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JOSEPHSON LOSSLESS TRANSMISSION LINES WITH NONLINEAR *R* ELEMENT

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ABSTRACT: *The present paper is devoted to the investigation of lossless transmission lines terminated by a nonlinear Josephson junction circuit. Such lines are described by a first order hyperbolic system of partial differential equations with sine-nonlinearity. We formulate a mixed problem for this system with nonlinear boundary conditions generated by a nonlinear resistive circuit. In contrast of our previous result [1] here we cannot reduce the mixed problem to an initial value one on the boundary because the hyperbolic system is not linear one. We extend results from [2] and present in an operator form the mixed problem in question. Then we cut off the domain and show that operator defined is contractive one. Its unique fixed point is an approximated solution of the mixed problem.*

KEYWORDS: *Transmission lines, Josephson junction, Superconductivity, Nonlinear circuits, sine-Gordon equation.*

INTRODUCTION

A lot of papers have been devoted to the investigation of lossless transmission lines terminated by nonlinear loads and their applications to *RF*-circuits (cf. [3]-[14]). Here we consider a lossless transmission line with Josephson junction (cf. [15]-[18]). We proceed from a lossless transmission line system with sine-nonlinearity. Unlike [19] here we consider a hyperbolic system with boundary conditions generated by Josephson junction with nonlinear resistive element. It generates a nonlinear term of polynomial type in the operator equation.

Here we do not follow the usually accepted approach based on the known sine-Gordon equation (cf. [15]). So we start with a brief derivation of sine-Gordon equation. We proceed from the system

$$\frac{\partial u(x,t)}{\partial x} = -L\frac{\partial i(x,t)}{\partial t},$$

$$\frac{\partial i(x,t)}{\partial x} = -C\frac{\partial u(x,t)}{\partial t} - j_0 \sin\frac{2\pi\Phi(x,t)}{\Phi_0},$$

$$\frac{\partial \Phi(x,t)}{\partial t} = u(x,t),$$

(1.1)

$$(x,t) \in \Pi = \{(x,t) \in R^2 : (x,t) \in [0,\Lambda] \times [0,T]\}$$

where $u(x,t)$, $i(x,t)$ and $\Phi(x,t)$ are unknown functions − voltage, current and Josephson flux, L and C are prescribed specific parameters of the line, $\Lambda>0$ is its length, j_0 is maximal Josephson current per unit length and $\Phi_0 = \hbar/(2e) = 2/10^{-15} W/m^2$ is flux induction quant, $\hbar$ is the Planck constant.

The commonly accepted approach to derive sine-Gordon equation is the following. Beginning from (1.1) we obtain

$$\frac{\partial^2\Phi(x,t)}{\partial x\partial t} = \frac{\partial u(x,t)}{\partial t} \Rightarrow \frac{\partial^2\Phi(x,t)}{\partial x\partial t} = -L\frac{\partial i(x,t)}{\partial t} \Rightarrow \frac{\partial^3\Phi(x,t)}{\partial x^2\partial t} + L\frac{\partial^2 i(x,t)}{\partial t\partial x} = 0\ ;$$

$$\frac{\partial i(x,t)}{\partial x} = -C\frac{\partial^2\Phi(x,t)}{\partial t^2} - j_0\sin\frac{2\pi\Phi(x,t)}{\Phi_0} \Rightarrow L\frac{\partial^2 i(x,t)}{\partial x\partial t} = -LC\frac{\partial^3\Phi(x,t)}{\partial t^3} - Lj_0\frac{\partial}{\partial t}\left(\sin\frac{2\pi\Phi(x,t)}{\Phi_0}\right)$$

Therefore

$$\frac{\partial^3\Phi(x,t)}{\partial x^2\partial t} - LC\frac{\partial^3\Phi(x,t)}{\partial t^3} - Lj_0\frac{\partial}{\partial t}\left(\sin\frac{2\pi\Phi(x,t)}{\Phi_0}\right) = 0$$

or

$$\frac{\partial^2\Phi(x,t)}{\partial x^2} - LC\frac{\partial^2\Phi(x,t)}{\partial t^2} - Lj_0\sin\frac{2\pi\Phi(x,t)}{\Phi_0} = 0\ . \tag{1.2}$$

So we have showed that if (1.1) has a solution then (1.2) is satisfied. It is quite obvious that the inverse assertion could not be proved without additional assumptions. That is why we consider the original first order lossless transmission line system. Taking into account

$$\Phi(x,t) = 2\pi K_J\int_0^t u(x,s)ds\ .$$

we obtain from (1.1) a nonlinear hyperbolic system with two unknown functions:

$$\frac{\partial u(x,t)}{\partial t} + \frac{1}{C}\frac{\partial i(x,t)}{\partial x} = -\frac{j_0}{C}\sin\left(2\pi K_J\int_0^t u(x,s)ds\right), \tag{1.3}$$

$$\frac{\partial i(x,t)}{\partial t} + \frac{1}{L}\frac{\partial u(x,t)}{\partial x} = 0.$$

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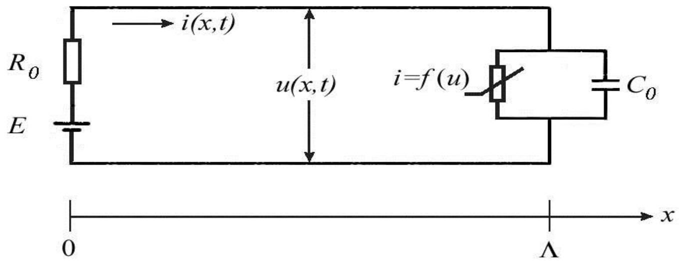
The Josephson junction is described by a circuit at right-hand end (cf. Fig.1). Here $K_J = 1/\Phi_0$ is Josephson constant. In contrast of [19] as we have already mentioned the transmission line is terminated by a circuit with nonlinear resistive element with polynomial characteristic of the type $i = \sum_{n=1}^{m} g^{(n)} u^n$.

Fig. 1. Lossless transmission line with Josephson circuit containing nonlinear resistive element



For (1.3) one can formulate the following mixed (initial-boundary value) problem: to find the unknown functions $u(x,t)$ and $i(x,t)$ in Π satisfying initial conditions and boundary conditions

$$u(x,0) = u_0(x),\ i(x,0) = i_0(x),\ x \in [0,\Lambda] \qquad (1.4)$$

$$E(t) - u(0,t) - R_0 i(0,t) = 0, \quad , \qquad C_0 \frac{du(\Lambda,t)}{dt} = i(\Lambda,t) - \sum_{n=1}^{m} g^{(n)} u^n(\Lambda,t),\ t \in [0,T]. \qquad (1.5)$$

Here $i_0(x), u_0(x)$ are prescribed initial functions − the current and voltage at the initial instant (cf. Fig.1), $E(t)$ is a prescribed source function, R_0 is the resistance of the source, C_0 is the capacity of the linear element, and $i = \sum_{n=1}^{m} g^{(n)} u^n$ is the characteristic of the resistive element.

Choosing a suitable function spaces and introducing suitable weighted metrics we prove existence of generalized continuous solutions of (1.3)-(1.6) by fixed point method [21]. In this way we demonstrate of how to overcome the difficulty caused by sine function. For our fixed point method sine function is not a “bad” nonlinearity.

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Diagonalization of the hyperbolic system

Introducing denotations

$$U=\begin{bmatrix}u\\ i\end{bmatrix},\quad \frac{\partial U}{\partial t}\equiv U_t=\begin{bmatrix}u_t\\ i_t\end{bmatrix},\quad \frac{\partial U}{\partial x}\equiv U_x=\begin{bmatrix}u_x\\ i_x\end{bmatrix},\quad A=\begin{bmatrix}0 & 1/C\\ 1/L & 0\end{bmatrix},$$

$$\Gamma=\begin{bmatrix}-\frac{j_0}{C}\sin\left(2\pi K_J\int_0^t u(x,s)ds\right)\\ 0\end{bmatrix}$$

we can rewrite (1.3) as

$$\begin{bmatrix}u_t\\ i_t\end{bmatrix}+\begin{bmatrix}0 & 1/C\\ 1/L & 0\end{bmatrix}\begin{bmatrix}u_x\\ i_x\end{bmatrix}=\begin{bmatrix}-\frac{j_0}{C}\sin\left(2\pi K_J\int_0^t u(x,s)ds\right)\\ 0\end{bmatrix}$$

(2.1)

or

$$U_t+AU_x=\Gamma .$$

(2.2)

The first step is to transform the matrix $A=\begin{bmatrix}0 & 1/C\\ 1/L & 0\end{bmatrix}$ in a diagonal form. We solve the characteristic equation: $\begin{vmatrix}-\lambda & 1/C\\ 1/L & -\lambda\end{vmatrix}=0$ whose roots are $\lambda_1=1/\sqrt{LC},\lambda_2=-1/\sqrt{LC}$. Eigen-vectors are solutions of the systems:

$$\left|\begin{array}{ll}-1/\left(\sqrt{LC}\right)\xi_1^{(1)} & +1/C\xi_2^{(1)}=0\\ 1/L\xi_1^{(1)} & -1/\left(\sqrt{LC}\right)\xi_2^{(1)}=0\end{array}\right. \quad\text{and}\quad \left|\begin{array}{ll}1/\left(\sqrt{LC}\right)\xi_1^{(2)} & +1/C\xi_2^{(2)}=0\\ 1/L\xi_1^{(2)} & +1/\left(\sqrt{LC}\right)\xi_2^{(2)}=0\end{array}\right.,$$

that is, the eigen-vectors are $\left(\xi_1^{(1)},\xi_2^{(1)}\right)=\left(\sqrt{C},\sqrt{L}\right),\left(\xi_1^{(2)},\xi_2^{(2)}\right)=\left(-\sqrt{C},\sqrt{L}\right)$.

Denote by H the matrix formed by eigen-vectors $H=\begin{bmatrix}\sqrt{C} & \sqrt{L}\\ -\sqrt{C} & \sqrt{L}\end{bmatrix}$. Its inverse one is

$$H^{-1}=\frac{1}{2}\begin{bmatrix}1/\sqrt{C} & -1/\sqrt{C}\\ 1/\sqrt{L} & 1/\sqrt{L}\end{bmatrix}.\ \text{Then}\ A^{\text{can}}=HAH^{-1}=\begin{bmatrix}1/\sqrt{LC} & 0\\ 0 & -1/\sqrt{LC}\end{bmatrix}.$$

Introduce new variables

$$Z=\begin{bmatrix}V(x,t)\\ I(x,t)\end{bmatrix},\quad U=\begin{bmatrix}u(x,t)\\ i(x,t)\end{bmatrix},\ Z=HU,\ (U=H^{-1}Z).$$

In an explicit form they are

$$\begin{cases} V(x,t) = \sqrt{C}\ u(x,t) + \sqrt{L}\ i(x,t) \\ I(x,t) = -\sqrt{C}\ u(x,t) + \sqrt{L}\ i(x,t) \end{cases} \quad \text{or} \quad \begin{cases} u(x,t) = \frac{1}{2\sqrt{C}} V(x,t) - \frac{1}{2\sqrt{C}} I(x,t) \\ i(x,t) = \frac{1}{2\sqrt{L}} V(x,t) + \frac{1}{2\sqrt{L}} I(x,t). \end{cases}$$

(2.3)

Substituting $U = H^{-1}Z$ in (2.2) we obtain $\frac{\partial(H^{-1}Z)}{\partial t} + A\frac{\partial(H^{-1}Z)}{\partial x} = \Gamma$. But H^{-1} is a constant matrix that implies $H^{-1}Z_t + (AH^{-1})Z_x = \Gamma$. After multiplication from the left by H we obtain

$$Z_t + A^{\text{can}} Z_x = H\Gamma .$$

(2.4)

Since

$$H\Gamma = \begin{bmatrix} \sqrt{C} & \sqrt{L} \\ -\sqrt{C} & \sqrt{L} \end{bmatrix} . \begin{bmatrix} -\frac{j_0}{C}\sin\left(2\pi K_J \int_0^t u(x,s)ds\right) \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{j_0}{\sqrt{C}}\sin\left(\frac{\pi K_J}{\sqrt{C}}\int_0^t (V(x,s)-I(x,s))ds\right) \\ -\frac{j_0}{\sqrt{C}}\sin\left(\frac{\pi K_J}{\sqrt{C}}\int_0^t (V(x,s)-I(x,s))ds\right) \end{bmatrix}$$

then (2.4) can be rewritten as:

$$\begin{bmatrix} \frac{\partial V}{\partial t} \\ \frac{\partial I}{\partial t} \end{bmatrix} + \begin{bmatrix} \frac{1}{\sqrt{LC}} & 0 \\ 0 & -\frac{1}{\sqrt{LC}} \end{bmatrix} \begin{bmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial I}{\partial x} \end{bmatrix} = \begin{bmatrix} -\frac{j_0}{\sqrt{C}}\sin\left(\frac{\pi K_J}{\sqrt{C}}\int_0^t (V(x,s)-I(x,s))ds\right) \\ -\frac{j_0}{\sqrt{C}}\sin\left(\frac{\pi K_J}{\sqrt{C}}\int_0^t (V(x,s)-I(x,s))ds\right) \end{bmatrix}$$

or

$$\frac{\partial V}{\partial t} + \frac{1}{\sqrt{LC}}\frac{\partial V}{\partial x} = -\frac{j_0}{\sqrt{C}}\sin\left(\frac{\pi K_J}{\sqrt{C}}\int_0^t (V(x,s)-I(x,s))ds\right)$$

$$\frac{\partial I}{\partial t} - \frac{1}{\sqrt{LC}}\frac{\partial I}{\partial x} = -\frac{j_0}{\sqrt{C}}\sin\left(\frac{\pi K_J}{\sqrt{C}}\int_0^t (V(x,s)-I(x,s))ds\right).$$

(2.5)

The new initial conditions become:

$$V(x,0) = \sqrt{C}\ u(x,0) + \sqrt{L}\ i(x,0) = \sqrt{C}\ u_0(x) + \sqrt{L}\ i_0(x) \equiv V_0(x), \qquad (2.6)$$

$$I(x,0) = -\sqrt{C}\ u(x,0) + \sqrt{L}\ i(x,0) = -\sqrt{C}u_0(x) + \sqrt{L}\ i_0(x) \equiv I_0(x),\ x \in [0,\Lambda].$$

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We have to obtain new boundary conditions substituting $u(x,t)$ and $i(x,t)$ from (2.3) into (1.5).

Indeed, in view of

$$\left| \begin{array}{l} u(0,t)=\dfrac{V(0,t)}{2\sqrt{C}}-\dfrac{I(0,t)}{2\sqrt{C}} \\ i(0,t)=\quad \dfrac{V(0,t)}{2\sqrt{L}}+\dfrac{I(0,t)}{2\sqrt{L}} \end{array} \right. , \quad \left| \begin{array}{l} u(\Lambda,t)=\dfrac{V(\Lambda,t)}{2\sqrt{C}}-\dfrac{I(\Lambda,t)}{2\sqrt{C}} \\ i(\Lambda,t)=\quad \dfrac{V(\Lambda,t)}{2\sqrt{L}}+\dfrac{I(\Lambda,t)}{2\sqrt{L}} \end{array} \right. \text{ and } Z_0=\sqrt{L/C}$$

we have

$$V(0,t)=\frac{2Z_0\sqrt{C}}{Z_0+R_0}E(t)+\frac{Z_0-R_0}{Z_0+R_0}I(0,t),$$

$$\frac{dI(\Lambda,t)}{dt}=\frac{dV(\Lambda,t)}{dt}-\frac{V(\Lambda,t)+I(\Lambda,t)}{C_0Z_0}+\frac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}g^{(n)}\left(\frac{V(\Lambda,t)-I(\Lambda,t)}{2\sqrt{C}}\right)^n .$$

If we assume that the following condition is satisfied (cf. [2]):

$$V_0(0)=I_0(0)=I_0(\Lambda)=V_0(\Lambda)=0,\ E(0)=0$$

then the following conformity condition **(CC)** is fulfilled:

$$V(0,0)=\frac{2Z_0\sqrt{C}}{Z_0+R_0}E(0)+\frac{Z_0-R_0}{Z_0+R_0}I(0,0);$$

$$I(\Lambda,0)=V(\Lambda,0)-\int_0^0\frac{V(\Lambda,s)+I(\Lambda,s)}{C_0Z_0}ds+\frac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}g^{(n)}\int_0^0\left(\frac{V(\Lambda,s)-I(\Lambda,s)}{2\sqrt{C}}\right)^n ds .$$

Introducing denotations $\alpha=\dfrac{2Z_0}{Z_0+R_0}$, $\beta=\dfrac{Z_0-R_0}{Z_0+R_0}$ we obtain the following boundary conditions:

$$V(0,t)=\alpha\sqrt{C}E(t)+\beta I(0,t),$$
(2.7)

$$I(\Lambda,t)=V(\Lambda,t)-\frac{1}{C_0Z_0}\int_0^t\left(V(\Lambda,s)+I(\Lambda,s)\right)ds+\frac{1}{C_0}\sum_{n=1}^{m}g^{(n)}\int_0^t\frac{\left(V(\Lambda,s)-I(\Lambda,s)\right)^n}{\left(2\sqrt{C}\right)^{n-1}}ds .$$

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An operator formulation of the mixed problem

The mixed problem for the new variables is: to find a solution $\left(V(x,t), I(x,t)\right)$ of the following system

$$\frac{\partial V}{\partial t}+\frac{1}{\sqrt{LC}}\frac{\partial V}{\partial x}=-\frac{j_0}{\sqrt{C}}\sin\left(\frac{\pi}{\Phi_0\sqrt{C}}\int_0^t \left(V(x,s)-I(x,s)\right)ds\right) \qquad (3.1\text{-}1)$$

$$\frac{\partial I}{\partial t}-\frac{1}{\sqrt{LC}}\frac{\partial I}{\partial x}=-\frac{j_0}{\sqrt{C}}\sin\left(\frac{\pi}{\Phi_0\sqrt{C}}\int_0^t \left(V(x,s)-I(x,s)\right)ds\right).$$

satisfying initial conditions

$$V(x,0)=V_0(x),\ I(x,0)=I_0(x),\ x\in[0,\Lambda] \qquad (3.1\text{-}2)$$

and boundary conditions

$$V(0,t)=\alpha\sqrt{C}E(t)+\beta I(0,t),\quad x=0,\ t\in[0,T]; \qquad (3.1\text{-}3)$$

$$I(\Lambda,t)=V(\Lambda,t)-\frac{1}{C_0 Z_0}\int_0^t \left(V(\Lambda,s)+I(\Lambda,s)\right)ds+\frac{1}{C_0}\sum_{n=1}^{m} g^{(n)}\int_0^t \frac{\left(V(\Lambda,s)-I(\Lambda,s)\right)^n}{\left(2\sqrt{C}\right)^{n-1}}ds.$$

Prior to define an operator corresponding to the mixed problem we consider Cauchy problem for the characteristics (cf. [2]) ($v=1/\sqrt{LC}$):

$$\frac{d\xi}{d\tau}=\lambda_V(x,t)=v,\ \xi(t)=x \text{ for each } (x,t)\in\Pi \Rightarrow \varphi_V(\tau;x,t)=v\tau+x-vt, \qquad (3.2)$$

$$\frac{d\xi}{d\tau}=\lambda_I(x,t)=-v,\ \xi(t)=x \text{ for each } (x,t)\in\Pi \Rightarrow \varphi_I(\tau;x,t)=-v\tau+x+vt. \qquad (3.3)$$

Functions $\lambda_V(x,t)=v>0$ and $\lambda_I(x,t)=-v<0$ are continuous and imply a uniqueness to the left from t_0 of the solution $x=\varphi_V(t;x_0,t_0)$ for $dx/dt=v$; $x(t_0)=x_0$ and respectively $x=\varphi_I(t;x_0,t_0)$ for $dx/dt=-v$; $x(t_0)=x_0$.

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Denote by $\chi_V(x,t)$ the smallest value of τ such that the solution $\varphi_V(\tau;x,t)=v\tau+x-vt$ of (3.2) still belongs to Π and respectively by $\chi_I(x,t)$ − for the solution $\varphi_I(\tau;x,t)=-v\tau+x+vt$ of (3.3). If $\chi_V(x,t)>0$ then $\varphi_V(\chi_V(x,t);x,t)=0$ or $\varphi_V(\chi_V(x,t);x,t)=\Lambda$ and respectively if $\chi_I(x,t)>0$ then $\varphi_I(\chi_I(x,t);x,t)=0$ or $\varphi_I(\chi_I(x,t);x,t)=\Lambda$. In our case

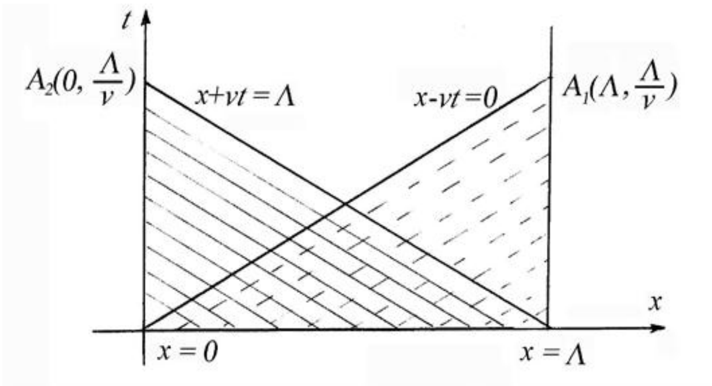
$$\chi_V(x,t)=\begin{cases} t-\dfrac{x}{v} & \text{for } vt-x>0 \\ 0 & \text{for } vt-x\le 0\end{cases};\qquad \chi_I(x,t)=\begin{cases} t+\dfrac{x-\Lambda}{v} & \text{for } vt+x-\Lambda>0 \\ 0 & \text{for } vt+x-\Lambda\le 0\end{cases}$$

Remark 3.1. We notice that $\chi_V(x,t)$ and $\chi_I(x,t)$ are retarded functions, that is, $0\le\chi_V(x,t)<t,\quad 0\le\chi_I(x,t)<t$.

It is easy to see that $\varphi_V(\tau;x,t)=v\tau+x-vt \;\Rightarrow\; \varphi_V(0;x,t)=x-vt$ and

$\varphi_I(\tau;x,t)=-v\tau+x+vt \;\Rightarrow\; \varphi_I(0;x,t)=x+vt$.

Fig. 2. Characteristics of the hyperbolic system



Introduce the sets

$$\Pi_{in,V}=\{(x,t)\in\Pi:\ \chi_V(x,t)=0\}\equiv\{(x,t)\in\Pi: x-vt\ge 0\},$$

$$\Pi_{in,I}=\{(x,t)\in\Pi:\ \chi_I(x,t)=0\}\equiv\{(x,t)\in\Pi: x+vt-\Lambda\le 0\},$$

$$\Pi_{0V}=\{(x,t)\in\Pi:\ \chi_V(x,t)>0,\ \varphi_V(\chi_V(x,t);x,t)=v(vt-x)/v+x-vt=0\},$$

$$\Pi_{0I}=\{(x,t)\in\Pi:\ \chi_I(x,t)>0,\ \varphi_I(\chi_I(x,t);x,t)=-v(vt+x-\Lambda)/v+x+vt=0\}=\varnothing,$$

$$\Pi_{\Lambda V}=\{(x,t)\in\Pi:\chi_V(x,t)>0,\ \varphi_V(\chi_V(x,t);x,t)==v(vt-x)/v+x-vt=\Lambda\}=\varnothing,$$

$$\Pi_{\Lambda I}=\{(x,t)\in\Pi:\ \chi_I(x,t)>0,\ \varphi_I(\chi_I(x,t);x,t)=-v(vt+x-\Lambda)/v+x+vt=\Lambda\}.$$

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Prior to present problem (3.1) in an operator form we introduce

$$\Phi_V(V,I)(x,t) = \begin{cases} V_0(x-vt)), \ (x,t)\in\Pi_{in,V} \\ \Phi_{0V}(V,I)(\chi_V(x,t)), (x,t)\in\Pi_{0V} \end{cases},$$

and

$$\Phi_I(V,I)(x,t) \begin{cases} I_0(x+vt)), \ (x,t)\in\Pi_{in,I} \\ \Phi_{\Lambda I}(V,I)(\chi_I(x,t)), \ (x,t)\in\Pi_{\Lambda I} \end{cases}$$

or

$$\Phi_V(V,I)(x,t) = \begin{cases} V_0(x-vt), \ (x,t)\in\Pi_{in,V} \\ \alpha\sqrt{C}E(\chi_V)+\beta I(0,\chi_V), \ \ (x,t)\in\Pi_{0V} \end{cases},$$

$$\Phi_I(V,I)(x,t) =$$

$$= \begin{cases} I_0(x+vt)), \ (x,t)\in\Pi_{in,I} \\ V(\Lambda,\chi_I) - \dfrac{1}{C_0Z_0}\displaystyle\int_0^{\chi_I} V(\Lambda,s)ds - \dfrac{1}{C_0Z_0}\int_0^{\chi_I} I(\Lambda,s)ds + \dfrac{1}{C_0}\sum_{n=1}^{m} g^{(n)}\int_0^{\chi_I} \dfrac{(V(\Lambda,s)-I(\Lambda,s))^n}{(2\sqrt{C})^{n-1}}ds, \ (x,t)\in\Pi_{\Lambda I}. \end{cases}$$

So we assign to the above mixed problem the following system of operator equations

$$V(x,t) = \Phi_V(V,I)(x,t) - \frac{j_0}{\sqrt{C}}\int_{\chi_V(x,t)}^{t} \sin\left(\frac{\pi K_J}{\sqrt{C}}\int_0^{\tau}(V(x,s)-I(x,s))ds\right)d\tau,$$

$$I(x,t) = \Phi_I(V,I)(x,t) - \frac{j_0}{\sqrt{C}}\int_{\chi_I(x,t)}^{t} \sin\left(\frac{\pi K_J}{\sqrt{C}}\int_0^{\tau}(V(x,s)-I(x,s))ds\right)d\tau$$

or in an explicit form

$$V(x,t) = \begin{cases} V_0(x-vt), \ (x,t)\in\Pi_{in,V} \\ \alpha\sqrt{C}E(\chi_V(x,t)) + \beta I(0,\chi_V(x,t)) - \dfrac{j_0}{\sqrt{C}}\displaystyle\int_{\chi_V(x,t)}^{t} \sin\left(\dfrac{\pi}{\Phi_0\sqrt{C}}\int_0^{\tau}(V(x,s)-I(x,s))ds\right)d\tau, (x,t)\in\Pi_{0V} \end{cases}$$

$$I(x,t) = \begin{cases} I_0(x+vt)), \ (x,t)\in\Pi_{in,I} \\ V(\Lambda,\chi_I) - \dfrac{1}{C_0Z_0}\displaystyle\int_0^{\chi_I} V(\Lambda,s)ds - \dfrac{1}{C_0Z_0}\int_0^{\chi_I} I(\Lambda,s)ds + \dfrac{1}{C_0}\sum_{n=1}^{m} g^{(n)}\int_0^{\chi_I} \dfrac{(V(\Lambda,s)-I(\Lambda,s))^n}{(2\sqrt{C})^{n-1}}ds - \end{cases}$$

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$$-\frac{j_0}{\sqrt{C}}\int_{\chi_I(x,t)}^{t}\sin\left(\frac{\pi}{\Phi_0\sqrt{C}}\int_0^{\tau}(V(x,s)-I(x,s))ds\right)d\tau\ ,\quad (x,t)\in\Pi_{\Lambda I}\,.$$

MAIN RESULT

To get a contractive operator we cut the domain $\Pi=\{(x,t)\in R^2 : x\in[0,\Lambda],\ t\in[0,T]\}$ in the following way $\Pi_\varepsilon=\{(x,t)\in R^2 : x\in[0,\Lambda-\varepsilon],\ t\in[0,T]\}$ for every $0<\varepsilon<\Lambda$.

Introduce the sets

$$M_V=\{V\in C(\Pi_\varepsilon):|V(x,t)|\le U_0e^{\mu t},(x,t)\in\Pi_\varepsilon\},$$
$$M_I=\{I\in C(\Pi_\varepsilon):|I(x,t)|\le J_0e^{\mu t},(x,t)\in\Pi_\varepsilon\},$$

where $U_0, J_0, \mu, \varepsilon$ are positive constants chosen below.

It is easy to verify that $M_V\times M_I$ turns out into a complete metric space with respect to the metrics: $\rho_\mu((V,I),(\bar V,\bar I))=\max\{\rho(V,\bar V),\rho(I,\bar I)\}$, where

$$\rho(V,\bar V)=\operatorname{ess\,sup}\{e^{-\mu t}|V(x,t)-\bar V(x,t)| : x\in[0,\Lambda-\varepsilon],\ t\in[0,T]\},$$

$$\rho(I,\bar I)=\operatorname{ess\,sup}\{e^{-\mu t}|I(x,t)-\bar I(x,t)| : x\in[0,\Lambda-\varepsilon],\ t\in[0,T]\}.$$

Prior to introduce an operator formulation for the mixed problem we redefine all domains $\Pi_{in,V}$, Π_{0V}, $\Pi_{in,I}$, $\Pi_{\Lambda I}$ proceeding from Π_ε and reformulation of conformity conditions.

Now we define the operator $B=(B_V,B_I):M_U\times M_I\to M_U\times M_I$, where $B_V=B_V(V,I)$, $B_I=B_I(V,I)$ by the formulas

$$B_V(V,I)(x,t):=\begin{cases}V_0(x-vt),\ (x,t)\in\Pi_{in,V}\\ \alpha\sqrt{C}E(\chi_V(x,t))+\beta I(0,\chi_V(x,t))-\end{cases}$$

$$-\frac{j_0}{\sqrt{C}}\int_{\chi_V(x,t)}^{t}\sin\left(\frac{\pi}{\Phi_0\sqrt{C}}\int_0^{\tau}(V(x,s)-I(x,s))ds\right)d\tau,\quad (x,t)\in\Pi_{0V};$$

$$B_I(V,I)(x,t):=$$
$$:=\begin{cases}I_0(x+vt)),\ (x,t)\in\Pi_{in,I}\\ V(\Lambda,\chi_I)-\dfrac{1}{C_0Z_0}\displaystyle\int_0^{\chi_I}V(\Lambda,s)ds-\dfrac{1}{C_0Z_0}\int_0^{\chi_I}I(\Lambda,s)ds+\dfrac{1}{C_0}\sum_{n=1}^{m}g^{(n)}\int_0^{\chi_I}\dfrac{(V(\Lambda,s)-I(\Lambda,s))^n}{(2\sqrt{C})^{n-1}}ds-\end{cases}$$

$$-\frac{j_0}{\sqrt{C}}\int_{\chi_I(x,t)}^{t}\sin\left(\frac{\pi}{\Phi_0\sqrt{C}}\int_0^{\tau}(V(x,s)-I(x,s))ds\right)d\tau,(x,t)\in\Pi_{\Lambda I}\,.$$

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Theorem 4.1. Let the following conditions be fulfilled:

$$|E(t)| \le E_0, |E'(t)| \le E_0^1,\ t \in [0,T];\ |V_0(x)| \le U_{00};\ |I_0(x)| \le J_{00},\ x \in [0,\Lambda];$$

$$\max\{U_{00}; \alpha\sqrt{C}\,E_0 + |\beta| J_0\} + \frac{j_0\pi(U_0 + J_0)}{\mu^2\Phi_0 C} \le U_0;$$

$$\max\left\{J_{00};\ e^{\frac{-\mu\varepsilon}{v}}\left(U_0 + \frac{U_0 + J_0}{\mu C_0 Z_0} + \frac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}\frac{|g^{(n)}|(U_0 + J_0)^n}{n\mu(2\sqrt{C})^n} e^{(n-1)\mu\left(T - \frac{\varepsilon}{v}\right)}\right)\right\} + \frac{j_0\pi(U_0 + J_0)}{\mu^2\Phi_0 C} \le J_0.$$

Then there exists a unique generalized solution $(V, I) \in C(\Pi_\varepsilon)$ of (3.1-1)-(3.1-3).

Proof: We establish that the operator B maps the set $M_V \times M_I$ into itself. We notice that $B_V(x,t)$ and $B_I(x,t)$ are continuously differentiable functions and we have to show that

$$|B_V(V,I)(x,t)| \le U_0,\ |B_I(V,I)(x,t)| \le J_0.$$

Indeed

$$|\Phi_V(V,I)(x,t)| \le \begin{cases} |V_0(x - vt)| \\ \alpha\sqrt{C}|E(\chi_V(x,t))| + |\beta||I(0,\chi_V(x,t))| \end{cases} \le$$

$$\le \begin{cases} U_{00} \\ \alpha\sqrt{C}E_0 + |\beta| J_0 \end{cases} \le \begin{cases} U_{00} \\ \alpha\sqrt{C}E_0 + |\beta| J_0 \end{cases} \le \max\{U_{00}; \alpha\sqrt{C}E_0 + |\beta| J_0\}.$$

Then

$$|B_V(V,I)(x,t)| \le |\Phi_V(x,t)| + \frac{j_0}{\sqrt{C}}\int_{\chi_V(x,t)}^{t}\left|\sin\left(\frac{\pi}{\Phi_0\sqrt{C}}\int_0^{\tau}(V(x,s) + I(x,s))ds\right)\right| d\tau \le$$

$$\le |\Phi_V(x,t)| + \frac{j_0\pi}{\Phi_0 C}\int_{\chi_V(x,t)}^{t}\int_0^{\tau}(|V(x,s)| + |I(x,s)|)ds d\tau \le$$

$$\le |\Phi_V(x,t)| + \frac{j_0\pi(U_0 + J_0)}{\Phi_0 C}\int_0^t\int_0^{\tau} e^{\mu s} ds d\tau \le$$

$$\le \left(\max\{U_{00};\ \alpha\sqrt{C}\,E_0 + |\beta| J_0\} + \frac{j_0\pi(U_0 + J_0)}{\mu^2\Phi_0 C}\right)e^{\mu t} \le U_0 e^{\mu t}.$$

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Further on we have:

$$|\Phi_I(V,I)(x,t)| \le$$

$$\le \left\{ \begin{array}{l} |I_0(x+vt))| \\ |V(\Lambda,\chi_I)| + \dfrac{1}{C_0Z_0}\displaystyle\int_0^{\chi_I}|V(\Lambda,s)|ds + \dfrac{1}{C_0Z_0}\displaystyle\int_0^{\chi_I}|I(\Lambda,s)|ds + \dfrac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}\left|g^{(n)}\right|\int_0^{\chi_I}\left(\dfrac{|V(\Lambda,s)|+|I(\Lambda,s)|}{2\sqrt{C}}\right)^n ds \end{array}\right. \le$$

$$\le \left\{ \begin{array}{l} |I_0(x+vt))| \\ U_0 e^{\mu\chi_I(x,t)} + \dfrac{1}{C_0Z_0}\displaystyle\int_0^{\chi_I}U_0e^{\mu s}ds + \dfrac{1}{C_0Z_0}\displaystyle\int_0^{\chi_I}J_0e^{\mu s}ds + \dfrac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}\dfrac{\left|g^{(n)}\right|}{\left(2\sqrt{C}\right)^n}\int_0^{\chi_I}\left(U_0e^{\mu s}+J_0e^{\mu s}\right)^n ds \end{array}\right. \le$$

$$\le \left\{ \begin{array}{l} |I_0(x+vt))| \\ U_0 e^{\mu\chi_I(x,t)} + \dfrac{U_0}{C_0Z_0}\dfrac{e^{\mu\chi_I}-1}{\mu} + \dfrac{J_0}{C_0Z_0}\dfrac{e^{\mu\chi_I}-1}{\mu} + \dfrac{2\sqrt{C}}{C_0}\displaystyle\sum_{n=1}^{m}\dfrac{\left|g^{(n)}\right|\left(U_0+J_0\right)^n}{\left(2\sqrt{C}\right)^n}\int_0^{\chi_I}e^{n\mu s}ds \end{array}\right. \le$$

$$\le \left\{ \begin{array}{l} |I_0(x+vt))| \\ U_0 e^{\mu\chi_I(x,t)} + \dfrac{U_0}{C_0Z_0}\dfrac{e^{\mu\chi_I}-1}{\mu} + \dfrac{J_0}{C_0Z_0}\dfrac{e^{\mu\chi_I}-1}{\mu} + \dfrac{2\sqrt{C}}{C_0}\displaystyle\sum_{n=1}^{m}\dfrac{\left|g^{(n)}\right|\left(U_0+J_0\right)^n}{\left(2\sqrt{C}\right)^n}\dfrac{e^{n\mu\chi_I}-1}{n\mu} \end{array}\right. \le$$

$$\le \left\{ \begin{array}{l} |I_0(x+vt))| \\ U_0 e^{\mu\chi_I} + e^{\mu\chi_I}\dfrac{U_0}{\mu C_0Z_0} + e^{\mu\chi_I}\dfrac{J_0}{\mu C_0Z_0} + \dfrac{2\sqrt{C}}{C_0}\displaystyle\sum_{n=1}^{m}\dfrac{\left|g^{(n)}\right|\left(U_0+J_0\right)^n}{\left(2\sqrt{C}\right)^n}\dfrac{e^{n\mu\chi_I}}{n\mu} \end{array}\right. \le$$

$$\le \left\{ \begin{array}{l} |I_0(x+vt))| \\ U_0 e^{\mu\left(t+\frac{x-\Lambda}{v}\right)} + e^{\mu\left(t+\frac{x-\Lambda}{v}\right)}\dfrac{U_0}{\mu C_0Z_0} + e^{\mu\left(t+\frac{x-\Lambda}{v}\right)}\dfrac{J_0}{\mu C_0Z_0} + e^{\mu\left(t+\frac{x-\Lambda}{v}\right)}\dfrac{2\sqrt{C}}{C_0}\displaystyle\sum_{n=1}^{m}\dfrac{\left|g^{(n)}\right|\left(U_0+J_0\right)^n}{n\mu\left(2\sqrt{C}\right)^n}e^{(n-1)\mu\left(t+\frac{x-\Lambda}{v}\right)} \end{array}\right. \le$$

$$\le e^{\mu t}\left\{ \begin{array}{l} |I_0(x+vt))| \\ U_0 e^{\mu\frac{x-\Lambda}{v}} + e^{\mu\frac{x-\Lambda}{v}}\dfrac{U_0}{\mu C_0Z_0} + e^{\mu\frac{x-\Lambda}{v}}\dfrac{J_0}{\mu C_0Z_0} + e^{\mu\frac{x-\Lambda}{v}}\dfrac{2\sqrt{C}}{C_0}\displaystyle\sum_{n=1}^{m}\dfrac{\left|g^{(n)}\right|\left(U_0+J_0\right)^n}{n\mu\left(2\sqrt{C}\right)^n}e^{(n-1)\mu\left(T+\frac{x-\Lambda}{v}\right)} \end{array}\right. \le$$

$$\le e^{\mu t}\left\{ \begin{array}{l} |I_0(x+vt))| \\ U_0 e^{\mu\frac{\Lambda-\varepsilon-\Lambda}{v}} + e^{\mu\frac{\Lambda-\varepsilon-\Lambda}{v}}\dfrac{U_0}{\mu C_0Z_0} + e^{\mu\frac{\Lambda-\varepsilon-\Lambda}{v}}\dfrac{J_0}{\mu C_0Z_0} + e^{\mu\frac{\Lambda-\varepsilon-\Lambda}{v}}\dfrac{2\sqrt{C}}{C_0}\displaystyle\sum_{n=1}^{m}\dfrac{\left|g^{(n)}\right|\left(U_0+J_0\right)^n}{n\mu\left(2\sqrt{C}\right)^n}e^{(n-1)\mu\left(T+\frac{\Lambda-\varepsilon-\Lambda}{v}\right)} \end{array}\right. \le$$

$$\le e^{\mu t}\left\{ \begin{array}{l} J_{00} \\ e^{\frac{-\mu\varepsilon}{v}}\left(U_0 + \dfrac{U_0+J_0}{\mu C_0Z_0} + \dfrac{2\sqrt{C}}{C_0}\displaystyle\sum_{n=1}^{m}\dfrac{\left|g^{(n)}\right|\left(U_0+J_0\right)^n}{n\mu\left(2\sqrt{C}\right)^n}e^{(n-1)\mu\left(T-\frac{\varepsilon}{v}\right)}\right) \end{array}\right.$$

and

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$$\left|B_I(V,I)(x,t)\right| \le \left|\Phi_I(V,I)(x,t)\right| + \frac{j_0}{\sqrt{C}}\left|\int_{\chi_{|I}(x,t)}^{t} \sin\left(\frac{\pi}{\Phi_0\sqrt{C}}\int_0^{\tau}(V(x,s)-I(x,s))ds\right)d\tau\right| \le$$

$$\le e^{\mu t}\left\{ \begin{matrix} J_{00} \\ e^{\frac{-\mu\varepsilon}{v}}\left(U_0 + \frac{U_0+J_0}{\mu C_0 Z_0} + \frac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}\frac{\left|g^{(n)}\right|(U_0+J_0)^n}{n\mu\left(2\sqrt{C}\right)^n}e^{(n-1)\mu\left(T-\frac{\varepsilon}{v}\right)}\right) \end{matrix}\right. +$$

$$+\frac{j_0}{\sqrt{C}}\left|\int_{\chi_{|I}(x,t)}^{t} \sin\left(\frac{\pi}{\Phi_0\sqrt{C}}\int_0^{\tau}(V(x,s)-I(x,s))ds\right)d\tau\right| \le$$

$$\le e^{\mu t}\max\left\{J_{00};\, e^{\frac{-\mu\varepsilon}{v}}\left(U_0 + \frac{U_0+J_0}{\mu C_0 Z_0} + \frac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}\frac{\left|g^{(n)}\right|(U_0+J_0)^n}{n\mu\left(2\sqrt{C}\right)^n}e^{(n-1)\mu\left(T-\frac{\varepsilon}{v}\right)}\right)\right\} +$$

$$+\frac{j_0}{\sqrt{C}}\frac{\pi}{\Phi_0\sqrt{C}}\int_{\chi_{|I}(x,t)}^{t}\int_0^{\tau}\left(\left|V(x,s)\right| + \left|I(x,s)\right|\right)ds\,d\tau \le$$

$$\le e^{\mu t}\max\left\{J_{00};\, e^{\frac{-\mu\varepsilon}{v}}\left(U_0 + \frac{U_0+J_0}{\mu C_0 Z_0} + \frac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}\frac{\left|g^{(n)}\right|(U_0+J_0)^n}{n\mu\left(2\sqrt{C}\right)^n}e^{(n-1)\mu\left(T-\frac{\varepsilon}{v}\right)}\right)\right\} +$$

$$+\frac{j_0\pi(U_0+J_0)}{\Phi_0 C}\int_{\chi_{|I}(x,t)}^{t}\int_0^{\tau}e^{\mu s}ds\,d\tau \le$$

$$\le e^{\mu t}\left[\max\left\{J_{00};\, e^{\frac{-\mu\varepsilon}{v}}\left(U_0 + \frac{U_0+J_0}{\mu C_0 Z_0} + \frac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}\frac{\left|g^{(n)}\right|(U_0+J_0)^n}{n\mu\left(2\sqrt{C}\right)^n}e^{(n-1)\mu\left(T-\frac{\varepsilon}{v}\right)}\right)\right\} + \frac{j_0\pi(U_0+J_0)}{\mu^2\Phi_0 C}\right] \le e^{\mu t}J_0.$$

It remains to show that (B_V, B_I) is contractive operator.

For the first component $B_V(V,I)$ we have

$$\left|B_V(V,I)(x,t) - B_V(\bar{V},\bar{I})(x,t)\right| \le \left|\Phi_V(V,I)(x,t) - \Phi_V(\bar{V},\bar{I})(x,t)\right| +$$

$$+\frac{j_0\pi}{\Phi_0 C}\int_{\chi_V(x,t)}^{t}\left|\left(\int_0^{\tau}\left|V(x,s)-\bar{V}(x,s)\right|ds\right) + \left(\int_0^{\tau}\left|I(x,s)-\bar{I}(x,s)\right|ds\right)\right|d\tau \le$$

$$\le |\beta|\left|I(0,\chi_V(x,t)) - \bar{I}(0,\chi_V(x,t))\right| + \frac{j_0\pi}{\Phi_0 C}\left(\rho(V,\bar{V}) + \rho(I,\bar{I})\right)\int_{\chi_I(x,t)}^{t}\int_0^{\tau}e^{\mu s}ds\,d\tau \le$$

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$$\leq |\beta|\rho(I,\bar{I})e^{\mu\chi_I} + e^{\mu t}\frac{j_0\pi\left(\rho(V,\bar{V})+\rho(I,\bar{I})\right)}{\mu^2\Phi_0 C} \leq$$

$$\leq |\beta|\rho(I,\bar{I})e^{\mu\left(t+\frac{x-\Lambda}{v}\right)} + \frac{j_0\pi\left(\rho(V,\bar{V})+\rho(I,\bar{I})\right)}{\mu^2\Phi_0 C} \leq e^{\mu t}\left(e^{\frac{-\mu\varepsilon}{v}}|\beta| + \frac{2j_0\pi}{\mu^2\Phi_0 C}\right)\rho_\mu\left((V,I),(\bar{I},\bar{V})\right).$$

It follows

$$\rho(B_V(V,I),B_V(\bar{V},\bar{I})) \leq \left(e^{\frac{-\mu\varepsilon}{v}}|\beta| + \frac{2j_0\pi}{\mu^2\Phi_0 C}\right)\rho_\mu((V,I),(\bar{V},\bar{I}) \equiv K_V\rho_\mu((V,I),(\bar{V},\bar{I}).$$

For the second component $B_I(V,I)$ we have

$$\left|B_I(V,I)(x,t) - B_I(\bar{V},\bar{I})(x,t)\right| \leq \left|\Phi_I(V,I)(x,t) - \Phi_I(\bar{V},\bar{I})(x,t)\right| +$$

$$+\frac{j_0}{\sqrt{C}}\int_{\chi_I(x,t)}^{t}\left|\sin\left(\frac{\pi}{\Phi_0\sqrt{C}}\int_0^\tau (V(x,s)-I(x,s))ds\right) - \sin\left(\frac{\pi}{\Phi_0\sqrt{C}}\int_0^\tau (\bar{V}(x,s)-\bar{I}(x,s))ds\right)\right|d\tau \leq$$

$$\leq \left|V(\Lambda,\chi_I) - \bar{V}(\Lambda,\chi_I)\right| + \frac{1}{C_0 Z_0}\int_0^{\chi_I}\left|V(\Lambda,s)-\bar{V}(\Lambda,s)\right|ds + \frac{1}{C_0 Z_0}\int_0^{\chi_I}\left|I(\Lambda,s)-\bar{I}(\Lambda,s)\right|ds +$$

$$+\frac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}\frac{n\left|g^{(n)}\right|\left(U_0+J_0\right)^{n-1}}{\left(2\sqrt{C}\right)^n}\int_0^{\chi_I}e^{(n-1)\mu s}\left(\left|V(\Lambda,s)-\bar{V}(\Lambda,s)\right| + \left|I(\Lambda,s)-\bar{I}(\Lambda,s)\right|\right)ds +$$

$$+\frac{j_0}{\sqrt{C}}\frac{\pi}{\Phi_0\sqrt{C}}\int_{\chi_I(x,t)}^{t}\left(\int_0^\tau\left|V(x,s)-\bar{V}(x,s)\right|ds + \int_0^\tau\left|I(x,s)-\bar{I}(x,s)\right|ds\right)d\tau \leq$$

$$\leq \rho(V,\bar{V})e^{\mu\chi_I} + \frac{\rho(V,\bar{V})+\rho(I,\bar{I})}{C_0 Z_0}\int_0^{\chi_I}e^{\mu s}ds + \frac{\rho(V,\bar{V})+\rho(I,\bar{I})}{C_0}\sum_{n=1}^{m}\frac{n\left|g^{(n)}\right|\left(U_0+J_0\right)^{n-1}}{\left(2\sqrt{C}\right)^{n-1}}\int_0^{\chi_I}e^{n\mu s}ds +$$

$$+\frac{j_0\pi\left(\rho(V,\bar{V})+\rho(I,\bar{I})\right)}{\Phi_0 C}\int_{\chi_I(x,t)}^{t}\int_0^\tau e^{\mu s}dsd\tau \leq$$

$$\leq \rho(V,\bar{V})e^{\mu\chi_I} + \frac{\rho(V,\bar{V})+\rho(I,\bar{I})}{C_0 Z_0}\int_0^{\chi_I}e^{\mu s}ds + \frac{\rho(V,\bar{V})+\rho(I,\bar{I})}{C_0}\sum_{n=1}^{m}\frac{n\left|g^{(n)}\right|\left(U_0+J_0\right)^{n-1}}{\left(2\sqrt{C}\right)^{n-1}}\int_0^{\chi_I}e^{n\mu s}ds +$$

$$+\frac{j_0\pi\left(\rho(V,\bar{V})+\rho(I,\bar{I})\right)}{\Phi_0 C}\int_{\chi_I(x,t)}^{t}\int_0^\tau e^{\mu s}dsd\tau \leq$$

$$\leq \rho(V,\bar{V})e^{\mu\chi_I} + \frac{\rho(V,\bar{V})+\rho(I,\bar{I})}{\mu C_0 Z_0}e^{\mu\chi_I} + e^{\mu\chi_I}\frac{\rho(V,\bar{V})+\rho(I,\bar{I})}{\mu C_0}\sum_{n=1}^{m}\frac{\left|g^{(n)}\right|\left(U_0+J_0\right)^{n-1}e^{(n-1)\mu\chi_I}}{\left(2\sqrt{C}\right)^{n-1}} +$$

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$$+e^{\mu t}\frac{j_0\pi\left(\rho(V,\bar{V})+\rho(I,\bar{I})\right)}{\mu^2\Phi_0 C}\le$$

$$\le e^{\mu t}\left(e^{\frac{-\mu\varepsilon}{v}}+e^{\frac{-\mu\varepsilon}{v}}\frac{2}{\mu C_0 Z_0}+e^{\frac{-\mu\varepsilon}{v}}\frac{2}{\mu C_0}\sum_{n=1}^{m}\frac{\left|g^{(n)}\right|\left(U_0+J_0\right)^{n-1}e^{(n-1)\mu\left(T-\frac{\varepsilon}{v}\right)}}{\left(2\sqrt{C}\right)^{n-1}}+\frac{2j_0\pi}{\mu^2\Phi_0 C}\right)\rho_\mu((V,I),(\bar{V},\bar{I}))\equiv$$

$$\equiv e^{\mu t}K_I\rho_\mu((V,I),(\bar{V},\bar{I}))$$

or

$$\rho(B_I(V,I),B_I(\bar{V},\bar{I}))\le K_I\rho_\mu((V,I),(\bar{V},\bar{I}))\,.$$

Consequently

$$\rho_\mu\left((B_V(V,I),B_I(V,I)),(B_V(\bar{V},\bar{I}),B_I(\bar{V},\bar{I}))\right)\le K\rho_\mu((V,I),(\bar{V},\bar{I}))\,,$$

where $K=\max\{K_V,K_I\}<1$ for sufficiently large μ. Consequently the operator B is contractive one. Its fixed point is a generalized continuous solution $\left(V_\varepsilon,I_\varepsilon\right)$ of the mixed problem (3.1).

Theorem 4.1 is thus proved.

CONCLUSION REMARKS

1) We have obtained a family of approximated solutions $\bigcup_{\varepsilon>0}\left(V_\varepsilon,I_\varepsilon\right)$. But in general the limit $\lim_{\varepsilon\to 0}\left(V_\varepsilon,I_\varepsilon\right)$ may not exist. Then we can proceed as in [20] to choose a convergent subsequence whose limit we can call a generalized solution of the above mixed problem.

2) Here we collect all inequalities from the proof of the last theorem. We would like to point out that all conditions of the main theorem are applicable to real problems.

Indeed in view for sufficiently small U_{00}, J_{00} we have:

$$\alpha\sqrt{C}\,E_0+|\beta|J_0+\frac{j_0\pi\left(U_0+J_0\right)}{\mu^2\Phi_0 C}\le U_0\,;$$

$$e^{\frac{-\mu\varepsilon}{v}}\left(U_0+\frac{U_0+J_0}{\mu C_0 Z_0}+\frac{2\sqrt{C}}{C_0}\sum_{n=1}^{m}\frac{\left|g^{(n)}\right|\left(U_0+J_0\right)^n}{n\mu\left(2\sqrt{C}\right)^n}e^{(n-1)\mu\left(T-\frac{\varepsilon}{v}\right)}\right)+\frac{j_0\pi\left(U_0+J_0\right)}{\mu^2\Phi_0 C}\le J_0\,;$$

$$K_V=e^{\frac{-\mu\varepsilon}{v}}|\beta|+\frac{2j_0\pi}{\mu^2\Phi_0 C}<1\,;$$

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$$K_I = e^{\frac{-\mu\varepsilon}{v}} + e^{\frac{-\mu\varepsilon}{v}} \frac{2}{\mu C_0 Z_0} + e^{\frac{-\mu\varepsilon}{v}} \frac{2}{\mu C_0} \sum_{n=1}^{m} \frac{\left|g^{(n)}\right| (U_0 + J_0)^{n-1} e^{(n-1)\mu\left(T - \frac{\varepsilon}{v}\right)}}{\left(2\sqrt{C}\right)^{n-1}} + \frac{2 j_0 \pi}{\mu^2 \Phi_0 C} < 1.$$

Let us consider a Josephson transmission line (cf. [16]-[19]) with $L = 2{,}6.10^{-9}\, H/m$, $C = 1{,}2.10^{-6}\, F/m$, length $\Lambda = 10^{-3} m$. Then

$$\sqrt{LC} = \sqrt{31{,}2.10^{-16}} \approx 5{,}9.10^{-8} \Rightarrow v = 1\sqrt{LC} \approx 1{,}8.10^{7}, \ T = \left(10^{-3}\right)/\left(1{,}8.10^{7}\right) \approx 5{,}5.10^{-11} \ \sec,$$

$$Z_0 = \sqrt{L/C} = \sqrt{21{,}6.10^{-4}} \approx 4{,}65.10^{-2}\Omega, \ \Phi_0 = 2.10^{-15} W/m^2; \ j_0 = 1{,}9\, A/m.$$

Let us take $C_0 = 5.10^{-10} F$, $R_0 = Z_0 = 4{,}65.10^{-2}\ \Omega$, $C_0 Z_0 = 5.10^{-10}.4{,}65.10^{-2} \approx 2{,}325.10^{-11}$. Then $\alpha = 2Z_0/(Z_0 + R_0) = 1, \beta = (Z_0 - R_0)/(Z_0 + R_0) = 0$.

Let us choose $\mu = 10^{11}$ and $\varepsilon = 10^{-4}$. Then $\sqrt{C} = \sqrt{1{,}2.10^{-6}} = 1{,}1.10^{-3}$,
$\Phi_0 C = 2.10^{-15}.1{,}2.10^{-6} \approx 2{,}4.10^{-21}$; $\mu \Phi_0 C = 10^{11}.2.10^{-15}.10^{-6} \approx 2.10^{-10}$; $j_0 \pi \approx 6$;

$$g^{(1)} = 0{,}12; \ g^{(2)} = 0; \ g^{(3)} = -0{,}1; \ e^{\frac{-\mu\varepsilon}{v}} = e^{-\frac{10^{12}.10^{-4}}{1{,}8.10^{7}}} \approx e^{-5{,}5};$$

$$e^{\mu\left(T - \frac{\varepsilon}{v}\right)} = e^{10^{11}\left(5{,}5.10^{-11} - \frac{10^{-4}}{1{,}8.10^{7}}\right)} \approx e^{\left(5{,}5 - \frac{1}{1{,}8.}\right)} \approx e^{4{,}94} = 140.$$

Then the above inequalities become:

$$1{,}1.10^{-3} E_0 + \frac{1{,}9(U_0 + J_0)}{10^{24} 2{,}4.10^{-21}} \le U_0$$

$$e^{-5{,}5}\left(U_0 + \frac{U_0 + J_0}{50.4{,}65.10^{-2}} + \frac{U_0 + J_0}{10^{11}.5.10^{-10}}\left(0{,}12 + \frac{0{,}1(U_0 + J_0)^2}{14{,}4.10^{-6}} 140^2\right)\right) + \frac{1{,}9(U_0 + J_0)}{10^{24} 2{,}4.10^{-21}} \le J_0;$$

$$K_V = \frac{3{,}8}{10^{24} 2{,}4.10^{-21}} < 1;$$

$$K_I = e^{-5{,}5}\left(1 + \frac{2}{50.4{,}65.10^{-2}} + \frac{2}{5.10}\left(0{,}12 + \frac{0{,}1(U_0 + J_0)^2 140^2}{4{,}8.10^{-6}}\right)\right) + \frac{3{,}8}{2.10} < 1$$

or

$$1{,}1.10^{-3} E_0 + 7{,}92.10^{-4}(U_0 + J_0) \le U_0;$$

$$4.10^{-3} U_0 + 2{,}5216.10^{-3}(U_0 + J_0) + 10{,}88.10^{3}(U_0 + J_0)^3 \le J_0;$$

$$K_V = 1{,}59.10^{-3} < 1;$$

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$$K_I = 4.10^{-3}\left(1{,}865 + 16{,}3.10^6 (U_0 + J_0)^2\right) + 0{,}19 < 1.$$

It should be noted that the actual physical quantities (voltage and current) should be calculated by the formulas

$$u(x,t) = V(x,t)/\left(2\sqrt{L}\right) + I(x,t)/\left(2\sqrt{L}\right);$$
$$i(x,t) = \quad V(x,t)/\left(2\sqrt{C}\right) - I(x,t)/\left(2\sqrt{C}\right).$$

Since $L = 2{,}6.10^{-9} \Rightarrow \sqrt{L} = \sqrt{0{,}26.10^{-8}} \approx 0{,}5.10^{-4}$ we have

$$u(x,t) = \frac{V(x,t)}{2\sqrt{L}} + \frac{I(x,t)}{2\sqrt{L}} \approx \left(V(x,t) + I(x,t)\right)10^4 \; .$$

Therefore to obtain voltage of order $u(x,t) \approx 10^0$ we have to take $U_0 + J_0 \approx 10^{-4}$.
Consequently in view of $E_0 \approx U_0 \approx 10^{-4}$ the above inequalities become

$$1{,}1.10^{-3}.10^{-4} + 7{,}92.10^{-8} \le 10^{-4} \Leftrightarrow 1{,}1.10^{-3} + 7{,}92.10^{-4} \le 1;$$

$$4.10^{-3}.10^{-4} + 2{,}5216.10^{-3}.10^{-4} + 10{,}88.10^3\left(10^{-4}\right)^3 \le 10^{-4} \;\Leftrightarrow\; 4.10^{-3} + 2{,}5216.10^{-3} + 10{,}88.10^{-5} \le 1$$
;

$$K_V = 0{,}00159 < 1;$$

$$K_I = 4.10^{-3}\left(1{,}865 + 16{,}3.10^6\left(10^{-4}\right)^2\right) + 0{,}19 = 0{,}198112 < 1.$$

REFERENCES

1. Angelov V.G., A Method for Analysis of Transmission Lines Terminated by Nonlinear Loads. *Nova Science, New York, 2014.*

2. Abolinya, W.E., A.D. Myshkis, On the mixed problem for almost linear hyperbolic system on the plain. *Math. Sbornik*. 1960, vol. 50 (92), No 4, 432-442. (in Russian)

3. Ishimaru A., Electromagnetic Wave Propagation Radiation and Scattering. *Prentice-Hall, Inc., New Jersey, 1991*.

4. Pozar D., Microwave Engineering. *John Wiley & Sons Inc., New York, 1998*.

5. Paul C.R., Analysis of Multi-Conductor Transmission Lines. *A Wiley-Inter science Publication, John Wiley & Sons, New York, 1994.*

6. Ramo S., J.R. Whinnery, T. van Duzer, Fields and Waves in Communication Electronics. *John Wiley & Sons Inc., New York, 1994.*

7. Rosenstark S., Transmission lines in computer engineering. *Mc Grow-Hill, New York, 1994*.

ISSN 2053-4108(Print) ISSN 2053-4116(Online)

8. Vizmuller P., RF Design Guide Systems, Circuits and Equations. *Artech House, Inc., Boston • London, 1995.*

9. Magnusson P.C., G.C. Alexander, V.K. Tripathi, Transmission Lines and Wave Propagation. *3rd ed., CRC Press. Boka Raton, 1992*.

10. Dunlop J., D.G. Smith, Telecommunications Engineering. *Chapman & Hall, London, 1994*.

11. Maas S.A., Nonlinear Microwave and RF Circuits. *Second Edition, Artech House, Inc., Boston • London, 2003.*

12. Misra D.K., Radio-Frequency and Microwave Communication Circuits. Analysis and Design. *2nd ed., University of Wisconsin-Milwaukee, John Wiley & Sons, Inc., Publication, 2004*.

13. Miano G., A. Maffucci, Transmission Lines and Lumped Circuits. *Academic Press, New York, 2001, 2nd ed., 2010.*

14. Angelov V.G., Lossless transmission lines terminated by *L*-load in series connected to parallel connected *GL*-loads, British Journal of Mathematics & Computer Science, 3(3), **(**2013), 352-389.

15. Scott A., Active and Nonlinear Wave Propagation in Electronics. *Wiley Int. Publications, John Wiley & Sons, New York, 1970.*

16. Barone A., G. Paterno, Physics and Applications of the Josephson Effect. *Wiley-Int. Publications, John Wiley & Sons, New York, 1982.*

17. Scott A.C., F. Chu, S. Reible, Magnetic-flux propagation on a Josephson transmission line. *J. Applied Physics*, vol. 47, No.7, July (1976), 3272-3286.

18. Swihart J.C., Field solution for a thin-film superconducting strip transmission line. *J. Applied Physics*, vol. 32, No.3, March, (1961), 461-469.

19. Angelov V.G., Lossless transmission lines with Josephson junction – approximated continuous generalized solutions. *Journal of Multidisciplinary Engineering Science and Technology* (JMEST), vol. 2, Issue 1, January, (2015), 291-298.

20. Angelov V.G., Lossy transmission lines with Josephson junction - continuous generalized solutions. *Communication in Applied Analysis, (2015), (to appear).*

21. Angelov V.G., Fixed Points in Uniform Spaces and Applications. *Cluj University Press, Cluj-Napoca, Romania, 2009.*

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