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IMPACT ASSESSMENT OF THE NATURAL LOGARITHM TRANSFORMATION ON THE ERROR COMPONENT OF THE MULTIPLICATIVE ERROR MODEL

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ABSTRACT: *In this study we examine the implication of natural logarithm transformation on the two most popular distributions (Gamma and Weibull) of the error component of the multiplicative error model. The k^{th} moment (k =1, 2, 3, …) of the logarithm transformed Gamma distribution was established while that of the log-transformed Weibull distribution was found not to be solvable in its closed form and therefore further investigations were limited to the Gamma distributed error component. The mean of the log-transformed Gamma distribution as required in statistical modeling was found to exist for $\alpha = \beta < 1$ while its variance exits for $\alpha = \beta \leq 0.49$. However using simulations the region for successful application of log-transformed distribution was found to be $0 < \alpha = \beta < 0.48$. Furthermore, it was discovered that the log-transform led to a significant reduction of the variance of the distribution, however the expected zero-mean assumption after linearing a multiplicative model with a logarithm transformation is not met even though there were decreases in the mean values after the transformation. Finally as a result of the findings of this study, we recommend in statistical modeling, that natural logarithm transformation is not appropriate in a multiplicative error model (with a unit mean error component) for either linearizing or stabilizing the variance of the model or both since it leads to a distribution whose k^{th} moment (k = 1, 2, 3, . .) is not solvable in a closed form (for the Weibull distribution) or whose mean is not zero (for the Gamma) as required after transformation.*

KEYWORDS: Multiplicative Error Model; Logarithm Transformation; Gamma Distribution; Weibull Distribution; Moments

INTRODUCTION

Multiplicative error model (MEM) are generally used for modeling non-negative valued discrete time stochastic processes. Brownlees et al., (2011) specified a MEM as

$$X_{t,t\in N} = X_t = \mu_t \xi_t \tag{1}$$

where $X_{t,t\in N}$ is a real-valued, discrete time stochastic process defined on [0, + ∞), μ_t , defined conditionally on $\Psi_{t-1} = \mu(\theta, \Psi_{t-1})$ is a positive quantity that evolves deterministically according to the parameter vector, θ. Ψ_{t-1} is the information available for forecasting $X_{t,t\in N}$ and ξ_t is a random variable with a probability density function defined over a [0, +∞) support with unit mean and unknown constant variance, σ_1^2. That is

$$\xi_t \sim D_1^+\left(1, \sigma_1^2\right) \tag{2}$$

There are many distributions having the distributional characteristics given in (2) among which are; Exponential, Gamma, Inversted Gamma, Raleigh, Weibull, Lognormal and so on. However the most common distributions used in modeling (1) are Gamma and Weibull as respectively suggested by Engle and Gallo (2006) and Bauwens and Giot (2000) and as a result our study distributions would be the Gamma and Weibull distributions.

One of the popular methods of handling (1) is to linearize by taking logarithm transformation. That is

$$Y_t = \log_e X_t = \mu_t^* + \xi_t^* \tag{3}$$

where, $\mu_t^* = \log_e X_t$, $\xi_t^* = \log_e \xi_t$ and

$$\xi_t^* \sim D_2^+\left(0, \sigma_2^2\right) \tag{4}$$

Logarithm transformation is one of the popular transformations used for variance stabilization and conversion of the error of any purely multiplicative error model component to an additive structure.

In statistical modeling of (1) where unit-mean and homogeneity of variance of the error component are required for the application of parametric method of statistical analysis, the presence of outliers and some other factors may yield a data set with a non-constant variance error structure. For a data set where the assumption of constant variance is far from being true, there is need for the application of a variance-stabilizing transformation.

According to Vidakovic (2012), "it is not an overstatement to say that statistics is based on various data transformations. Basic statistical summaries such as sample mean, variance, z-scores, histograms, etc., are all transformed data. Some more advanced summaries such as principal components, periodograms, empirical characteristics functions, etc., are also examples of transformed data. Vidakovic (2012) went further to say that "transformations in statistics are utilized for several reasons, but unifying arguments are that transformed data"; (i) are easier to report, store and analyze (ii) comply better with a particular modeling framework and (iii) allow for additional insight to the phenomenon not available in the domain of non-transformed data. For example, variance stabilizing transformations, symmetrizing transformations, transformations to additivity, laplace, Fourier, Wavelet, Gabor, Wigner-Ville, Hugh, Mellin, transforms all satisfy one or more of points listed in (i – iii).

Data transformations are the applications of mathematical modifications to the values of a variable and the most common variance stabilizing transformations are the power transformations namely; logarithm, inverse, inverse-square, inverse-square-root, square and inverse-square (Iwueze et al., (2011). In practice, a data transformation where absolutely necessary must be successful. Ohakwe et al.,(2012) defined a successful transformation for (1) as one where the unit mean and constant variance assumptions are not violated after transformation. Except the logarithm transformation every other power transformation leaves (1) multiplicative and as such the distributional characteristics of ξ_t^* still follow (2), while it follows (4) for the logarithm transformation. For model (1) Ohakwe et al., (2012) had studied the implication of square root transformation on the unit mean and constant variance assumptions of the error component of model (1) whose distributional characteristics belong to the Generalized Gamma Distribution under the various forms; Chi-square, Exponential, Gamma (a, b, 1), Weibull, Maxwell and Rayleigh distributions. From the results of the study, the unit mean assumption is approximately maintained for all the given distributional forms of the GGD. However there were reductions in the variances of the distributions except those of the Gamma (a, b, 1), for a > 1, Rayleigh and Maxwell that increased, hence they concluded that square-root transformation is not appropriate for multiplicative error model with a Gamma (a, b, 1) for a > 1 or Rayleigh or Maxwell distributed error component. Finally, Ohakwe et al., (2012) recommended that square-root transformation, where applicable for a multiplicative error model is successful for the studied distributions if the variance of the transformed error component < 0.5.

Also in this area of study, Ohakwe and Chikezie (2013) had investigated the implications of power transformations namely, inverse-square-root, inverse, inverse-square and square transformations on the unit-mean and constant variance assumptions of the error component of the multiplicative error model. Here the distributions of the error component studied were the various forms of the generalized gamma distribution namely Gamma (a, b, 1), Chi-square, Exponential, Weibull, Rayleigh and Maxwell distributions. The purpose of their study were to investigate whether the unit-mean and constant variance assumptions necessary for modeling using the multiplicative error model are either violated or retained after the various power transformations. From the results of the study, Ohakwe and Chikezie (2013) discovered the following; (i) For the inverse-square-root transformation, the unit-mean and constant variance assumptions are approximately maintained for all the distributions under study except the Chi-square distribution where the unit mean assumption was violated. (ii) For the inverse transformation, the unit-mean assumptions are violated after transformation except for the Rayleigh and Maxwell distributions. (iii) For the inverse-square transformation, the unit-mean assumption is violated for all the distributions under study. (iv) For the square transformation, it is only the Maxwell distribution that maintained the unit-mean assumption. (v) For all the studied transformations the variances of the transformed distributions were found to be greater than those of the untransformed distribution.

The popular power transformations frequently used in statistical time series data modeling are logarithm, inverse, inverse-square-root, square-root, square and inverse-square and based on the literature review, there is no question that out of the six transformations only the logarithm transformation is yet to be explored. Therefore the essence of this paper is to study the implication of logarithm transformation of model (1) with the overall aim of establishing if the said transformation will lead to a zero-mean distributed error component with a constant variance $\sigma_2^2 < \infty$ as it is expected even though σ_1^2 may not be equal σ_2^2. The study distributions in this paper would be the Gamma and Weibull distributions which are the most favoured distributions of the error component of the MEM. The paper is organized as follows: Section one contains the introduction while the moments of the logarithm transformed distributions are given in section two. Numerical results and discussions are contained in Section three while the conclusion and references are respectively contained in Sections four and five.

Distributional Characteristics of the Gamma and Weibull Distributions

In this Section we give the probability density function (pdf) and moments of the untransformed distribution. Furthermore in this Section, we would also obtain the moments of the Gamma and Weibulll distributions under natural logarithm transformation.

The pdf of a Gamma random variable, X and its kth moment as contained in Walck (2000) is given by

$$f(x) = \frac{\beta^{\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha)}, \; x > 0 \tag{5}$$

with

$$E\left(X^k\right) = \frac{\Gamma(\alpha + k)}{\beta^k \Gamma(\alpha)}, \; k = 1, 2, 3, \ldots \tag{6}$$

$$E(X) = \frac{\alpha}{\beta} \tag{7}$$

$$E\left(X^2\right) = \frac{\alpha(\alpha+1)}{\beta^2} \tag{8}$$

and

$$\operatorname{Var}(X) = \sigma_1^2 = \frac{\alpha}{\beta^2} \tag{9}$$

Similarly for the Weibull distributed random variable whose pdf is also given in Walck (2000) as

$$f(x) = \frac{\eta}{\sigma}\left(\frac{x}{\sigma}\right)^{\eta-1} e^{-\left(\frac{x}{\sigma}\right)^{\eta}}, \ x > 0, \sigma > 0 \text{ and } \eta > 0 \tag{10}$$

with

$$E\left(X^k\right) = \sigma^k \Gamma\left(1 + \frac{k}{\eta}\right) \tag{11}$$

$$E(X) = \sigma\Gamma\left(1 + \frac{1}{\eta}\right) = \frac{\sigma}{\eta}\Gamma\left(\frac{1}{\eta}\right) \tag{12}$$

$$E\left(X^2\right) = \sigma^2\Gamma\left(1 + \frac{2}{\eta}\right) = \frac{2\sigma^2}{\eta}\Gamma\left(\frac{2}{\eta}\right) \tag{13}$$

and

$$\operatorname{Var}(X) = \sigma_1^2 = \frac{\sigma^2}{\eta}\left[2\Gamma\left(\frac{2}{\eta}\right) - \frac{1}{\eta}\left(\Gamma\left(\frac{1}{\eta}\right)\right)^2\right] \tag{14}$$

Logarithm Transformation of the Gamma and the Weibull Distributions

Applying the following substitutions in (5) and (10);

$$\left.\begin{aligned} & y = e^x, 1 < y < \infty \\ & \ln y = x \\ & \frac{dx}{dy} = \frac{1}{y} \end{aligned}\right\} \tag{15}$$

Where the pdf of $Y = f(y) = f(x = \ln x)\frac{dx}{dy}$ (Hogg and Craig (1978)), we obtain the following logarithm transformed pdfs of Gamma and Weibull;

For the Gamma;

$$f(y) = \frac{\beta^{\alpha}(\ln y)^{\alpha-1} e^{-\frac{\ln y}{\beta}}}{y\Gamma(\alpha)}, 1<y<\infty \tag{16}$$

and for the Weibull;

$$f(y) = \frac{\eta}{\sigma y}\left(\frac{\ln y}{\sigma}\right)^{\eta-1} e^{-\left(\frac{\ln y}{\sigma}\right)^{\eta}}, 1<y<\infty \tag{17}$$

Moments of Logarithm Transformed Distributions

To obtain the k^{th} moment $\left(E\left(Y^{k}\right), k=1,2,3,\ldots\right)$ of the logarithm transformed Gamma distribution, we make the following substitutions in (16);

$$\left.\begin{aligned} w &= \frac{\ln y}{\beta},\ 0< w < \infty \\ y &= e^{\beta w} \\ dy &= \beta e^{\beta w}\, dw \end{aligned}\right\} \tag{18}$$

hence

$$E\left(Y^{k}\right) = \int_{1}^{\infty} y^{k} f(y) dy = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\int_{1}^{\infty} y^{k-1}(\ln y)^{\alpha-1} e^{-\frac{\ln y}{\beta}} dy$$

$$= \frac{\beta^{\alpha}}{\Gamma(\alpha)}\int_{0}^{\infty} e^{\beta w(k-1)}(\beta w)^{\alpha-1} e^{-w}\beta e^{\beta w} dw = \frac{\beta^{2\alpha}}{\Gamma(\alpha)}\int_{0}^{\infty} w^{\alpha-1} e^{-w(1-\beta k)} dw \tag{19}$$

Also by change of variable method in (19), let

$$p = w(1-\beta k),\ 0 < p < \infty \tag{20}$$

then

$$w = \frac{p}{1-\beta k} \text{ and } dw = \frac{dp}{1-\beta k} \tag{21}$$

hence

$$E\left(Y^k\right)=\frac{\beta^{2\alpha}}{\Gamma(\alpha)}\left(\frac{1}{1-\beta k}\right)^{\alpha}\int_0^{\infty}p^{\alpha-1}e^{-p}dp=\frac{\beta^{2\alpha}}{\Gamma(\alpha)}\left(\frac{1}{1-\beta k}\right)^{\alpha}\Gamma(\alpha)=\left(\frac{\beta^2}{1-\beta k}\right)^{\alpha} \tag{22}$$

Based on the result of (22), the first and second moments are obtained as follows;

$$E(Y)=\left(\frac{\beta^2}{1-\beta}\right)^{\alpha} \tag{23}$$

$$E\left(Y^2\right)=\left(\frac{\beta^2}{1-2\beta}\right)^{\alpha} \tag{24}$$

and

$$\sigma_2^2=\left(\frac{\beta^2}{1-2\beta}\right)^{\alpha}-\left(\frac{\beta}{1-\beta}\right)^{2\alpha} \tag{25}$$

Similarly, for the k^{th} moment of the logarithm transformed Weibull distribution, we adopt the following substitution

$$\left.\begin{aligned} w &= \left(\frac{\ln y}{\sigma}\right)^{\eta},\ 0< w < \infty \\ w^{\frac{1}{\eta}} &= \left(\frac{\ln y}{\sigma}\right) \\ y &= e^{\sigma w^{\frac{1}{\eta}}} \\ dy &= \frac{\sigma}{\eta} w^{\frac{1}{\eta}-1} e^{\sigma w^{\frac{1}{\eta}}} dw \end{aligned}\right\} \tag{26}$$

therefore

$$E\left(Y^k\right)=\frac{\eta}{\sigma}\int_1^{\infty}y^k\frac{1}{y}\left(\frac{\ln y}{\sigma}\right)^{\eta-1}e^{-\left(\frac{\ln y}{\sigma}\right)^{\eta}}dy=\frac{\eta}{\sigma}\int_1^{\infty}y^{k-1}\left(\frac{\ln y}{\sigma}\right)^{\eta-1}e^{-\left(\frac{\ln y}{\sigma}\right)^{\eta}}dy$$

will yield

$$E\left(Y^k\right)=\int_0^{\infty}e^{-\left(w-\sigma k w^{\frac{1}{\eta}}\right)}dw \tag{27}$$

However (27) cannot be integrated in a closed form and this agrees with the comment of O'Brien(2010) therefore we will proceed our study using only the Gamma distribution.

Numerical Illustrations

It is important to recall that the unit-mean conditions (That is $\alpha = \beta$ for the Gamma distribution) had been given in Ohakwe et al., (2012). Also given in Ohakwe et al., (2012) is the variance of the study distribution resulting from the application of the unit-mean condition and it is as follows;

$$\sigma_1^2 = \frac{1}{\alpha}\left(\text{where } \alpha = \beta\right) \tag{28}$$

In this Section we shall first apply the unit mean conditions to the expressions for the mean and variance of the logarithm transformed Gamma distribution to obtain the following results;

$$\mathrm{E}\left(\mathrm{Y}\right) = \left(\frac{\alpha^2}{1-\alpha}\right)^{\alpha}, \alpha < 1 \tag{29}$$

$$\sigma_2^2 = \left(\frac{\alpha^2}{1-2\alpha}\right)^{\alpha} - \left(\frac{\alpha^2}{1-\alpha}\right)^{2\alpha}, \alpha < 0.5 \tag{30}$$

Secondly we would investigate the interval for successful logarithm transformation of the study distribution, by computing compute the values of (28), (29) and (30) for values of $\alpha = 0.01, 0.02, \ldots, 0.98, 0.99$. The results of the computations are given in Table 1. In Table 1 it is obvious that the mean of the logarithm transformed gamma distribution under the application of the unit-mean condition is less than unity but never zero $E\left(Y\right) < \left(E\left(X\right) = 1.0\right)$. The percentage decrease in mean were calculated for various values of α using $\left(E\left(X\right) - E\left(Y\right)\right)100\%$. Also in Table 1, there is a significant reduction in the variance of the logarithm transformed Gamma against the untransformed gamma distribution. As it is expected, the logarithm transformation reduced the spread of values of the distribution. The percentage reduction in variance were calculated for the various values of α using $\left(\frac{\sigma_2^2 - \sigma_1^2}{\sigma_1^2}\right)100\%$.

Mathematically when $\alpha = 0.5$, $E\left(Y^2\right) = \left(\frac{\alpha^2}{0}\right)^{\alpha}$ which is undefined and also for $\alpha > 0.5$, we have that $E\left(Y^2\right) = \left(\Phi\left(\alpha\right)\right)^{\alpha} \in R^-$ where $\Phi\left(\alpha\right)$, a function of α is negative-valued and this contradicts the fundamental result that $E\left(Y^2\right) = \frac{\sum_i^N Y_i^2}{N} \in R^+ \ \forall \ Y_i$, therefore logarithm transformation can be successfully applied to a multiplicative error model with a Gamma distributed error component for the purpose of either linearizing the model or stabilizing the variance or both when $0<\alpha \leq 0.48$ considering that there is an increase in variance for $\alpha = 0.49$ after the transformation.

CONCLUSIONS

In this study we examined the implication of logarithm transformation on the two most popular distributions (Gamma and Weibull) of the error component of the multiplicative error model. The k^{th} moment (k =1, 2, 3, …) of the logarithm transformed Gamma distribution was established while that of the log-transformed Weibull distribution was found not to be solvable in its closed form hence further investigations were limited to the Gamma distributed error component. The mean of the log-transformed Gamma distribution as required in statistical modeling was found to exist for $\alpha = \beta < 1$ while its variance exits for $\alpha = \beta \leq 0.49$. However using simulations the region for successful application of natural log-transformation for the distributed error component was found to be $0<\alpha = \beta < 0.48$. Furthermore, it was discovered that the log-transform led to a significant reduction of the variance of the distribution, however the expected zero-mean assumption after linearing a multiplicative model with a logarithm transformation is not met even though there were decreases in the mean values after the transformation.

Finally as a result of the findings of this study, we recommend in statistical modeling, that logarithm transformation is not appropriate in a multiplicative error model (with a unit mean error component) for either linearizing or stabilizing the variance of the model or both since it leads to a distribution whose k^{th} moment (k = 1, 2, 3, . .) is not solvable in a closed form (for the Weibull distribution) or whose mean is not zero (for the Gamma) as required after transformation.

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Table 1: Computations of the Moments of Logarithm Transformed Gamma Distribution, Variance of the untransformed Gamma Distribution and the PIV

α	$E(X)$	$E(Y)$	$E(Y^2)$	σ_1^2	σ_2^2	$\left(E(X)-E(Y)\right)100\%$	$\left(\frac{\sigma_2^2-\sigma_1^2}{\sigma_1^2}\right)100\%$
0.01	1.00	0.91	0.91	100.00	0.08	8.79	-99.92
0.02	1.00	0.86	0.86	50.00	0.12	14.45	-99.75
0.03	1.00	0.81	0.81	33.33	0.15	18.90	-99.54
0.04	1.00	0.77	0.78	25.00	0.18	22.58	-99.30
0.05	1.00	0.74	0.75	20.00	0.19	25.70	-99.04
0.06	1.00	0.72	0.72	16.67	0.21	28.39	-98.76
0.07	1.00	0.69	0.70	14.29	0.22	30.73	-98.48
0.08	1.00	0.67	0.68	12.50	0.23	32.80	-98.20
0.09	1.00	0.65	0.66	11.11	0.23	34.62	-97.91
0.10	1.00	0.64	0.65	10.00	0.24	36.24	-97.61
0.11	1.00	0.62	0.63	9.09	0.24	37.67	-97.32
0.12	1.00	0.61	0.62	8.33	0.25	38.95	-97.02
0.13	1.00	0.60	0.61	7.69	0.25	40.09	-96.71
0.14	1.00	0.59	0.60	7.14	0.26	41.10	-96.40
0.15	1.00	0.58	0.60	6.67	0.26	42.00	-96.09
0.16	1.00	0.57	0.59	6.25	0.26	42.80	-95.77
0.17	1.00	0.57	0.59	5.88	0.27	43.49	-95.44
0.18	1.00	0.56	0.58	5.56	0.27	44.10	-95.10
0.19	1.00	0.55	0.58	5.26	0.28	44.62	-94.76
0.20	1.00	0.55	0.58	5.00	0.28	45.07	-94.40
0.21	1.00	0.55	0.58	4.76	0.28	45.45	-94.03
0.22	1.00	0.54	0.58	4.55	0.29	45.75	-93.64

Table 1 Continues

α	$E(X)$	$E(Y)$	$E(Y^2)$	σ_1^2	σ_2^2	$(E(X)-E(Y))$*100%	$\left(\frac{\sigma_2^2-\sigma_1^2}{\sigma_1^2}\right)$*100%
0.23	1.00	0.54	0.59	4.35	0.29	45.99	-93.23
0.24	1.00	0.54	0.59	4.17	0.30	46.16	-92.80
0.25	1.00	0.54	0.59	4.00	0.31	46.27	-92.35
0.26	1.00	0.54	0.60	3.85	0.31	46.32	-91.87
0.27	1.00	0.54	0.61	3.70	0.32	46.32	-91.36
0.28	1.00	0.54	0.62	3.57	0.33	46.25	-90.81
0.29	1.00	0.54	0.63	3.45	0.34	46.13	-90.22
0.30	1.00	0.54	0.64	3.33	0.35	45.96	-89.59
0.31	1.00	0.54	0.65	3.23	0.36	45.72	-88.89
0.32	1.00	0.55	0.67	3.13	0.37	45.44	-88.13
0.33	1.00	0.55	0.69	3.03	0.39	45.09	-87.28
0.34	1.00	0.55	0.71	2.94	0.40	44.70	-86.35
0.35	1.00	0.56	0.73	2.86	0.42	44.24	-85.30
0.36	1.00	0.56	0.76	2.78	0.44	43.73	-84.12
0.37	1.00	0.57	0.79	2.70	0.47	43.15	-82.77
0.38	1.00	0.57	0.82	2.63	0.49	42.52	-81.23
0.39	1.00	0.58	0.87	2.56	0.53	41.82	-79.43
0.40	1.00	0.59	0.91	2.50	0.57	41.06	-77.31
0.41	1.00	0.60	0.97	2.44	0.62	40.24	-74.78
0.42	1.00	0.61	1.04	2.38	0.67	39.34	-71.70
0.43	1.00	0.62	1.13	2.33	0.75	38.38	-67.87
0.44	1.00	0.63	1.23	2.27	0.84	37.33	-62.97
0.45	1.00	0.64	1.37	2.22	0.97	36.21	-56.49
0.46	1.00	0.65	1.56	2.17	1.14	35.01	-47.47
0.47	1.00	0.66	1.85	2.13	1.41	33.72	-33.92
0.48	1.00	0.68	2.32	2.08	1.86	32.34	-10.74
0.49	1.00	0.69	3.38	2.04	2.90	30.87	42.19