

**LEARNING TRAJECTORIES IN MATHEMATICS AND CHEMISTRY ON
SECONDARY SCHOOL STUDENTS' UNDERSTANDING OF WORD PROBLEMS IN
RIVERS STATE, NIGERIA**

AMADI, J. C.

Department of Curriculum Studies and Educational Technology, University of Port Harcourt,
P.M.B 5323 CHOBA, Port Harcourt, Nigeria.

AROKUYO, A. A.

Department of Curriculum Studies and Educational Technology, University of Port Harcourt,
P.M.B 5323 CHOBA, Port Harcourt, Nigeria.

ABSTRACT: *The study investigated the effects of using Learning Trajectories (LT) in Mathematics and Chemistry on secondary students' understanding of word problems in Rivers State, Nigeria. The study used a quasi-experimental pre-test /post- test control design. A sample of 84 (54 male :30 female) senior secondary two (SS2) students was purposively selected from Federal College of Education (Technical) Demonstration secondary school Omoku in Ogba/Egbema/Ndoni local government area. Four research questions were answered and four research hypotheses were tested at 0.05 level of significance. Data was obtained through a Test of Basic Skills on Word Problems (TBSWP) designed by the researchers. The reliability coefficient index $r=0.82$. Mean scores were used in analyzing the research questions while z-test was employed in testing the hypotheses. The findings revealed that learning trajectories (LT) [rigid operational, flexible operational, basic operational levels etc] enhanced students' academic achievements in mathematics. Based on the findings, it was recommended that the use of learning trajectories to teach mathematics should be encouraged in senior secondary schools.*

KEYWORDS: Learning Trajectories, mathematics, students' understanding, Word problems.

INTRODUCTION

Algebra is not only an important area of school mathematics but a gatekeeper to future educational and employment opportunities. Reasoning developed through algebra has individual applications and also affects decisions making in many areas such as personal finance, travelling, cooking and real estate management, to name a few (Wiki Answers, 2010).. Thus, it can be argued that a better understanding of algebra improves decision making capabilities in the society.

Notwithstanding its importance, algebra continuous to be a struggle for many students, a fact that has led to first year algebra courses in the United States being characterized as an unmitigated disaster for most students, (National Research Council [NRC], 1998). This difficulties have been attributed to a variety of factors; among them are: Students' limited conceptions of what the equal sign ($=$) means which happens to be the major stumbling block in learning algebra (Carpenter, Franke & Levi, 2003), poor methods of teaching employed by mathematics teachers as well as teachers' non-use of relevant instructional materials in the teaching of mathematics and chemistry

concepts (Ezeugo & Agwagah, 2000). These factors interfere with students' ability to successfully solve and analyze equations (Carpenter et al., 2003).

Evidence of poor performance in mathematics and chemistry by senior secondary school students highlight the fact that the most desired technological, scientific and business application for mathematics cannot be sustained. This makes it paramount to seek for a strategy for teaching mathematics and chemistry that aims at improving their understanding and performance by students practically (Okigbo & Osuafor, 2008). In order to break this pattern, teaching should be designed to promote a relational understanding of mathematical equivalence (equality) in the context of solving, building and discussing about number sentence and there should be more emphasis on problem solving, applications and higher order thinking skills. Designing instructional activities that focused on enriching each student's proper understanding of equality contribute to students performing at a seemingly higher level, with respect to symbolic relational equivalence, (NCTM, 2000). NCTM (2000) believes that an effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well; and that understanding of the basic concepts in mathematics and chemistry in the secondary school is developed through an effective mathematics and chemistry program that includes quality instruction and an environment that fosters a community of mathematics and chemistry learners. According to (NCTM, 2000) improvement is achieved and sustained by the continuous, conscious efforts of teachers to assess their students' progress, identify areas for improvement, determine the instructional strategies to pursue in light of assessment data, implement those strategies, measure whether the strategies have been successful in addressing students' needs, and plan the next step in the instructional process. Thus, if students participate in learning activities that encourage them to reflect, manipulate, explore, construct, simulate, discuss, structure and practice, they will assimilate concepts, processes and strategies.

Learning trajectory affects classroom interactions with students during instruction. A trajectory can be viewed as possible pathways (non-linear) students can navigate during instruction (Confrey and Kazat, 2006). It begins with prior knowledge, the instructional tasks act as constraints on students' responses and the sequence of tasks encourage students' movement through the corridor. It sensitizes teachers to a variety of strategies students may use, provides a framework for using and sequencing students' work, and helps teachers connect and relate mathematical ideas across multiple examples of students' ideas. With learning trajectories pivotal tasks are designed and sequenced to support students' understanding at a conceptual level or benchmark within the developmental progression. The tasks are designed to help children learn the ideas and practice the skills needed to master that level.

According to (Clements and Sarama, 2009) a Learning Trajectory of a particular mathematical concept consists of three components:

- A specific mathematical goal;
- A developmental path along which students' thinking and comprehension develops; and
- A set of instructional activities that help students move along that path.

Statement of the problem

From years of experience as mathematics and chemistry teachers, the researchers observed that many students have limited understanding of equal sign and, this has influenced their learning of mathematics and chemistry. The equals sign though stands for relational equivalence but, many of the researchers' students misconstrued it in many different ways which adversely affected their efforts to proceed to advanced algebraic concepts such as mathematical and chemical equation solving, word problems, number sentences, chemical equivalence and so on. Since algebra is the gateway to higher mathematics, if students fail to grasp the meaning of equal sign in secondary school, it could affect their future academic achievement in mathematics and chemistry. These problems encountered by the students appeared to have connection with their lack of conceptual (relational) understanding which is the result of teaching experienced in learning Algebra at the primary school level. In addition, textbooks are also not presenting content in such an elaborate way that these could have provided sufficient room for students to develop their relational knowledge and conceptual understanding of equal sign. In the present circumstance, the researchers set out to determine how Learning Trajectories affect students' understanding of mathematical and chemical equality in solving word problems.

Objectives of the study

The present study focused on the effects of using Learning Trajectories in Mathematics on secondary school students' understanding of word problems. The specific objectives of this study are to:

- i. determine the academic achievement of both the experimental and control groups in students' pre-test on word problems.
- ii. investigate the effect of Learning Trajectories (LT) on students' academic achievement on understanding of equality signs and solving word problems;
- iii. examine the effect of Learning Trajectories (LT) on solving word problems on male and female students' academic achievement; and
- iv. explore the effect of using Learning Trajectories to teach students with different academic abilities.

Research questions

The following research questions were raised in the study:

- i. Is there any difference in the pre-test results of the experimental and control groups in terms of academic achievement?
- ii. What is the effect of Learning Trajectories (LT) on students' academic achievement in understanding equality sign and solving word problems?
- iii. What is the effect of Learning Trajectories on the academic achievement of males and females exposed to learning trajectories?

- iv. Does using Learning Trajectories to teach students of low and high abilities have any effect?

Research hypotheses

- i. There is no significant difference in the academic achievement of both the experimental and control groups in their pre-tests scores.
- ii. There is no significant difference between Learning Trajectories and students' academic achievement in understanding equality signs and solving word problems.
- iii. There is no significant difference between the academic achievement of males and females exposed to learning trajectories.
- iv. There is no significant difference between the academic performance of students with low and high abilities using learning trajectories.

METHODOLOGY

The researchers adopted quasi-experimental design (non-equivalent, pre-test/ post-test control group design) for the study. Students received treatment in their various classes (intact classes) this is used as control in their various classes without disorganizing the class setting. The population of the study consisted of all senior secondary students of Federal College of Education (Technical) Omoku Demonstration secondary school in Ogba/Egbema/Ndoni local government area of Rivers State, Nigeria. A sample of 84 (54 male + 30 female). Senior secondary two (SS2) A and B formed the experimental group (27 male + 15 female), and senior secondary two (SS2) C and D formed the control group (27 male + 15 female). The control group received instruction using the conventional method, while the experimental groups received instruction using Learning Trajectories. The study lasted for six weeks. The 2 groups were given pre-test and post-test. The mean score achievements of the students` were determined and correlated. The instrument used for the study was Mathematics Achievement Test developed by the researchers which was tagged Test of Basic Skills on Word Problems (TBSWP). This was a written assessment which targeted the students' understandings of various aspects of algebra, which includes terms like: open equation-solving, equation-structure, equal-sign-definition, and advanced relational reasoning. The test was designed to measure students' understanding of the equal and inequality sign; Interpretations of the equal sign, and ability to solve word problems of similar structure, which were represented in different formats. The face, content and construct-validity of the achievement tests (the instrument) were established by presenting the test questions to the experts in mathematics and chemistry education who have been teaching over ten years for appraisal. Comments made by the experts were taken into consideration and the necessary modifications and corrections were made before the production of the final copies that were administered in both the pre-test and post-test. The reliability of the instrument was established through the application of Kuder-Richardson, (KR-21) as a measure of the internal consistency. This was done by administering the test to a sample of 40 senior secondary students selected outside the target sample, then Kuder-Richardson formula 21 was used to calculate the reliability coefficient of the

test, which was 0.82. Then researchers used a construct map which was developed as a representation of the progression of knowledge levels to teach the lessons. At the end of the exercise, a post-test was administered to both the experimental and the control groups. This was scored and recorded before data analysis.

Data analysis:

Research questions were answered using mean (\bar{X}) and standard deviation (σ), while z-test of scores was used to determine the statistical significance of the research hypotheses at .05 level of significance.

The pre -test mean scores of the experimental and control groups of senior secondary two (SS2) mathematics students were analyzed by means of z-test statistics of significance for large independent variable samples at .05 level of significance as shown below:

Research Question 1: Is there any difference in the pre-test results of the experiment and control groups in terms of academic achievement?

Table 1: Pre-test Mean scores of students in the two groups

Group	N	Mean	SD	df	z-cal	z-critical	Decision
Experimental group	42	18.3	4.53	82	0.5331	1.98	Retained
Control group	42	21.6	5.87				

Table 1 shows that the experimental and control group of SS2 students did not differ significantly in their academic achievement in an algebra achievement test before treatment. The students were homogeneous in their entry behavior. This implies that any change that might occur in the behavior after treatment could be attributed to the effect of the treatment.

Research Hypothesis 1: There is no significant difference in the academic achievement of both the experimental and control groups in their pre-tests scores.

Table 2: Mean scores of students in the two groups before treatment

Group	N	Mean	SD	df	z-cal	z-critical	Decision
Experimental group	42	18.3	4.53	82	0.5331	1.98	Retained
Control group	42	21.6	5.87				

Not sig. at .05

The analysis in Table 2 shows that the calculated z-value of 0.5331 is less than z-critical of 1.98 (that is $0.5331 < 1.98$) at .05 level of significance and 82 degree of freedom. Therefore, there was no significant difference between the results of the experimental and control groups in terms of academic achievement before administering the treatment. The null hypothesis was therefore retained.

Research Question 2: What is the effect of Learning Trajectories (LT) on students' academic achievement in understanding equality signs and solving word problems?

Table 3: Gain scores of academic achievement of students taught using LT and CCT.

Group	N	Pre-test mean	SD	Post-test mean	SD	Mean score	Gain%
LT	42	21.6	5.87	28.3	4.49	6.70	31.02
CCT	42	18.3	4.53	19.8	5.11	1.5	8.20

LT: Learning Trajectories; CCT: Conventional Classroom Teaching.

Table 3 shows that the mean gain score of SS2 students in the Learning Trajectories group (6.7, 31.02%) is higher than that of their conventional classroom teaching counter parts (1.5, 8.20%) indicating that the Learning Trajectories was more beneficial in improving the students academic achievement than the conventional classroom teaching.

Research hypothesis 2: There is no significant difference between Learning Trajectories and students' academic achievement in understanding equality signs and solving word problems.

Table 4: z-test posttest mean scores of students in the two groups

Group	N	Mean	SD	Df	z-cal	z-critical	Decision
Experimental group	42	28.3	4.49	82	17.6	1.98	Reject
Control group	42	19.8	5.11				

Sig. at .05

From table 4, z-calculated is greater than z-critical (that is $17.6 > 1.98$) at .05 level of significance and 82 degree of freedom. Therefore, there was a significant difference between the results of the experimental and control groups in terms of academic performance after administering the treatment. The use of learning trajectories had a positive effect on the academic performance of the students.

Research Question 3: What is the effect of Learning Trajectories on the academic achievement of male and female exposed to learning trajectories?

Table 5: Mean score difference between boys and girls taught with LT and CCT

Gender	LT			CCT		MDMF	%MDMF
	N	Mean	SD	Mean	SD		
Males	27	27.9	4.29	19.7	5.17	8.2	41.62
Females	15	29	4.54	22.7	5.97	6.3	27.75

LT: Learning Trajectories; CCT: conventional classroom teaching; MDMF: Mean difference between male & female

Table 5 shows that there exists a mean difference between the post-test scores of males (27.9, 19.7) and females (29, 22.7) taught with Learning Trajectories (LT) and those taught with conventional classroom teaching (CCT) respectively.

Table 5 shows that the percentage means difference of the males is greater than that of the females (41.62% , 27.75%). This result indicates that although, both gender benefitted from the treatments, the males benefitted more.

Research Hypothesis 3: There is no significant difference in academic achievement of male and female students exposed to learning trajectories.

Table 6: Academic achievements of males and females exposed to Learning Trajectories.

Group	N	Mean	SD	Df	z-cal	z-critical	Decision
Male	27	27.9	4.54	40	1.60	2.02	retained
Female	15	29	4.16				

Not sig. at .05

The result in performance based on gender of students exposed to learning trajectories from Table 6 indicates that $z\text{-cal} < z\text{-critical}$ (that is $1.60 < 2.02$). Hence, the null hypothesis was retained, which implies that there was no significant difference between the academic achievements of boys and girls students exposed to learning trajectories in mathematics and chemistry based on gender.

Research Question 4: Does using Learning Trajectories to teach students of low and high abilities have any effect?

Table 7: The percentage means score difference of students with different ability level

Ability levels	Method	N	Mean	SD	Mean difference	%Mean difference
Class A 60% & above	LT	16	25.7	3.38	10.40	67.97
	CCT	16	15.3	1.83		
Class B 59% & below	LT	26	32.2	2.46	14.8	84.57
	CCT	26	17.5	2.14		
Class A	LT	16	25.7	3.38	6.5	25.29
Class B	LT	26	32.2	2.46		

Criteria: Class A score of 60% & above; Class B score of 59% & below

Table 6 indicates that the mean difference exists in the academic achievement of the Class A and Class B students taught selected algebraic concepts using Learning Trajectories (LT) and conventional classroom teaching (CCT) (10.40, 14.8). An observation of Table 6 shows that the mean score of students in Class B is greater than those in the Class A. The result shows that in each level, students who were taught with learning trajectories (LT) performed better than those who were taught with conventional classroom teaching (CCT) (84.57, 67.97%). This implies that learning trajectories enhanced students' academic achievement at high and low levels than

conventional method. The result also indicated that the experiment was more beneficial to the students of Class B (low level) than those in the class A (high level) groups who were exposed to learning trajectories, based on their post-test mean scores, mean difference and percentage mean difference.

Research Hypothesis 4: There is no significant difference between the academic performance of students with low and high abilities using learning trajectories.

Table 8: The mean score of students with different ability levels taught with LT

Group	N	Mean	SD	Df	z-cal	z-critical	Decision
Class A	16	25.7	3.38	40	12.07	2.02	Reject
Class B	26	32.3	2.46				

Sig. at.05

Data on Table 7 revealed that the calculated z-value of 12.07 is greater than the critical z-value of 2.02 at 0.05 level of significance and 82 degree of freedom. Therefore, the null hypothesis is rejected, which implies that there is a significant difference between the academic achievement of Class A and Class B students. The result indicates that students' exposure to learning trajectories has improved the performance of Class B students.

Summary of findings

1. There was no significant difference between the results of the experimental and control groups in terms of academic achievement before administering the treatment.
2. There was a significant difference between the results of the experimental and control groups in terms of academic performance after administering the treatment. The use of learning trajectories had a positive effect on the academic performance of the students.
3. There was no significant difference between the achievements of students exposed to learning trajectories in mathematics and chemistry based on gender. However, both sexes benefitted from the treatment.
4. There was a significant difference between the academic achievements of students with different ability levels. The result indicated that students' exposure to learning trajectories has improved the performance of Class B students.

Discussion of findings

Difficulty in solving word problems as well as poor performance of students in algebra has been attributed to ineffective teaching methods and student's misunderstanding of the equal sign (McNeil & Alibali, 2005); (Knuth, Stephens, McNeil, and Alibali (2006)) and (Molina & Ambrose 2008).

This research was an effort towards searching for a strategy that would increase and sustain interest in algebra for improved performances in mathematics and chemistry. It was also geared towards resolving the controversy in the existing literature on which methods of teaching leads to better

performance when teachers used Learning Trajectories and Conventional Classroom Methods. In all the findings, there were significant differences between Learning Trajectories and students' academic achievement in understanding equality signs and solving word problems. This confirms the earlier findings of (Knuth et al., 2006); (Molina & Ambrose, 2008) and (McNeil & Alibali, 2005) that students who interpret the equal sign in a relational manner successfully solve more word problems. The results show a strong positive relation between senior secondary school two students' equal sign understanding and their solving word problem performance. This result implies that understanding the concept of equality is an important prerequisite for students' abilities to solve word problems.

In all the variables tested the results showed that students who were taught with learning trajectories (LT) performed better than those who were taught with conventional classroom teaching (CCT). This was because Learning Trajectories was planned to give students the opportunity to work in activities involving multiple representations of the concepts of equality and word problems. The students' effective involvement in such activities helped them link the representations of these concepts and proceeded to solve word problems and to generalize on the idea of equality symbols.

Implications of the Findings

The findings of the study have some educational implications:

1. Learning trajectories has been found to be more effective in teaching mathematics and chemistry more than the conventional method, since students performed better when they were exposed to it. Therefore, for effective performance in mathematics and chemistry lessons; teachers should use Learning trajectories in their lesson preparation and presentation.
2. From the study, both high and low ability level students benefited from the treatment, this suggests that LT will increase students' performance in mathematics and chemistry in both internal and external examinations, thereby reducing students' tendencies for examination malpractices and saving examination bodies and the Government huge resources expended in curbing examination malpractices.

RECOMMENDATIONS

The following recommendations were made based on the findings of the study.

1. Mathematics teachers and educators should adopt the use of learning trajectories as a purposeful strategy to effectively teach word problems as well as other abstract concepts or other difficult mathematical concepts in order to make mathematics meaningful and interesting.
2. The Federal and State governments and other educational bodies should organize workshops and in-service training programs as well as seminars on a regular basis for

senior secondary mathematics teachers on the use of learning trajectories as an effective and efficient teaching strategy.

3. Curriculum planners should include a course on the use of learning trajectories in teacher training institutions as a compulsory course for all would be teachers of mathematics.
4. Education planners and other education stakeholders should encourage the use of learning trajectories by senior secondary school mathematics teachers.

CONCLUSION

On the basis of the findings of this study with respect to the problems investigated, the following statements of conclusions were derived:

1. That students taught with learning trajectories performed significantly better with higher achievement than those taught using conventional method. Therefore, the use of Learning trajectories as a true and innovative instructional strategy in senior secondary school mathematics teaching and learning is a way of ensuring meaningful teaching against learning by rote characteristics of conventional method.
2. There is no significant gender difference between the academic achievement of male and female students taught with learning trajectories. Both sexes benefitted from the exposure. Therefore, gender is an insignificant factor in the academic achievement of male and female students in this study.
3. The students in different ability levels (high and low) taught with learning trajectories differed significantly in performance than those taught with conventional method. Besides, those at the low ability level (class B) improved in their academic performance more than those at the high ability level (class A) when both groups were exposed to learning trajectories.
4. Another significant conclusion was that quality instructional practices contribute to students performing at a seemingly higher level.

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