{x}{\omega},u_1'=-\frac{\omega t'}{2\sqrt{\mu t}};\ u_2'=\frac{2l-\omega t'}{2\sqrt{\mu t}};$$

$$\int\frac{2}{\sqrt{\pi}}f_1\frac{1}{4\mu^{1/2}t^{3/2}}\left(1-e^{-\frac{\omega x'}{\mu}}\right)xe^{-\frac{x^2}{\mu t}}dx$$

$$=\left\{\left(-\frac{\sqrt{\mu}}{4\sqrt{\pi t}}\right)f_1\left(1-e^{-\frac{\omega x'}{\mu}}\right)e^{-\frac{x^2}{\mu t}}-\frac{\omega}{8}f_1e^{-\frac{4\omega x-\omega^2t^2}{4\mu t}}\left\{erf\left(\frac{2l-x}{2\sqrt{\mu t}}\right)+erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right\}\right\}$$

.... (3-66)

and

$$\int\frac{2}{\sqrt{\pi}}g_1\frac{1}{4\mu^{1/2}t^{3/2}}\left(e^{-\frac{\omega x'}{\mu}}-e^{-\frac{-\omega l}{\mu}}\right)x'e^{-\frac{x'^2}{\mu t}}dx$$

$$=\left\{\left(\frac{\sqrt{\mu}}{4\sqrt{\pi t}}\right)f_1\left(e^{-\frac{\omega x'}{\mu}}-e^{-\frac{-\omega l}{\mu}}\right)e^{-\frac{x^2}{\mu t}}-\frac{\omega}{8}f_1e^{-\frac{4\omega x-\omega^2t^2}{4\mu t}}\left\{erf\left(\frac{2l+x'}{2\sqrt{\mu t}}\right)+erf\left(\frac{x\prime}{2\sqrt{\mu t}}\right)\right\}\right\}$$

.... (3-67)

From continuity equation:

$$\frac{\partial y}{\partial t}+y\frac{\partial u}{\partial x}+u\frac{\partial y}{\partial x}=0$$

$$u(x,t)=\left(-\frac{1}{y}\right)\int\frac{\partial y}{\partial t}dx+u_0=\left(\frac{1}{y}\right)\int\left(-\frac{\partial y}{\partial t}\right)dx+C_c(y_0+h_0)^{1/2}S_0^{1/2}$$

and $y\fallingdotseq H_0+f_1R_1(x,t)+g_1R_2(x',t)$

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$u(x,t) =$

$$\frac{1}{y}\left\{\left(\frac{\sqrt{\mu}}{4\sqrt{\pi t}}\right) f_1 \left(1 - e^{-\frac{\omega x'}{\mu}}\right) e^{-\frac{x^2}{\mu t}} + \frac{\omega}{8} f_1 e^{-\frac{4\omega x - \omega^2 t^2}{4\mu t}} \left\{erf\left(\frac{2l-x}{2\sqrt{\mu t}}\right) + erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right\} -\right.$$

$$\left(\frac{\sqrt{u}}{4\sqrt{\pi t}}\right) g_1 \left(e^{-\frac{\omega x'}{\mu}} - e^{-\frac{\omega t}{\mu}} e^{\frac{x'^2}{\mu}} - \frac{\omega}{8} g_1 e^{-\frac{4\omega x - \omega^2 t^2}{4\mu t}}\right)\left\{erf\left(\frac{2l+x'}{2\sqrt{\mu t}}\right) + erf\left(x' \frac{1}{2\sqrt{\mu t}}\right)\right\} +$$

$$\left. C_c (H_0 + h_0)^{1/2} S_0^{1/2}\right\} \quad \text{...} \quad (3\text{-}68)$$

Solutions of Input Sine or Cosine Function from the Simplified Diffusion Equation

Let's talk about a little complicated condition with the upstream and downstream boundaries as Figure 3.1, if we set,

$$f_1 = y_P \left(1 - \cos\frac{2\pi t}{T_1}\right); \quad g_1 = y_s \left(1 - \cos\frac{2\pi t}{T_2}\right)$$

then: $\frac{\partial y}{\partial t} = \left(f_1 \frac{\partial R_1(x,t)}{\partial t} + g_1 \frac{\partial R_2(x',t)}{\partial t} + R_1(x,t)\frac{\partial f_1}{\partial t} + R_2(x',t)\frac{\partial g_i}{\partial t}\right)$............. (3-69)

We can directly use Eq.(3-63) with $-\int \left(\frac{\partial y}{\partial t}\right) dx = -(\text{Eq.}(3-66) + \text{Eq.}(3-67))$,

and

$$R_1(x,t)\frac{\partial f_1}{\partial t} = \left(1 - e^{-\frac{\omega x'}{\mu}}\right)\left[1 - erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right]\frac{2\pi y_p}{T_1} sin\frac{2\pi t}{T_1}$$

$$R_2(x',t)\frac{\partial g_1}{\partial t} = \left(e^{-\frac{\omega x'}{\mu}} - e^{-\frac{\omega l}{\mu}}\right)\left[1 - erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right]\frac{2\pi y_\delta}{T_2} sin\left(\frac{2\pi t}{T_2}\right)$$

$$erf\left(\frac{x}{2\sqrt{\mu t}}\right) dx = \frac{2}{\sqrt{\pi}} 2\sqrt{\mu t} \int \left(v - \frac{1}{3}v^3 + \frac{v^5}{5 \times 2\,!} - \frac{v^7}{7 \times 3\,!} + \cdots\right) dv$$

$$= \frac{4\sqrt{\mu t}}{\sqrt{\pi}} \sum_{n=0}^{\infty} \left[\frac{(-1)^n \left(\frac{x^2}{4\mu t}\right)^{n-1}}{(2n+1)(2n+2)n\,!}\right] \quad \text{...} \quad (3\text{-}70)$$

let $v = \frac{x}{2\sqrt{\mu t}}$, and the example of calculation $R_1(x,t)$, as Figure 3.2, if n=0,

$\int \left(R_1(x,t)\frac{\partial f_1}{\partial t}\right) dx =$

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$$\left(\frac{2\pi y_p}{T_1}\right) sin\left(\frac{2\pi t}{T_1}\right)\left\{x\left(1+\frac{\sqrt{\mu}e^{-\frac{\omega x'}{\mu}}}{\omega(\sqrt{\pi t})}\right)-e^{-\frac{\omega x'}{\mu}}\frac{\mu}{\omega}\left(1+\frac{\mu}{\omega\sqrt{\pi\mu t}}\right)-\frac{x^2}{2\sqrt{\pi\mu t}}\right\} \text{.... (3-71)}$$

$$R_2(x',t)\frac{\partial g_1}{\partial t}=\left(e^{-\frac{\omega x'}{\mu}}-e^{-\frac{\omega l}{\mu}}\right)\left(1-erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right)\frac{2\pi y_\delta}{T_2}sin\left(\frac{2\pi t}{T_2}\right) \text{.............. (3-72)}$$

$$\int\left(R_2(x',t)\frac{\partial g_1}{\partial t}\right)dx=-\int\left(R_2(x',t)\frac{\partial g_1}{\partial t}\right)dx' \text{ .... (3-73)}$$

if n=0, then

$$\int \left(R_2(x',t)\frac{\partial g_1}{\partial t}\right)dx=$$

$$\left(-\frac{2\pi y_\delta}{T_2}\right)\left(\sin\frac{2\pi t}{T_2}\right)\left\{(x')^2\left(\frac{1}{2\sqrt{\pi\mu t}}\right)e^{-\frac{\omega l}{\mu}}+(x')\left(\frac{\mu}{\omega\sqrt{\pi\mu t}}e^{-\frac{\omega x'}{\mu}}-e^{-\frac{\omega l}{\mu}}\right)-\right.$$
$$\left.\left(\frac{\mu}{\omega}\right)\left(\frac{e^{-\omega x'}}{\mu}\right)\left(\frac{\mu}{\omega\sqrt{\pi\mu t}}-1\right)\right\} \text{ .... (3-74)}$$

Therefore the complete solutions for both water depth and flow velocity are,

$$y_T(x,t)=y_0+y_p\left(1-\cos\frac{2\pi t}{T_1}\right)\left(1-e^{-\frac{\omega x'}{\mu}}\right)\left(1-erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right)+y_s\left(1-\right.$$
$$\left.\cos\frac{2\pi t}{T_2}\right)\left(e^{-\frac{\omega x'}{\mu}}-e^{-\frac{\omega t}{\mu}}\right)\left(1-erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right) \text{ .... (3-75.1)}$$

$$u_T(x,t)=$$

$$\frac{1}{y_T}\left\{\left(\frac{\sqrt{\mu}}{4\sqrt{\pi t}}\right)y_p\left(1-\cos\frac{2\pi t}{T_1}\right)\left(1-e^{-\frac{\omega x'}{\mu}}\right)\left(e^{-\frac{x^2}{\mu t}}\right)+\right.$$

$$\frac{\omega}{8}y_p\left(1-\cos\frac{2\pi t}{T_1}\right)\left(e^{-\frac{4l\omega t-\omega^2t^2}{4\mu t}}\right)\left[erf\left(\frac{2l-x}{2\sqrt{\mu t}}\right)+erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right]-$$

$$\left(\frac{2\pi y_p}{T_1}\right)sin\left(\frac{2\pi t}{T_1}\right)\left[x\left(1+\frac{\sqrt{\mu}e^{-\frac{\omega x'}{\mu}}}{\omega\sqrt{\pi t}}\right)-\frac{x^2}{2\sqrt{\pi\mu t}}-\left(\frac{\mu}{\omega}\right)\left(e^{-\frac{\omega x'}{\mu}}\right)\left(1+\frac{\mu}{\omega\sqrt{\pi\mu t}}\right)\right]-$$

$$\left(\frac{\sqrt{\mu}}{4\sqrt{\pi t}}\right)y_\delta\left(1-\cos\frac{2\pi t}{T_2}\right)\left(e^{-\frac{\omega x'}{\mu}}-e^{-\frac{\omega t}{\mu}}\right)\left(e^{-\frac{x'^2}{\mu t}}\right)-$$

$$\frac{\omega}{8}y_\delta\left(1-\cos\frac{2\pi t}{T_2}\right)\left(e^{-\frac{4l\omega t-\omega^2t^2}{4\mu t}}\right)\left[erf\left(\frac{2l+x'}{2\sqrt{\mu t}}\right)-erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right]+$$

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$$\left(\frac{2\pi y_\delta}{T_2}\right) sin\left(\frac{2\pi t}{T_2}\right)\Bigg\}\Bigg\{(x')^2\left(\frac{1}{2\sqrt{\pi\mu t}}\right)e^{-\frac{\omega l}{\mu}}+(x')\left(\frac{\mu}{2\sqrt{\pi\mu t}}\right)\left(e^{-\frac{\omega x'}{\mu}}-e^{-\frac{\omega l}{\mu}}\right)-$$

$$\left(\frac{\mu}{\omega}\right)\left(e^{-\frac{\omega x'}{\mu}}\right)\left(\frac{\mu}{\omega\sqrt{\pi\mu t}}-1\right)\Bigg\}+u_0 \quad \text{……………………………………….. (3-75.2)}$$

if we only consider the upstream control for y_s=0,

$$y_u(x,t)=y_0+y_p\left(1-\cos\frac{2\pi t}{T_1}\right)\left(1-e^{-\frac{\omega x'}{\mu}}\right)\left[1-erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right] \quad \text{……………. (3-76)}$$

$$u_u(x,t)=$$

$$\frac{1}{y}\left\{\left(\frac{\sqrt{\mu}}{4\sqrt{\pi t}}\right)y_p\left(1-\cos\frac{2\pi t}{T_1}\right)\left(1-e^{-\frac{\omega x'}{\mu}}\right)\left(e^{-\frac{x^2}{\mu t}}\right)\right\}+$$

$$\frac{\omega}{8}y_p\left(1-\cos\frac{2\pi t}{T_1}\right)\left(e^{-\frac{4l\omega t-\omega^2t^2}{4\mu t}}\right)\left[erf\left(\frac{2l-x}{2\sqrt{\mu t}}\right)+erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right]-$$

$$\left(\frac{2\pi y_p}{T_1}\right)sin\left(\frac{2\pi t}{T_1}\right)\left[x\left(1+\frac{\sqrt{u}e^{-\frac{\omega x'}{\mu}}}{\omega\sqrt{\pi t}}\right)-\frac{x^2}{2\sqrt{\pi\mu t}}-\left(\frac{\mu}{\omega}\right)\left(e^{-\frac{\omega x'}{\mu}}\right)\left(1+\frac{\mu}{\omega\sqrt{\pi\mu t}}\right)+\right.$$

$$\left.u_0\right] \quad \text{………………………………… (3-77)}$$

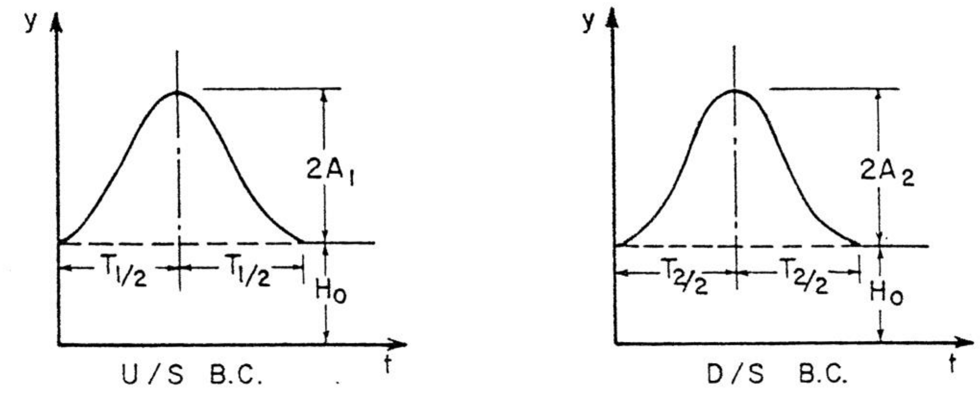

Figure 3.1 Boundary Conditions Considered in the Study

Numerical Solution Methods of the St Venant Equations

One-dimensional differential equations of gradually varied unsteady flow have been used in the following from:

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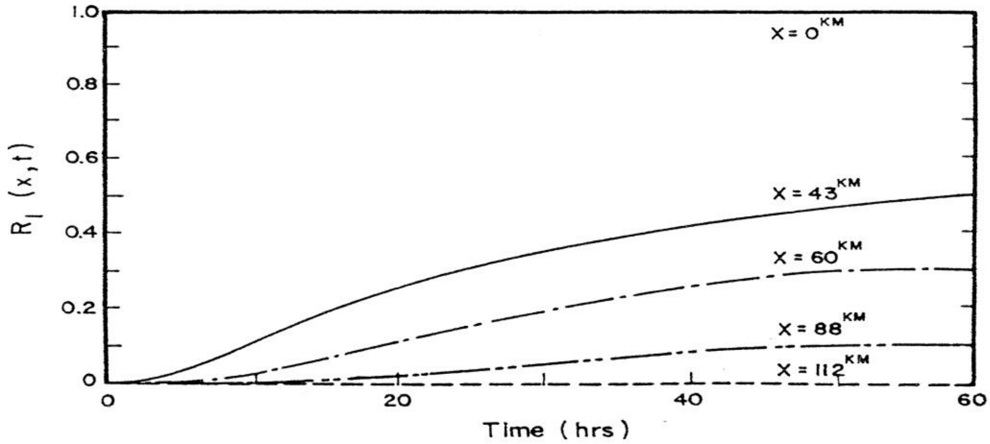

Figure 3.2 The U/S Effect, R1 (x,t) for ω= 0.788 m/s, μ= 10,000 m3/s

$$\frac{\partial Q}{\partial x} + B\frac{\partial Z}{\partial t} = 0 \quad \text{(3-78)}$$

$$\frac{\partial y}{\partial x} + S_f + \frac{1}{g}\left[\frac{\partial}{\partial t}\left(\frac{Q}{A}\right) + \frac{Q}{A}\frac{\partial}{\partial x}\left(\frac{Q}{A}\right)\right] = S_0 \quad \text{(3-79)}$$

A solution of the above equations attains when it is possible to determine the values of the unknowns y(x ,t) and Q(x , t), delimited by the horizontal line of the initial conditions, at t=t0, and the two vertical lines of the boundary conditions at the extreme sections.

1. Direct Difference Methods:

Direct difference methods are based on replacing the partial derivatives, can be written as:

$$Q \cong P_s P_t Q_l^{k+1} + (1-P_s)P_t Q_l^{k+1} + P_s(1-P_s)Q_{l+1}^{k} + (1-P_t)(1-P_t)Q_l^{k} \quad \text{... (3-80)}$$

$$\frac{\partial Q}{\partial x} \cong \frac{P_t}{\Delta t}\left(Q_{l-1}^{k+1} - Q_l^{k+1}\right) + \frac{1-P_t}{\Delta x}\left(Q_{l+1}^{k+1} - Q_l^{k}\right) \quad \text{(3-81)}$$

$$\frac{\partial Q}{\partial t} \cong \frac{P_S}{\Delta t}\left(Q_{l-1}^{k+1} - Q_{l+1}^{k}\right) + \frac{1-P_t}{\Delta x}\left(Q_l^{k+1} - Q_l^{k}\right) \quad \text{(3-82)}$$

in which the upper index, k, refers to time and the lower, l, refers to space. In these formulae P_t and P_S are suitable weighting coefficient between 0 and 1 with which the different variables and their derivatives are averaged in relation to space (P_S) and time (P_t).

2. Stability Analysis of the Adopting Numerical Scheme

The St Venant equations can be rewritten as following forms:

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$\frac{\partial J}{\partial t} = MJ$.. (3-83)

the two-dimensional vector J and the 2×2 matrix M are then expressed as:

$J = \binom{y}{u},\ M = \begin{pmatrix} -u\frac{\partial}{\partial x} - \frac{q}{By} & -\frac{A}{B}\frac{\partial}{\partial x} - \frac{1}{B}\frac{\partial A}{\partial x} \\ -g\left(\frac{\partial}{\partial x} + \frac{S_0 - S_f}{y}\right) & -u\frac{\partial}{\partial x} \end{pmatrix}$.. (3-84)

if we simplify the matrix M as the following:

$M1 = \begin{pmatrix} -u\frac{\partial}{\partial x} & -\frac{A}{B}\frac{\partial}{\partial x} \\ -g\left(\frac{\partial}{\partial x}\right) & -u\frac{\partial}{\partial x} \end{pmatrix}$ .... (3-85)

then Eq.(3-82) becomes

$\frac{\partial J}{\partial t} = M_1 J$ .... (3-86)

Using the finite difference approximation:

$\frac{J_{(x)}^{n+1} - J_{(x)}^{n}}{\Delta t} = L^*\left[\eta^* J_{(x)}^{n+1} + (1+\eta^*) J_{(x)}^{n}\right]$ .... (3-87)

η^* :the time weighting factor, and L^* a space differential operator. After some transformations, Eq.(3-87) can be written:

$J_{(x)}^{n+1} = CJ_{(x)}^{n}$ .... (3-88)

Following the Von Neumann:

$\hat{J}_{(k)}^{n+1} = G\hat{J}_{(k)}^{n}$ .... (3-89)

where G deduced from C, and amplification matrix, and $|G|$ the determination of Matrix $G \leq 1\ for\ \forall K$. Assuming Ps=0.5, for simplicity, Eq. (3-80), (3-81) and (3-82) can be analyzed if Pt=0.5, Eq.(3-87) and matrix G can be obtained:

$G = \begin{pmatrix} \frac{b_1 c_2 - c_1 b_2}{a_1 b_2 - a_2 b_1} & \frac{b_1 d_2 - b_2 d_1}{a_1 b_2 - a_2 b_1} \\ \frac{c_1 a_2 - c_2 a_1}{a_1 b_2 - a_2 b_1} & \frac{d_1 a_1 - d_2 a_1}{a_1 b_2 - a_2 b_1} \end{pmatrix}$ .... (3-90)

$$a_1 = 1 + p_t ur; \quad a_2 = p_t gr;$$

$b_1 = p_t yr; \quad b_2 = 1 + p_t ur;$

$c_1 = -1 + (1 - p_t)ur; \quad c_2 = (1 - p_t)gr;$

$d_1 = (1 - p_t)yr; \quad d_2 = -1 + (1 - p_t)ur;$

$r = i\frac{2\Delta t}{\Delta x} tg\frac{\Delta x}{2}$.. (3-91)

the two eigen-values of matrix G are:

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$$\lambda = \frac{1+(1-p_t)p_t r^2(gy-u^2)-ur+2p_t ur \pm r\sqrt{gy}}{1+p_t^2 r^2(gy-u^2)-2p_t ur} \quad \text{......... (3-92)}$$

$$\lambda = \frac{1+(1-p_t)p_t r^2(gy-u^2)-ur+2p_t ur \pm r\sqrt{gy}}{1+p_t^2 r^2(gy-u^2)-2p_t ur}$$

(a): $p_t = 0, (explicit\ scheme): \lambda = 1 - \mathrm{r}\left(u \mp \sqrt{gy}\right)$

$|\lambda| < 1\,, stable\colon\ |\lambda| \geq 1\,, unstable$

(b): $p_t = 1, (implicit\ scheme): \lambda = \frac{1}{1+\mathrm{r}\,(u \mp \sqrt{gy})}$

$|\lambda| \leq 1\,, for\ any\ quantities, it's\ stable.$

(c): $p_t = 0.5, (implicit\ scheme): \lambda = \frac{1+\frac{\mathrm{r}^2}{4}(gy-u^2) \pm r\sqrt{gy}}{1-\frac{\mathrm{r}^2}{4}(gy-u^2)+ru}$

.... (3-93)

for which: r is a purely imaginary quantities, it is easy to see that $|\lambda| = 1$ for any value of quantity.

The one-dimensional differential equation of gradually varied unsteady flow including the lateral inflows or outflows is:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_L$$

$$\bar{S}_f + \frac{\partial y}{\partial x} + \frac{1}{g}\left[\frac{\partial}{\partial t}\left(\frac{Q}{A}\right) + \frac{Q}{A}\frac{\partial}{\partial x}\left(\frac{Q}{A}\right)\right] = S_0 - \frac{q_L\left(\frac{Q}{A} - u^*\right)}{gA}$$

q_L: lateral inflow per unit length

$U = \left(\frac{Q}{A} - u^*\right)$: relative velocity component for lateral inflow

u^*: the stream-wise velocity component of the lateral inflow

The numerical solutions for the completely form of the St Venant Equation are:

$$\left(\frac{B_t^{k+t}+B_{t+1}^{k+1}}{2}\right)\left(\frac{\Delta x_t}{\Delta t}\right)\left\{p_s\left(Z_{t+1}^{k+1} - Z_{t+1}^{k}\right) + (1-p_s)\left(Z_t^{k+1} - Z_t^{k}\right)\right\} + \left\{p_t\left(Q_{t+1}^{k+1} + Q_t^{k+1}\right) + (1-p_t)\left(Q_{t+1}^{k} - Q_t^{k}\right)\right\} = q_L \quad \text{......... (3-94)}$$

$$\left(\frac{\Delta x_t}{2C_c^2}\right)\left\{\frac{\left(Q_t^{k+1}\right)^2}{\left(y_t^{k+1}\right)\left(A_t^{k+1}\right)^2} + \frac{\left(Q_{t+1}^{k+1}\right)^2}{\left(y_{t+1}^{k+1}\right)\left(A_{t+1}^{k+1}\right)^2}\right\} = S_0\Delta x - \left\{p_t\left(y_{t+1}^{k+1} - y_t^{k+1}\right) + (1-p_t)\left(y_{t+1}^{k} - y_t^{k}\right)\right\} - \frac{\Delta x_t}{g\Delta t}\left\{p_s\left(\frac{Q_{t+1}^{k+1}}{y_{t+1}^{k+1}B_{t+1}^{k+1}} - \frac{Q_{t+1}^{k}}{y_{t+1}^{k}B_{t+1}^{k}}\right) + (1-p_s)\left(\frac{Q_t^{k+1}}{y_t^{k+1}B_t^{k+1}} - \frac{Q_t^{k}}{y_t^{k}B_t^{k}}\right)\right\} - \frac{1}{2g}\left\{\frac{Q_{t+1}^{k+1}}{y_{t+1}^{k+1}B_{t+1}^{k+1}} + \frac{Q_t^{k+1}}{y_t^{k+1}B_t^{k+1}}\left[p_t\left(\frac{Q_{t+1}^{k+1}}{y_{t+1}^{k+1}B_{t+1}^{k+1}} - \frac{Q_t^{k+1}}{y_t^{k+1}B_t^{k+1}}\right)\right] + (1-p_t)\left(\frac{Q_{t+1}^{k}}{y_{t+1}^{k}B_{t+1}^{k}} - \frac{Q_t^{k}}{y_t^{k}B_t^{k}}\right)\right\} -$$

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$$\frac{q_L\left(\frac{Q_{t+1}^{k+1}}{y_{t+1}^{k+1}B_{t+1}^{k+1}}-\frac{Q_t^{k+1}}{y_t^{k+1}B_t^{k+1}}-2u^*\right)}{g\left(y_{t+1}^{k+1}B_{t+1}^{k+1}+y_t^{k+1}B_t^{k+1}\right)} \quad \text{(3-95)}$$

if there are no lateral inflow and B=constant, then Eqs.(3-94) and (3-95) become to:

$$\frac{\Delta x_t}{\Delta t}\left\{p_s\left(Z_{t+1}^{k+1}-Z_{t+1}^{k}\right)+(1-p_s)\left(Z_t^{k+1}-Z_t^k\right)\right\}+$$
$$\left\{p_t\left(q_{t+1}^{k+1}-q_t^{k+1}\right)+(1-p_t)\left(q_{t+1}^{k}-q_t^k\right)\right\}=$$
$$0 \quad \text{(3-96)}$$

$$\left(\frac{\Delta x_t}{2C_c^2}\right)\left\{\frac{\left(q_t^{k+1}\right)^2}{\left(y_t^{k+1}\right)^3}+\frac{\left(q_{t+1}^{k+1}\right)^2}{\left(y_{t+1}^{k+1}\right)^3}\right\}=S_0\Delta x-\left\{p_t\left(y_{t+1}^{k+1}-y_t^{k+1}\right)+(1-p_t)\left(y_{t+1}^{k}-y_t^k\right)\right\}-$$
$$\frac{\Delta x_t}{g\Delta t}\left\{p_s\left(\frac{q_{t+1}^{k+1}}{y_{t+1}^{k+1}}-\frac{q_{t+1}^{k}}{y_{t+1}^{k}}\right)+(1-p_s)\left(\frac{q_t^{k+1}}{y_t^{k+1}}-\frac{q_t^k}{y_t^k}\right)\right\}-\frac{1}{2g}\left\{\frac{q_{t+1}^{k+1}}{y_{t+1}^{k+1}}+\frac{q_t^{k+1}}{y_t^{k+1}}\left[p_t\left(\frac{q_{t+1}^{k+1}}{y_{t+1}^{k+1}}-\frac{q_t^{k+1}}{y_t^{k+1}}\right)\right]+\right.$$
$$\left.(1-p_t)\left(\frac{q_{t+1}^{k}}{y_{t+1}^{k}}-\frac{q_t^k}{y_t^k}\right)\right\} \quad \text{(3-97)}$$

COMPUTATION RESULTS AND COMPARISONS

COMPUTATION RESULTS

A. The properties of the Discharge Parameter, α_q of the Input (upstream control)

For the following set of equations:

$$y=y_0+y_p\left(1-cos\frac{2\pi}{T}\right)\left(1-e^{-\frac{\omega x'}{\mu}}\left[1-erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right]\right)$$

$$u=$$
$$\frac{1}{y}\left\{\left(\frac{\sqrt{\mu}}{4\sqrt{\pi t}}\right)y_p\left(1-\cos\frac{2\pi t}{T}\right)\left(1-e^{-\frac{\omega x'}{\mu}}\right)\left(e^{-\frac{x^2}{\mu t}}\right)\right\}+$$
$$\frac{\omega}{8}y_p\left(1-\cos\frac{2\pi t}{T_1}\right)\left(e^{-\frac{4l\omega t-\omega^2t^2}{4\mu t}}\right)\left[erf\left(\frac{2l-x}{2\sqrt{\mu t}}\right)+erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right]-$$
$$\left(\frac{2\pi y_p}{T}\right)sin\left(\frac{2\pi t}{T}\right)\left[x\left(1+\frac{\sqrt{\mu}e^{-\frac{\omega x'}{\mu}}}{\omega\sqrt{\pi t}}\right)-\frac{x^2}{2\sqrt{\pi\mu t}}-\left(\frac{\mu}{\omega}\right)\left(e^{-\frac{\omega x'}{\mu}}\right)\left(1+\frac{\mu}{\omega\sqrt{\pi\mu t}}\right)\right]+u_0$$

if we let:

$1-\ e^{-\frac{\omega x'}{\mu}}\doteq 1,\ then\ e^{-\frac{\omega x'}{\mu}}=0$ we have,

$$q-q_0=yu-y_0u_0=$$
$$\left\{\left(\frac{\sqrt{\mu}}{4\sqrt{\pi t}}\right)y_p\left(1-\cos\frac{2\pi t}{T}\right)\left(e^{-\frac{x^2}{\mu t}}\right)+\frac{\omega}{8}y_p\left(1-\cos\frac{2\pi t}{T_1}\right)\left(e^{-\frac{4l\omega t-\omega^2t^2}{4\mu t}}\right)\left[erf\left(\frac{2l-x}{2\sqrt{\mu t}}\right)+\right.\right.$$

$$erf\left(\frac{x}{2\sqrt{\mu t}}\right)\Big] - \left(\frac{2\pi y_p}{T}\right) sin\left(\frac{2\pi t}{T}\right)\Big[x - \frac{x^2}{2\sqrt{\pi\mu t}}\Big]\Big\} \quad \text{(4-1)}$$

for x=0

$$q_t(O,t) = q_p^* = y_p\left(1 - \cos\frac{2\pi t}{T}\right)\omega\left[\frac{\sqrt{\mu}}{(4\sqrt{\pi})(\omega\sqrt{t})} + \frac{1}{8}erf\left(\frac{l}{\sqrt{\mu t}}\right)\left(\frac{e^{-4l\omega t} - \omega^2 t^2}{4\mu t}\right)\right] \quad \text{(4-2)}$$

$$\frac{q(O,t) - q_0}{q_p} = \frac{q_p^*}{q_p} = \frac{q_t(O,t)}{y_p\omega} = 1 - \cos\frac{2\pi t}{t}\alpha_q \quad \text{(4-3)}$$

where

$$\alpha_q = \left\{\frac{\sqrt{\mu}}{(4\sqrt{\pi})(\omega t)} + \frac{1}{8}erf\left(\frac{1}{\sqrt{\mu t}}\right)\left(e^{-\frac{4l\omega t - \omega^2 t^2}{4\mu t}}\right)\right\} \quad \text{(4-4)}$$

Now we can group the dimensionless parameters:

$\pi_1 = \left(\frac{l}{\sqrt{\mu t}}\right)^{-1}$, $\pi_2 = \frac{\omega t}{l}$, $\pi_3 = \frac{x}{\sqrt{\mu t}}$, $\frac{\sqrt{\mu}}{4\omega\sqrt{\pi t}} = \left(\frac{1}{4\sqrt{\pi}}\right)\left(\frac{\pi_1}{\pi_2}\right)$, we got

$$y = y_0 + y_p\left(1 - cos\frac{2\pi t}{T}\right)\left(1 - e^{-\frac{\omega x'}{\mu}}\right)\left[1 - erf\left(\frac{x}{2\sqrt{\mu t}}\right)\right] = y_0 + y_p\left(1 - cos\frac{2\pi t}{T}\right)\left[1 - erf\left(\frac{\pi_3}{2}\right)\right] \quad \text{(4-5)}$$

$$u = u_0 + \frac{\omega y_p}{y}\left(1 - cos\frac{2\pi t}{T}\right)\left\{\left(\frac{1}{4\sqrt{\pi}}\right)\left(\frac{\pi_1}{\pi_2}\right)exp(-\pi_3^2) + \frac{1}{8}exp\left(\frac{-4\pi_2 + \pi_2^2}{4\pi_1^2}\right)\left[erf\left(\frac{1}{\pi_1} - \frac{\pi_3}{2}\right) + erf\left(\frac{\pi_3}{2}\right)\right] - \left(\frac{2\pi t}{T}\right)\left(\frac{x}{l}\right)\left(\frac{1}{\pi_2}\right)\left(\frac{sin\frac{2\pi t}{\tau}}{1 - cos\frac{2\pi t}{T}}\right)\left[1 - \frac{\pi_3}{2\sqrt{\pi}}\right]\right\} \quad \text{(4-6)}$$

$$q = yu = q_0 + q_t \quad \text{(4-7)}$$

$$\frac{y(x,t) - y_0}{y_p} = \left(1 - cos\frac{2\pi t}{T}\right)\left[1 - erf\left(\frac{\pi_3}{2}\right)\right] = Y^* \quad \text{(4-8)}$$

$$\frac{q(x,t) - q_0}{y_p\omega} = q^* = \left(1 - cos\frac{2\pi t}{T}\right)\left\{\left(\frac{1}{4\sqrt{\pi}}\right)\left(\frac{\pi_1}{\pi_2}\right)exp(-\pi_3^2) + \frac{1}{8}exp\left(\frac{-4\pi_2 + \pi_2^2}{4\pi_1^2}\right)\left[erf\left(\frac{1}{\pi_1} - \frac{\pi_3}{2}\right) + erf\left(\frac{\pi_3}{2}\right)\right] - \left(\frac{2\pi t}{T}\right)\left(\frac{x}{l}\right)\left(\frac{1}{\pi_2}\right)\left(\frac{sin\frac{2\pi t}{T}}{1 - cos\frac{2\pi t}{T}}\right)\left[1 - \frac{\pi_3}{2\sqrt{\pi}}\right]\right\} \quad \text{(4-9)}$$

the computation results of Eqs. (4-8) and (4-9) are shown in Figures 4.1 and 4.2

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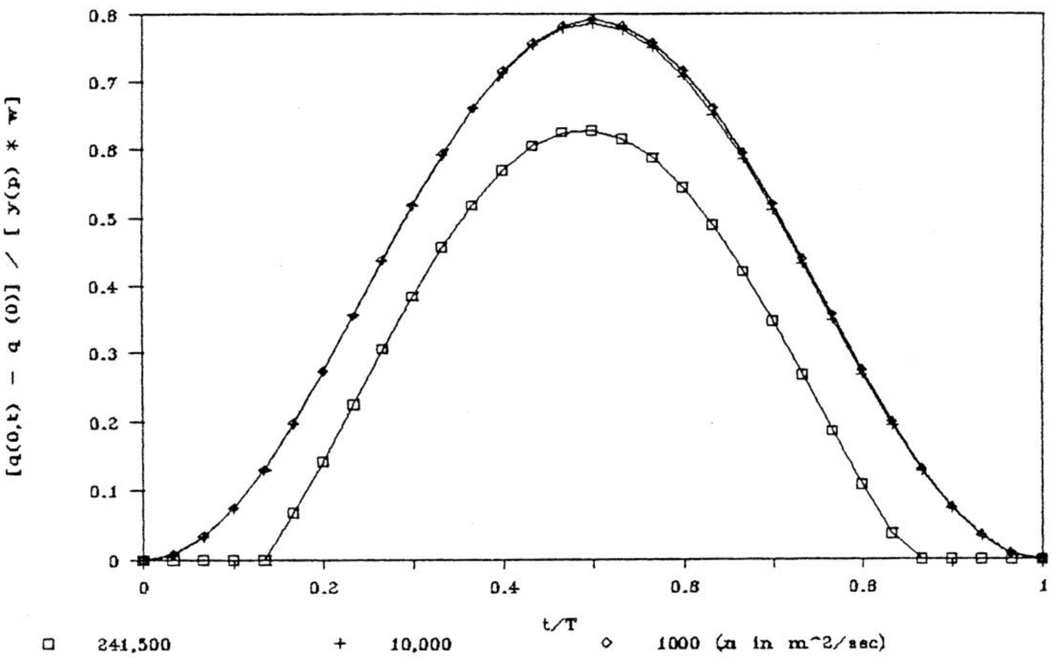

Fig. 4.1 Relationship between the Analytical Results of the Dimensionless Upstream Input Discharge and Dimensionless Period for Different μ and Periods 15 days

if x=0, then: π_3=0; we can obtain that

$$\frac{y(O,t)-y_0}{y_p} = \left(1 - cos\frac{2\pi t}{T}\right) \;,\; \frac{q(O,t)-q_0}{y_p\omega} = \left(1 - cos\frac{2\pi t}{T}\right)\alpha_q \;,$$

$$\alpha_q = \left\{\left(\frac{1}{4\sqrt{\pi}}\right)\left(\frac{\pi_1}{\pi_2}\right) + \frac{1}{8}exp\left(\frac{\pi_2^2-4\pi_2}{4\pi_1^2}\right)erf\left(\frac{1}{\pi_1}\right)\right\} \text{ with } exp\left(\frac{\pi_2^2-4\pi_2}{4\pi_1^2}\right) \le 1 \quad .. (4\text{-}10)$$

In order to determine the limit time of upstream to downstream effect,

$if\ \ \pi_1 \le 2.0,\ when\ \ \pi_2 \dashrightarrow 4.0,\ then\ \ \alpha_q \fallingdotseq 0.14$

$if\ \ \pi_1 > 2.0, when\ \ \pi_2 \dashrightarrow 4.0,\ then\ \ \alpha_q = \alpha_{min}$ .... (4-11)

the computation results of Eq. (4-10) and (4-9) are presented in Figures 4.3

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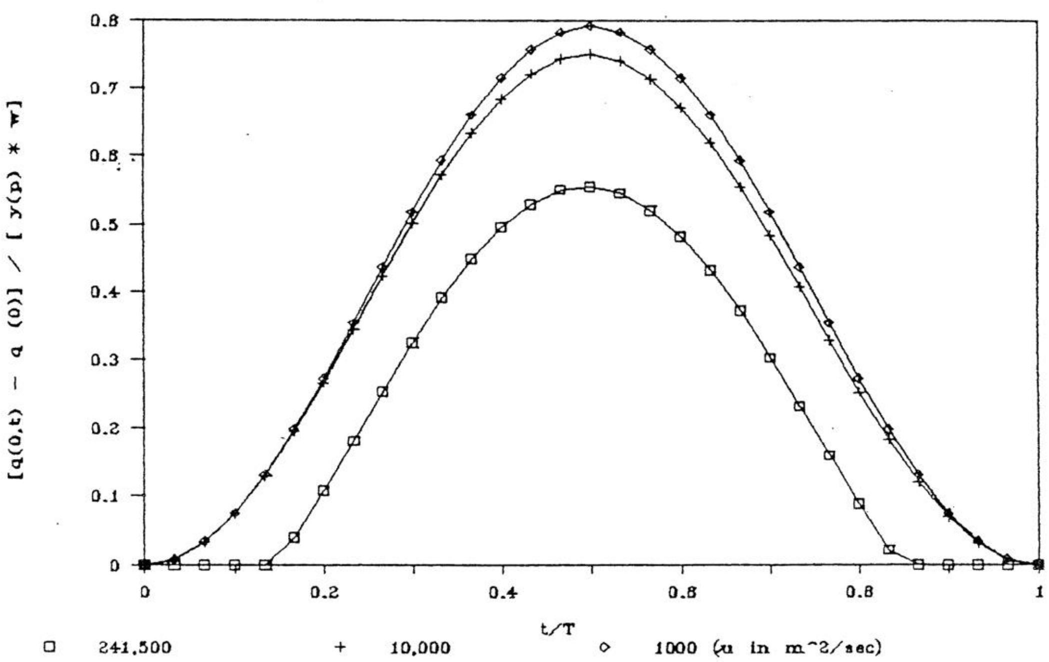

Fig. 4.2 Relationship between the Analytical Results of the Dimensionless Upstream Input Discharge and Dimensionless Period for Different μ and Period 60 days

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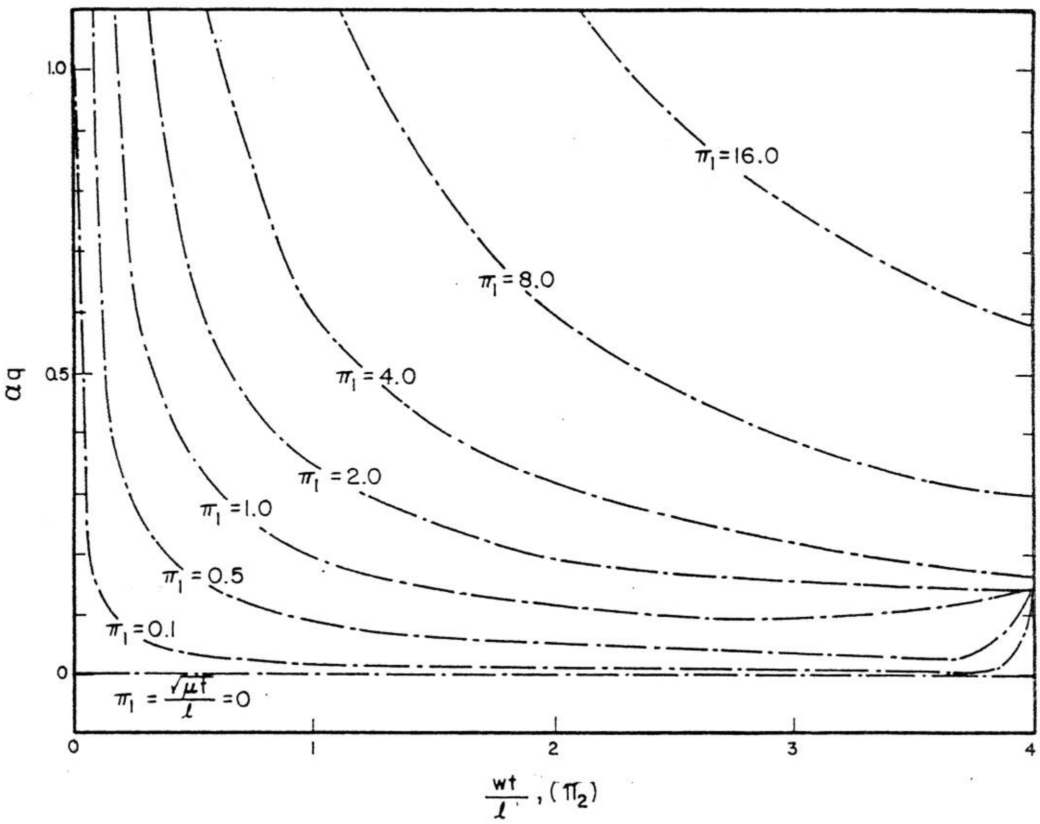

Fig. 4.3 Relationship between the Input Station Discharge Coefficient αq, and Non-dimensional Parameter, $\pi_1 (= \frac{\sqrt{\mu t}}{\ell})$ and $\pi_2 (= wt/\ell)$

B. The properties of the Amplitude Parameter, π_4, of the Downstream Control

$$y = y_d = y_0 + y_s \left(1 - cos\frac{2\pi t}{T}\right)\left(e^{-\frac{\omega x'}{\mu}} - e^{-\frac{\omega l}{\mu}}\right)\left[1 - erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right] \quad \text{............ (4-12)}$$

let assume that $I - e^{-\frac{\omega l}{\mu}} \doteq 1, then\ e^{-\frac{\omega l}{\mu}} = 0$ .... (4-13)

and $\frac{y_d - y_0}{y_s\left(1 - cos\frac{2\pi t}{T}\right)} = Y^{**} = \left(e^{-\frac{\omega x'}{\mu}}\right)\left[1 - erf\left(\frac{x'}{2\sqrt{\mu t}}\right)\right]$.. (4-14)

$$\mu_\omega^{**} = \frac{\omega x'}{\mu};\ \pi_4 = \sqrt{\frac{\omega^2 t}{\mu}} = \frac{\mu_\omega^{**}}{\left(\frac{x'}{2\sqrt{\mu t}}\right)};\ let\ \pi_5 = \frac{x'}{\sqrt{\mu t}}\ then\ Y^{**} = \left(e^{-\mu_\omega^{**}}\right)\left[1 - erf\left(\frac{\pi_5}{2}\right)\right]$$

The result between μ_ω^{**} and Y^{**} could be expressed as following Figure 4.4,

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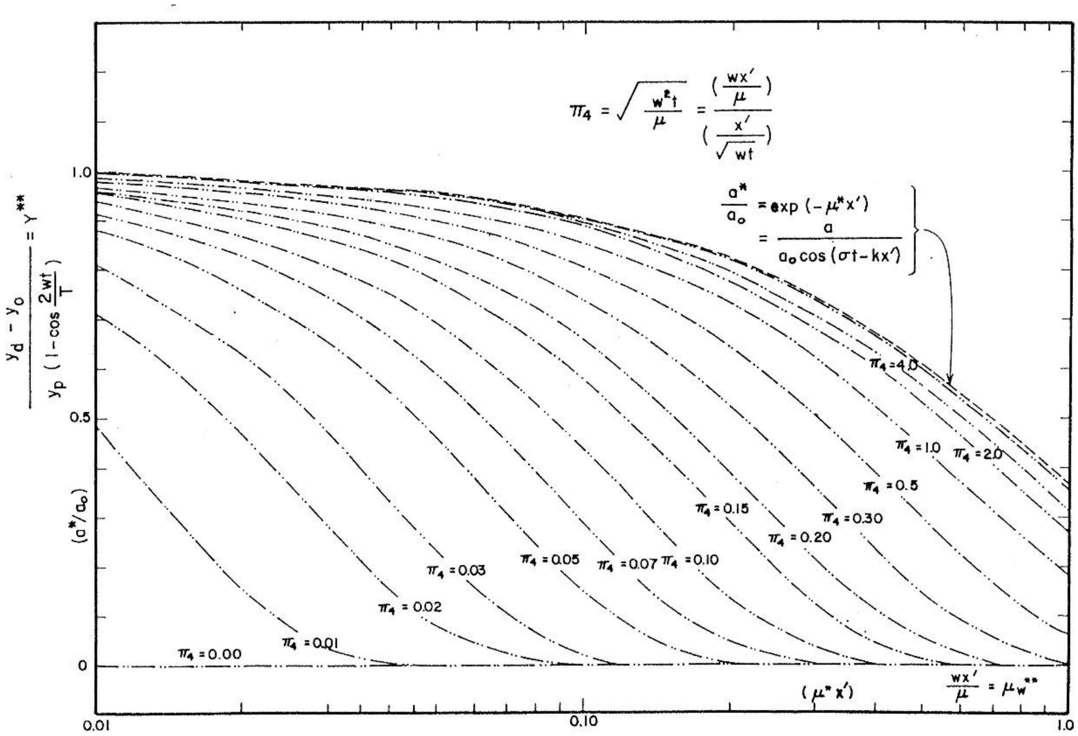

Fig. 4.4 Relationship between the Dimensionless Amplitude, Y**, and Dimensionless dispersion Parameter, μ**

C. Results of Relationship between the Analytical Results of Depth or Velocity with given Bed Slope, Dispersion Parameter and Time Period

The calculation results with Eqs. (3-76) and (3-77) with different bed slopes, dispersion parameter and time periods are presented as the following Figures 4.5 and 4.6.

D. Results of Rating Curves for the Analytical Results between Depth, velocity and the Discharge per Unit width with different Bed Slope, Dispersion Parameter and Time Period

The calculations results of combining Eqs. (3-76) and (3-77) for the given with bed slopes, dispersion parameter, and time periods are expressed in Figures 4.7 and 4.8.

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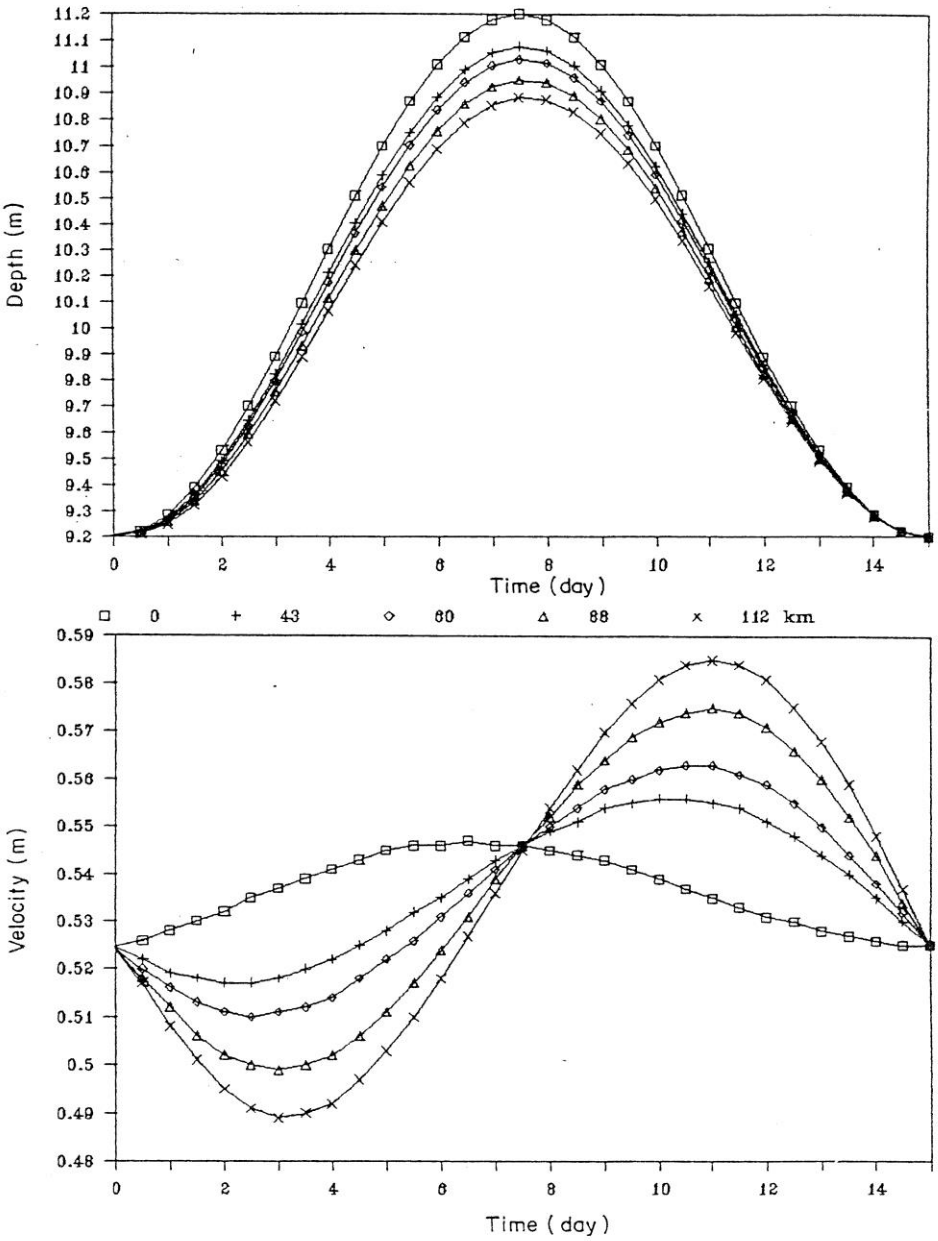

Fig. 4.5 Relationship between the Analytical Results of Depth, Velocity and Time for μ= 241,500 m^2/s Periods 15 days and $S_0=10^{-5}$

E. Results of Rating Curves for the Analytical Results between the Dimensionless Depth and the Dimensionless Discharge for different Bed Slope, Dispersion Parameter and Time Period

The calculations results of combining Eqs. (4-8) and (4-9) for the given bed slopes, diffusion coefficient, and time periods are expressed in Figures 4.9 and 4.14. Obviously seeing, the influences of different bed slope, dispersion parameter and time period on the

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shape of the rating curves between dimensionless depth and dimensionless discharge are really important.

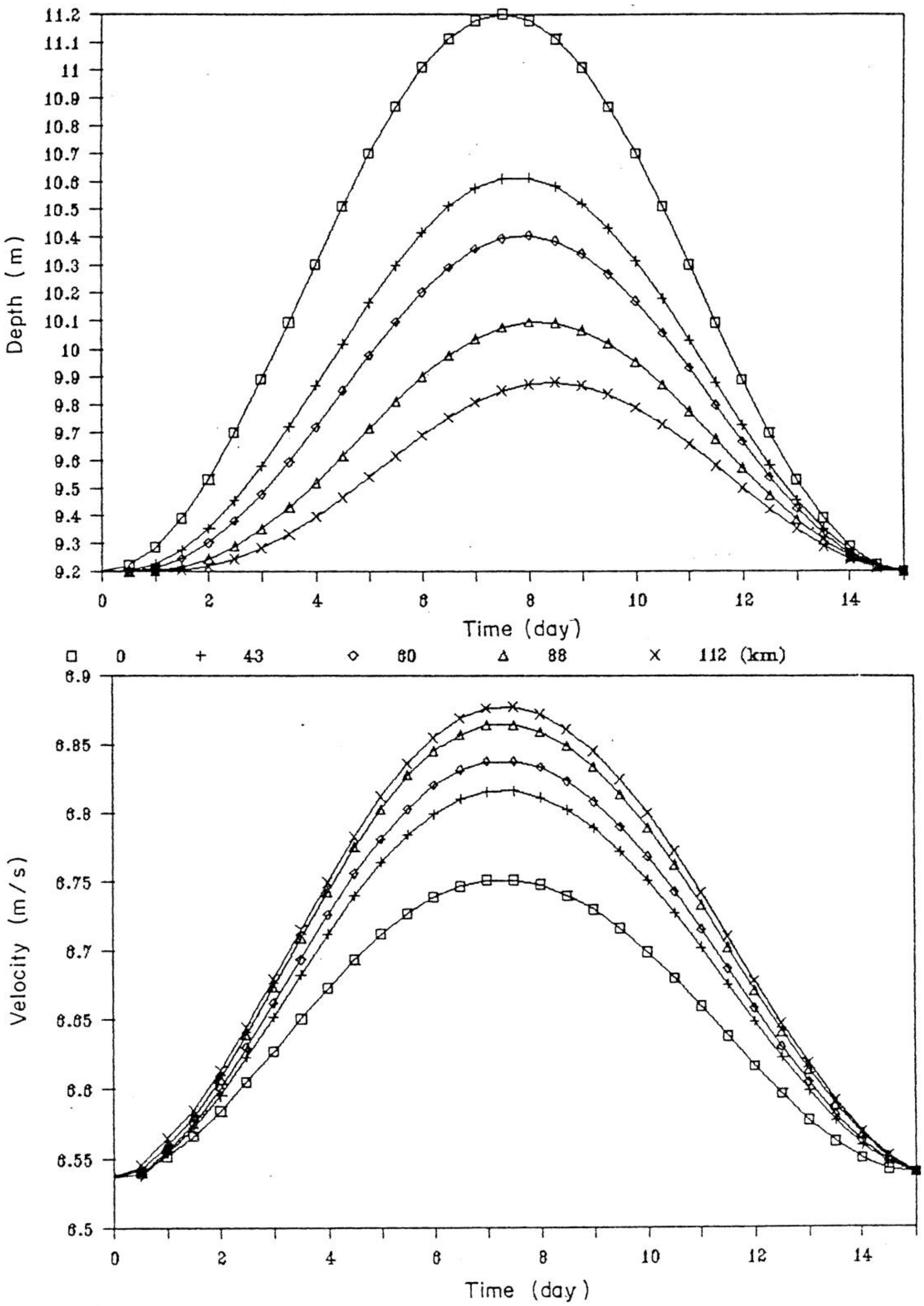

Fig. 4.6 Relationship between the Analytical Results of Depth, Velocity and Time for μ=

10,000 m^2/s Periods 15 days and $S_0=1.55\times10^{-3}$

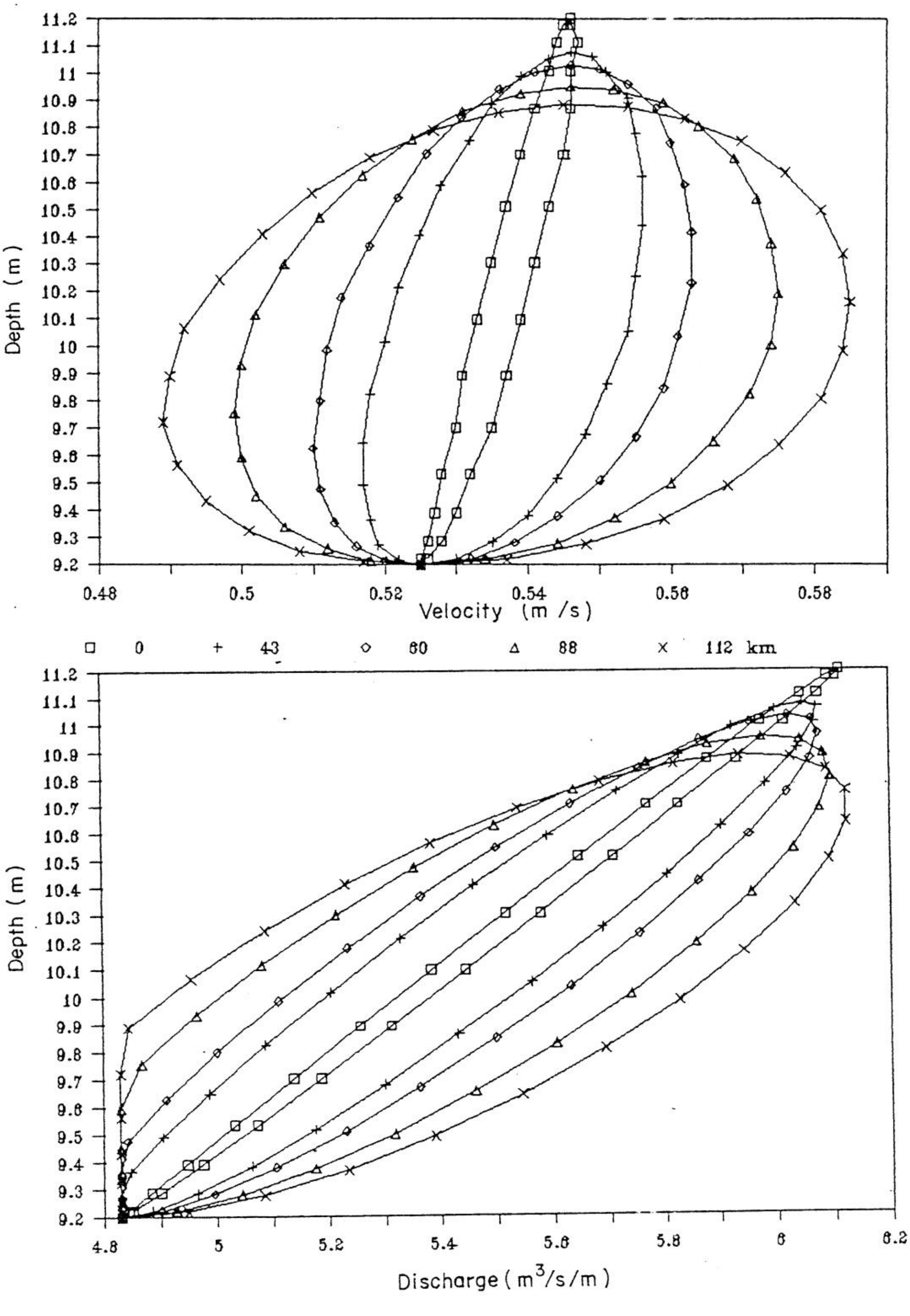

Fig. 4.7 Rating Curves of the Analytical Results between Depth, Velocity and Discharge per Unit Width for μ= 241,500 m^2/s Periods 15 days and $S_0=10^{-5}$

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F. Results from Hydrodynamic, Analytical Model: Water Level and Horizontal Velocity at a Given Station with Different Fresh Water Discharges from Upstream and with Varied Downstream Water Level

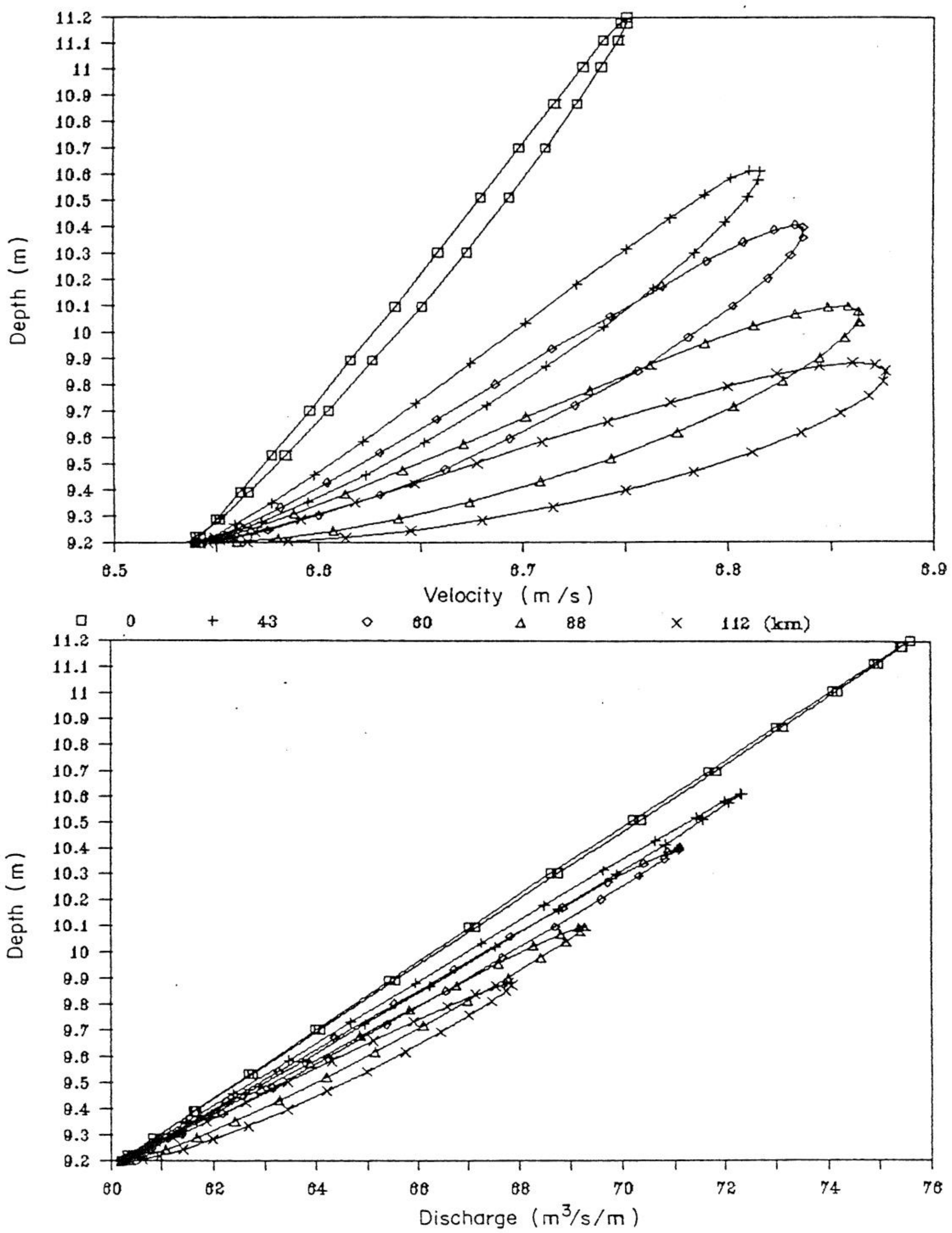

Fig. 4.8 Rating Curves of the Analytical Results between Depth, Velocity and Discharge per Unit width for μ= 10,000 m^2/s Periods 15 days and S_0=1.55×10^{-3}

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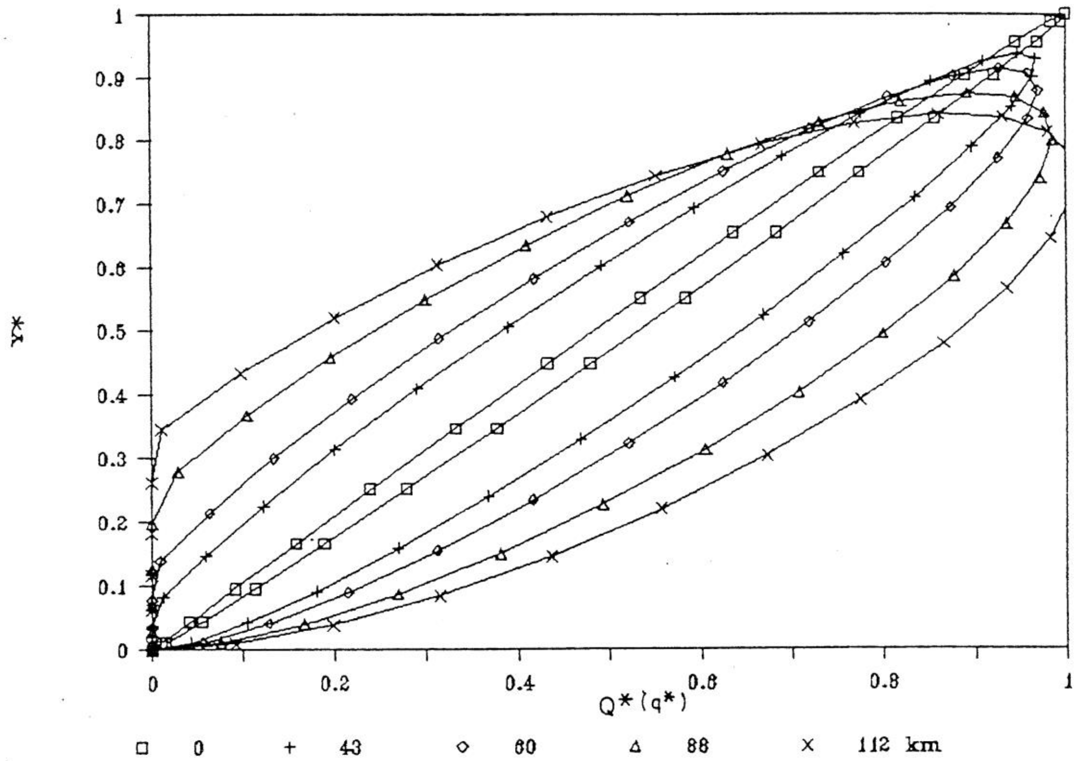

Fig. 4.9 Rating Curves of the Analytical Results between the Dimensionless Depth and the Dimensionless Discharge for μ= 241,500 m^2/s Periods 15 days and $S_0=10^{-5}$

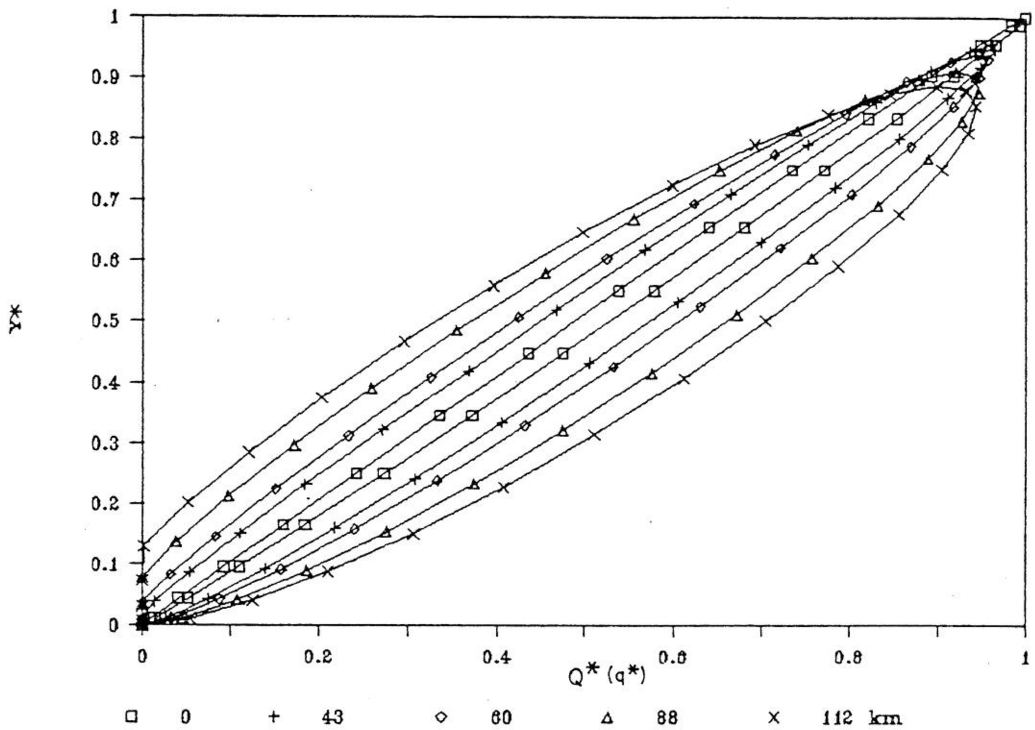

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Fig. 4.10 Rating Curves of the Analytical Results between the Dimensionless Depth and the Dimension less Discharge for μ= 241,500 m^2/s Periods 30 days and $S_0=10^{-5}$

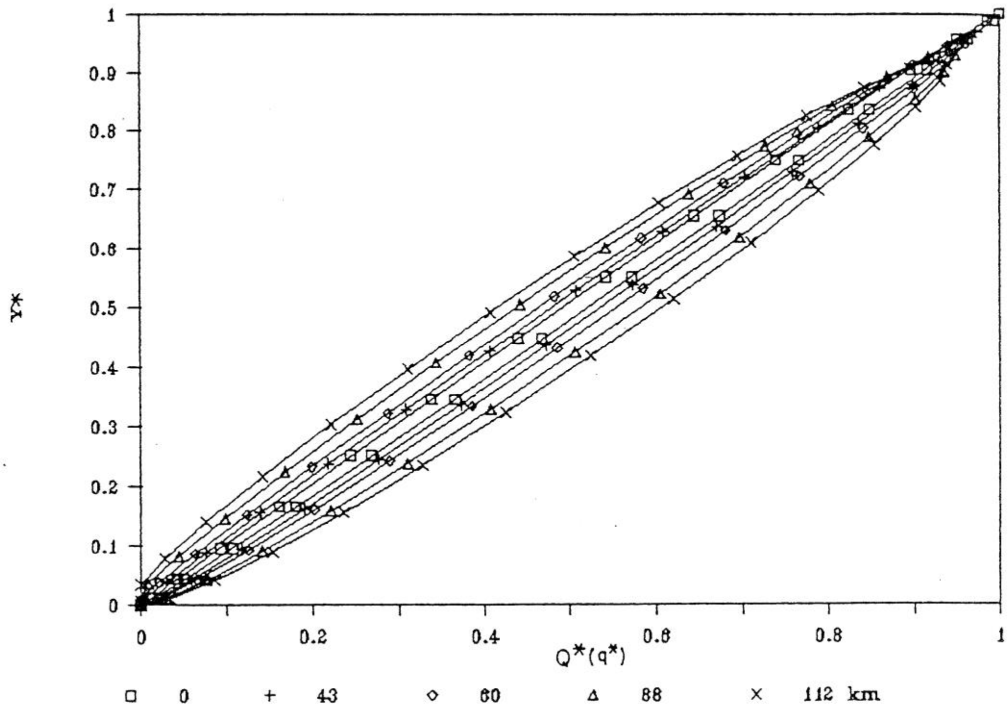

Fig. 4.11 Rating Curves for the Analytical Results between the Dimensionless Depth and the Dimensionless Discharge for μ= 241,500 m^2/s Periods 60 days and $S_0=10^{-5}$

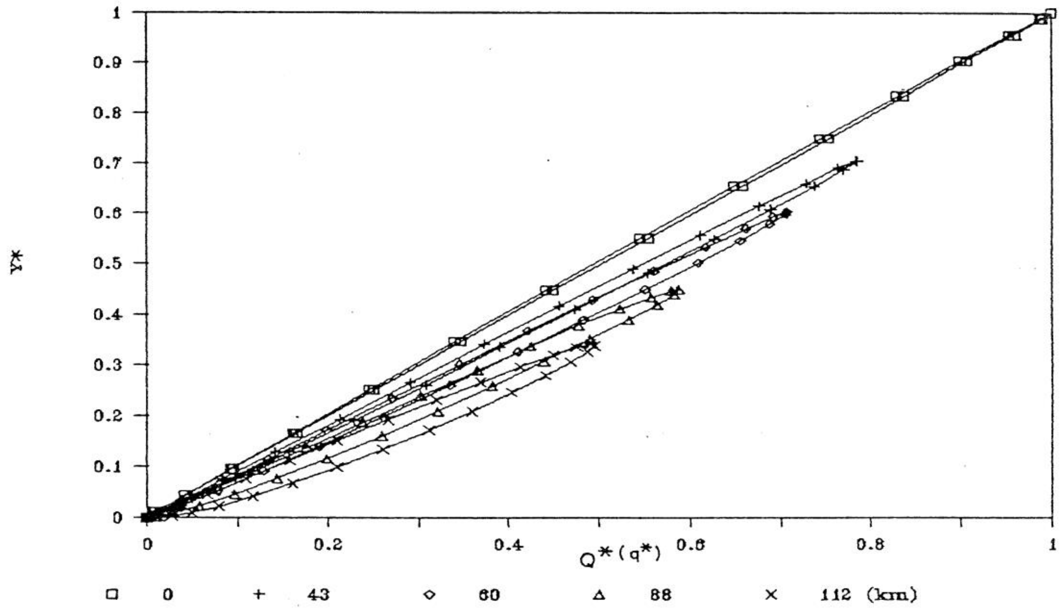

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Fig. 4.12 Rating Curves of the Analytical Results between the Dimensionless Depth and the Dimensionless Discharge for μ= 10,000 m^2/s Periods 15 days and $S_0=1.55\times10^{-3}$

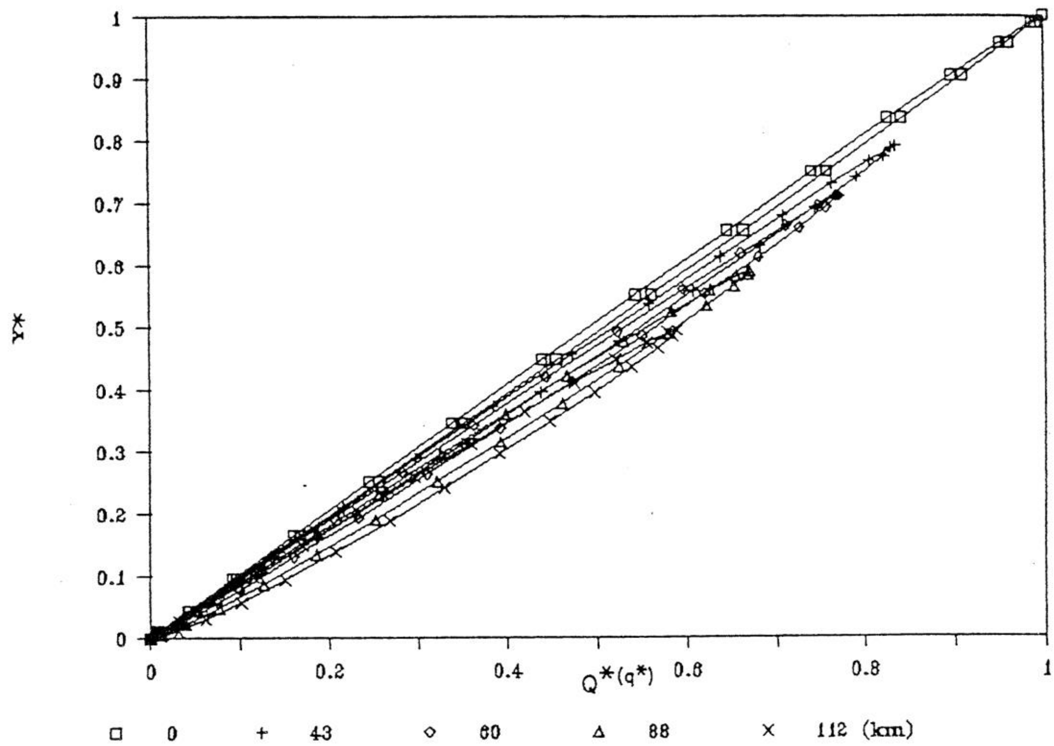

Fig. 4.13 Rating Curves of the Analytical Results between the Dimensionless Depth and the Dimensionless Discharge for μ= 10,000 m^2/s Periods 30 days and $S_0=1.55\times10^{-3}$

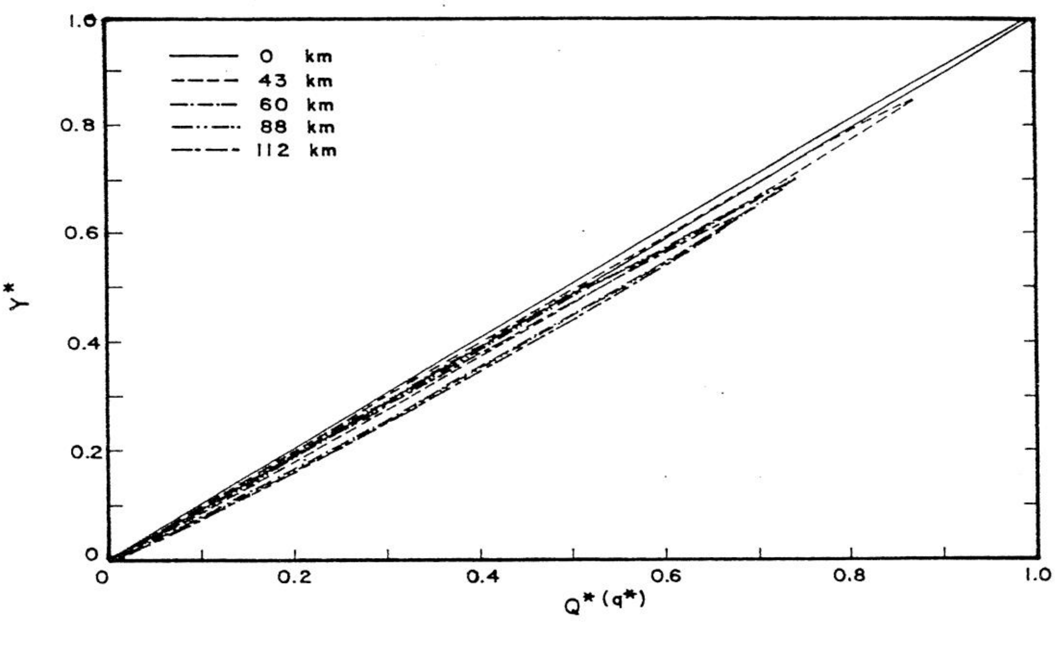

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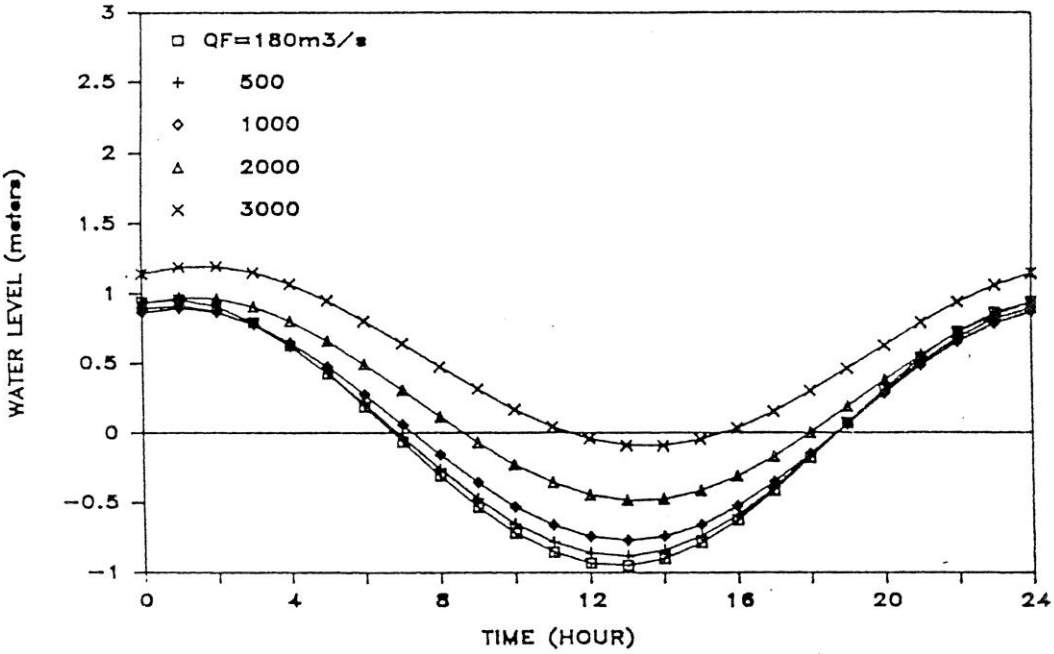
Fig. 4.14 Rating Curves of the Analytical Results between the Dimensionless Depth and the Dimensionless Discharge for μ= 10,000 m^2/s Periods 60 days and $S_0=1.55\times10^{-3}$



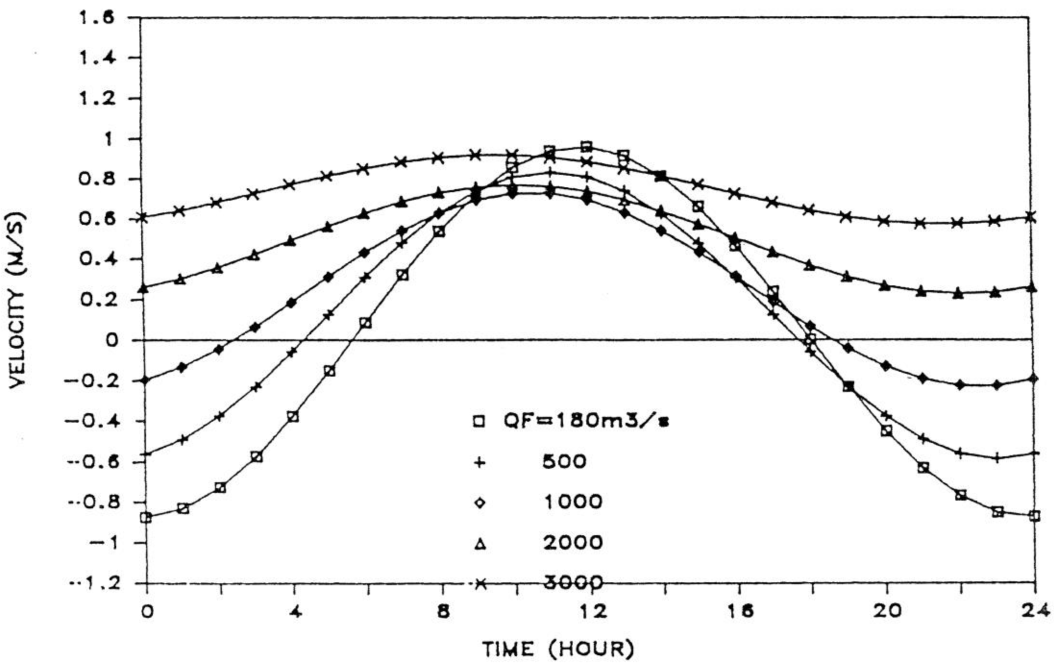


Fig. 4.15 Results from Hydrodynamic, Analytical Model: Water Depth and Horizontal Velocity at Station 24 Km with Different Fresh Water Discharges

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Comparisons the Results between Analytical and Numerical Modellings

A. The Depth-Time for a given station and Time Period with Different Bed Slopes and Weighting Factors on the upstream discharge with a given downstream water level

The comparisons are given in Figures 4.16 and 4.17.

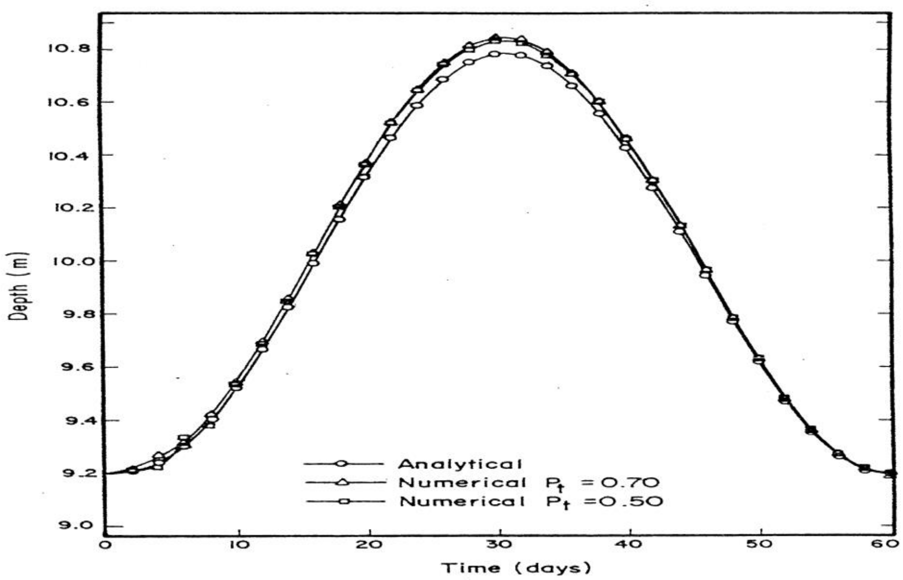

Fig. 4.16 Comparison of Analytical and Numerical Results at the Station X=60km, $S_0=10^{-5}$, T=60 days P_t=0.70 and P_t=0.50

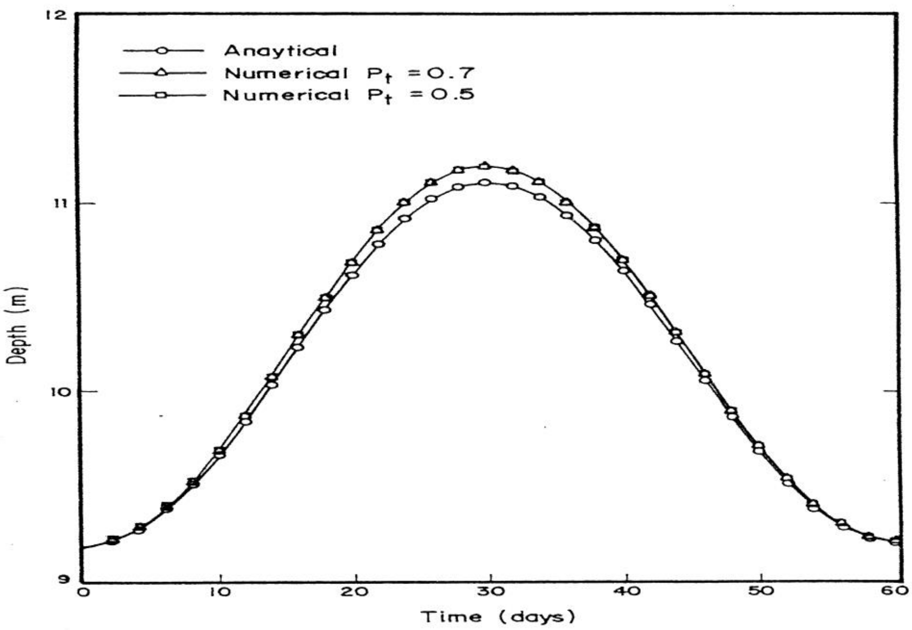

Fig. 4.17 Comparison of Analytical and Numerical Results at the X=60km,
$S_0=10^{-3}$, T=60 days P_t=0.70 and P_t=0.50

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B. The Discharge-Time for a given station and Time Period with Different Bed Slopes and Weighting Factors on the upstream discharge with a given downstream water level control

The comparing results are presented in Figures 4.18 and 4.19.

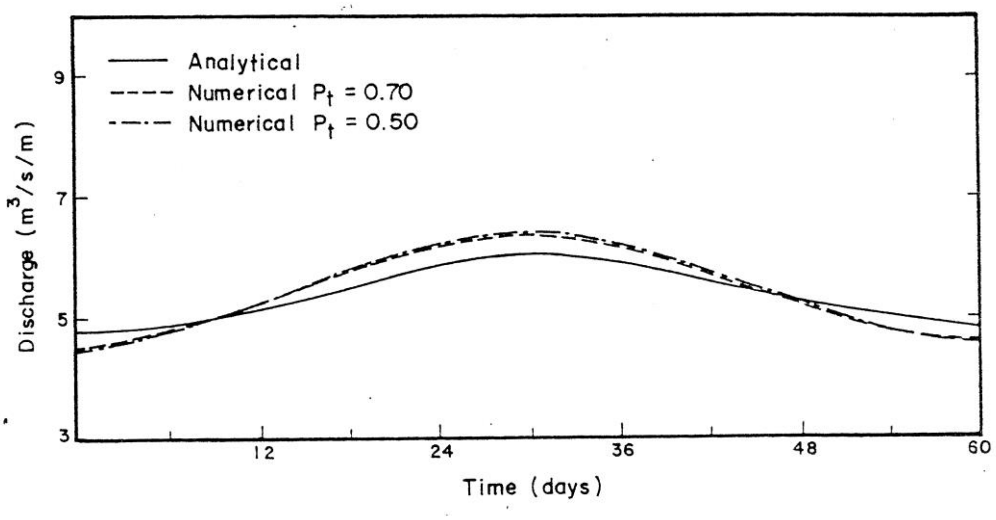

Fig. 4.18 Comparison of the Discharge between the Analytical and Numerical Results at X=60km, for different time Weighting Factor and $S_0=10^{-5}$

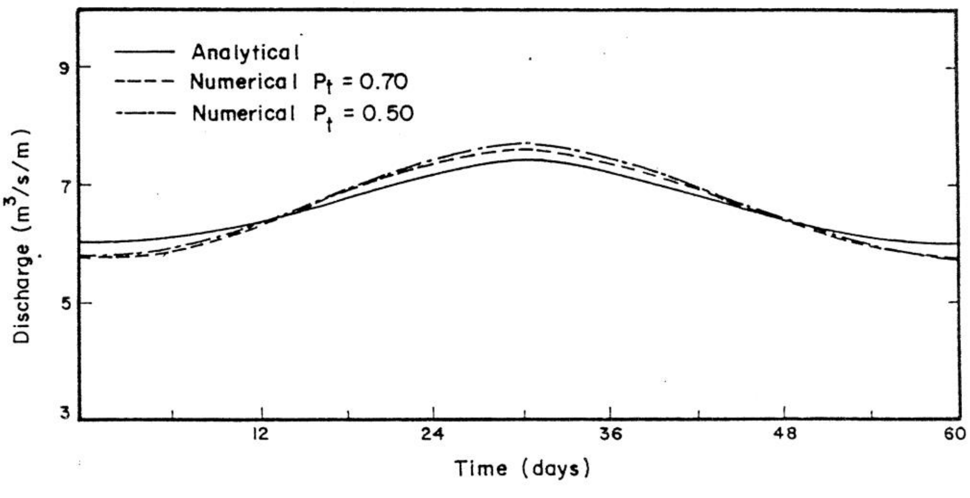

Fig. 4.19 Comparison of the Discharge between the Analytical and Numerical Results at X=60km, $S_0=1.55\times10^{-3}$ for different Weighting Factor

C. Comparison of Analytical and Numerical Results between Depth and Discharge for a given station, Time Period and Weighting Factor with Different Bed Slopes on the upstream discharge with a given downstream water level

The combinations of the analytical and numerical results the given station X=60 km at the

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same period T= 60 days, and same weighting factor, 0.7, with different bed slope, are given in Figures 4.20 and 4.21.

D. Comparison of Analytical and Numerical Results (Pt = 0.7) between Water Depth and Horizontal Velocity at a Given Station with Different Upstream Fresh Water Discharges and Varied Downstream Water Depth

The comparisons of Water Depth and Horizontal Velocity with Time between the analytical and numerical results, weighting factor, 0.7, for the given station X=60 km with Different Upstream Fresh Water Discharges and Varied Downstream Water Depth are given in Figures 4.22.

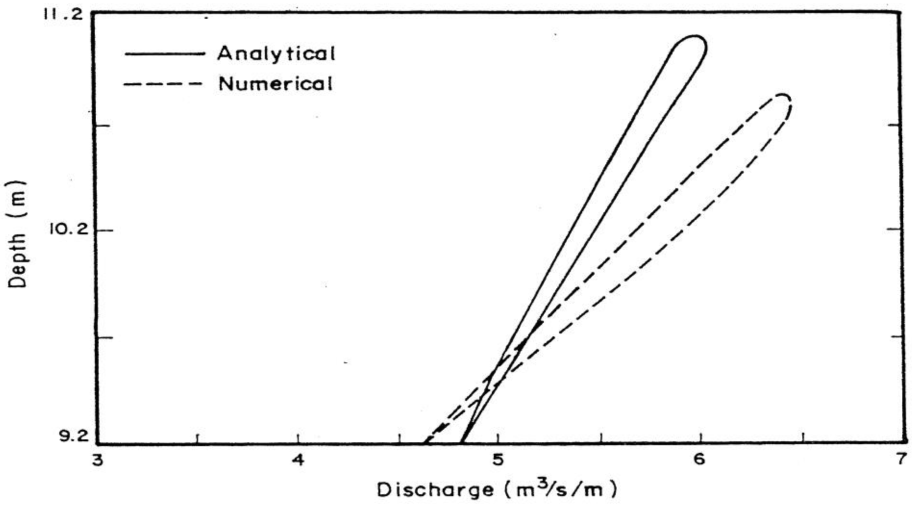

Fig. 4.20 Comparison of Analytical and Numerical Results between Depth and Discharge at X=60km, $S_0=10^{-5}$, T=60 days, Weighting Factor, P_t=0.70

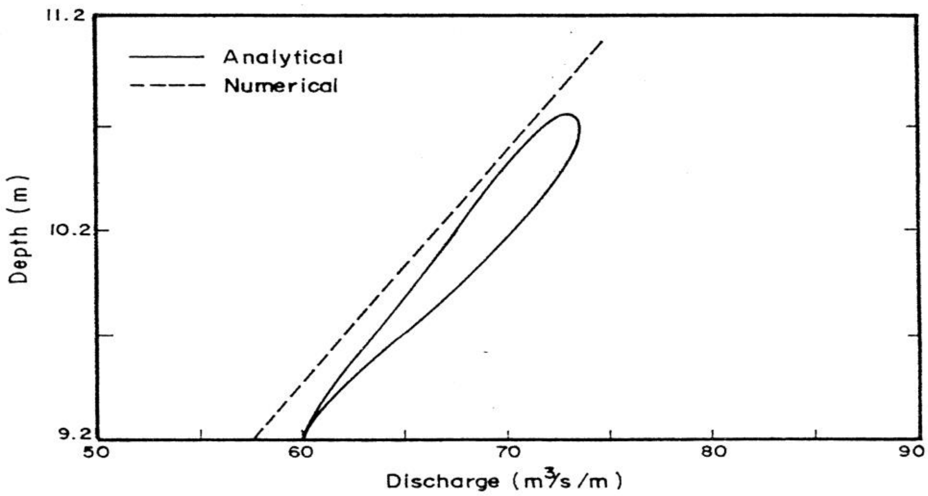

Fig. 4.21 Comparison of Analytical and Numerical Results between Depth and Discharge

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at X=60km, S_0=1.55×10^{-3}, T=60 days. Weighting Factor, P_t=0.70

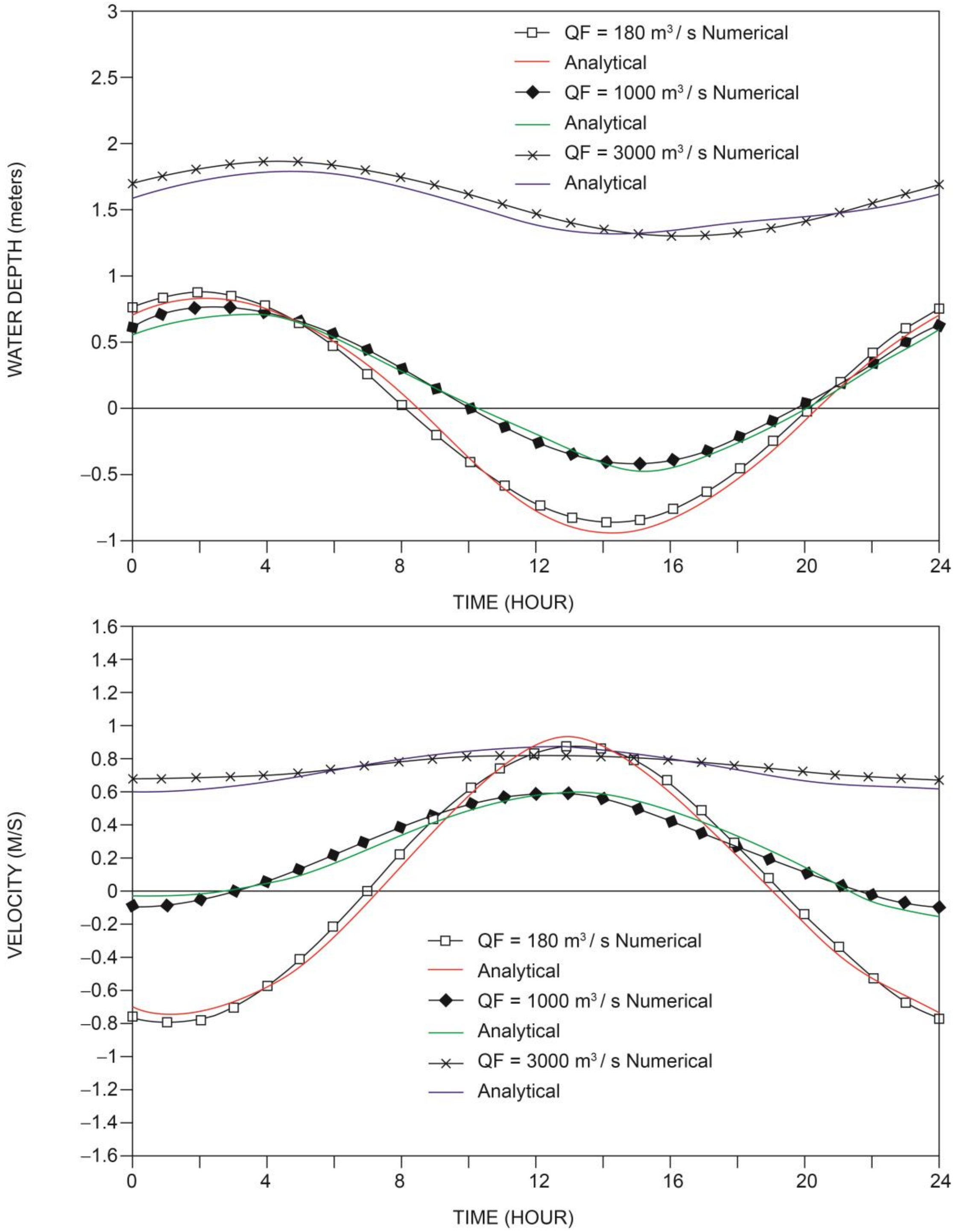

Fig. 4.22 Comparisons of Analytical and Numerical (Pt = 0.7) Results: Water Depth and Horizontal Velocity at Station 60Km with Different Upstream Fresh Water Discharges and Varied Downstream Water Depth

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DISCUSSION AND CONCLUSIONS

Discussion

The expression R_1 (x, t) represents the upstream effect. It is dimensionless and is a function of ω, μ, t and x only, where ωand μ are functions of stage hydrograph. In other words, it represents the fluctuation in depth at a given station or location x corresponding to the fluctuation represented by a series unit rise or a sine or cosine hydrograph in the depth at the upstream end x = 0, when there is no lateral inflow and when uniform condition exists at the downstream end. For a given ω and μ, R_1 (t) at any station x, tends to be a constant with t increasing. The same for R_2 (x', t), for a given ω and μ, R2 (t) at any station x' (= 1-x) , tends to be a constant as t increases. And R_2 (x', t) represents the effect from the downstream end of the reach. It represents the fluctuation in depth at a given location x' corresponding to a fluctuation represented a hydrograph in depth, when there exists uniform condition at the upstream end, and when there is no lateral inflow. If we only consider the upstream flood hydrograph then y_s can be set equal to zero. The same for y_p equal to zero with downstream tide control only.

By Janes formula (RATKY, *et al* 2001) , $\frac{Q}{Q_0} = \sqrt{1 - \frac{1}{S_0}\frac{\partial y}{\partial x}}$ or $\frac{Q}{Q_0} = \sqrt{1 + \frac{1}{S_0 \omega}\frac{\partial y}{\partial t}}$ the rating curve of q - y (ZHENG, *et al* 2012) can be constructed by the two methods, however there are some differences:

(1) The dispersion parameter, μ, is not considered in Janes formula.

(2) The time of peak flow for each station x, will occur at the same time which is not reasonable. The time of peak flow for each different location must be shifting due to the wave propagation or kinematic wave velocity, and the property can be expressed by this analytical method.

(3) By using Janes formula, the rating curves of q - y for different locations will complete a loop. It looks like peacock tail feather. In fact, when we consider ω and μ, the rating curves will shift and they will not complete an enveloped loop, and the results of the rating curves look like the spreading fingers.

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(4) The exact peak flow time can be solved by tp= T/2 + x/ω with tp, time of peak flow for station x and T, period of flood wave. The simplified form of diffusion equation in which dispersion parameter is function of bed slope, mean velocity, water depth, and Chezy roughness coefficient. It is obvious that μ increases with slope decreasing and ω decreasing for a given water depth and Chezy roughness coefficient; on the other hand, when bed slope increases, then kinematic wave velocity increases, and they should result in the decreasing ofμ. After getting the exact peak flow time and substituting it, the exact peak flow discharge and depth can be obtained.

(5) When slope increases, then $q = q_o$ by using Janes formula, it is also not reasonable. Even for a steep slope, the discharge will not be the same. By using the analytical method of this study, we can find that it is more beneficial than the Janes formula.

(6) Certain procedure for the channel schematization of this study could be developed a little for the flood routing of the irregular channels in the natural river. And further modifications might have to be carried out if multiple linearization is to be replaced single linearization.

(7) Application of the models to the single flood have been studied. The ability of the model to route flood sequences is of prime importance in river and reservoir regulation scheme as well as in flood peak production of the discharge calculations.

(8) For numerical model, the time increment, in the approximation of the input hydrographs (RODNEY, 2001) by a series of rectangles for regular flood waves or irregular or discrete flood waves, must be short enough so that this approximation does not cause significant errors. The program can be modified to include a means of altering the time increment to suit the requirements of the problem.

(9) Basing on certain procedure in mathematical derivation of solutions from wave hydrodynamic equations, an appropriate model representing the propagation of tidal waves, or say downstream control, for the irregular channels should be developed, taking into account the changing of width of the so that the analytical models will become a much more powerful tools to do researches on estuarine problems, such as:

(a) Unsteady salinity intrusion in estuaries of variable cross-section.

(b) Suspended sediment modelling in estuaries with unsteady flow characteristics.

(c) Pollution in estuary with complicated pattern of waste load from the factories.

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CONCLUSIONS

(A) Upstream Control (Flood Wave)

1. The model developed provided a simple, rapid and accurate means of tracing the course of flood wave resulting from the variation in the stage at upstream, and even though at downstream the model first computed the coefficient of upstream effect R_1 (x, t) and downstream effect R_2 (x', t) . These coefficients were then multiplied by the representative hydrograph ordinates and the simple addition of these lead to the overall effect. And R_1 (x, t) or R_2 (x', t) approached to a constant as time increased to t is greater or equal to $4l/\omega$.

2. For a given bed slope, S_o, Chezy's roughness coefficient, C_C, and normal depth, y_o, then ω and μ could be calculated. The steeper the bed slope, the larger ω, the kinematic wave velocity, and the smaller the dispersion parameter, μ. If the channel could not be schematized to have only one value of bed slope, and Chezy's roughness coefficient, then the calibration was required for both ω and μ.

3. The water depth, y, would be distorted due to the complementary error function, which was included in R_1 (x, t) or R_2 (x', t). For the same μ at the same dimensionless period t/T, the shorter the period, the more the distortion of the depth. Meanwhile, for the same t, the steeper the bed slope, the more the distortion of the depth.

4. There are several new interesting phenomena for the rating curves :

(1) The loop was wider for the milder bed slope and it was narrow for the steeper bed slope. Even though for the same bed slope, the shorter the flood period, the bigger the rating curve.

(2) The peak flow time at the different locations would be shifted a little due to the kinematic wave velocity, and when the dispersion parameter was considered, the rating curves for different stations would spread as the fingers, not complete a loop like peacock tail feather.

(3) The dispersion parameter, μ, would be equal to zero for the supercritical flow; it existed only for subcritical flow. The rating curves would be below the line of $y^* = q^*$ for

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the subcritical flow and they would be above the line of $y^{*}=q^{*}$ when the flow condition was supercritical flow.

(B) Downstream Control (Tide)

The majority of this present study for tide involved with the mathematical derivation of solutions from wave hydrodynamic equations. The tidal mathematical models describing the propagation of tides into a shallow river were appropriately developed by considering the effect of freshwater discharge, interaction. The general conclusion could be drawn:

(1) With the inclusion of convective and nonlinear terms in the governing equations, the equations were then solved by introducing an approximation method of perturbation. The linear friction which was achieved by linearizing the friction term in the unsteady flow equations.

(2) The assumed solutions were developed by including the effect of freshwater discharge velocity in consistent with the solutions from straight-forward mathematical derivation. The relations of dimensionless parameters of damped tides were achieved. It was seen from the expressions that propagation and attenuation were affected evidently by freshwater discharge. The discharge caused the friction of the river and mean water level to increase considerably which resulted in more energy dissipation.

(3) The interaction of flood wave and tide could be directly obtained or it also could be done by combining Eqs. (3-75.1) and (3-75.2).

(C) Numerical Solutions

(1) Comparison between the analytical model and the finite difference model with an implicit scheme showed beyond doubt that the analytical model could be applied in the river reach where there was moderate backwater effect. Even though the time weighting factor $p_t = 0.5$ and 0.7, the results were quite good.

(2) By using the depths as the boundary conditions, the results of the depths for the other stations were in good agreement, even if $p_t = 0.7$; for $p_t = 0.5$, there was a small deviation of the results of the discharge. Because the difference of the results of the discharge was small for pt- 0.5, if we took the smaller time interval, Δt, then the fluctuation was reduced, and the smooth results could be obtained.

(D) Application of Analytical Model

(1) The analytical results and numerical results are in good agreement, not only for the weighting factor, $p_t = 0.7$ but also for $p_t = 0.5$. The results of discharge for $p_t = 0.5$ from

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numerical solution by using the depth being a boundary condition is a little fluctuation, but when the time interval reducing, the fluctuation is also reducing. The results for the numerical solution for $p_t = 0.7$ and $p_t = 0.5$ are practically the same, the percentage error of the depth and discharge between the analytical and numerical results is less than 5%.

(2) This analytical model could be used without any sophisticated computing machine. In fact, a simple desk calculator and a table of error function were sufficient in carrying out the computation.

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