

HIGH SCHOOL TEACHERS' KNOWLEDGE FOR TEACHING ALGEBRA: A COMPARISON OF GHANA AND THE US.

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ABSTRACT: *Ghanaians complain about falling standards of their students in mathematics and point, for instance, to the poor performance of their eighth graders in the Trends in International Mathematics and Science Study (TIMSS) since 2003 when Ghana's eighth graders began participating. Since student performance has been known to be related to teacher knowledge, this study was set up to investigate the knowledge base of senior high school teachers in Ghana for teaching algebra and compare it to that of their counterparts in the US. In all 339 and 3,841 of teachers in Ghana and US respectively agreed to participate in the study. Analysis of the performance of the two group indicated that the US teachers performed significantly better than their Ghanaian counterparts. Implications of this finding have been discussed. In addition, recommendations for practice and further research have also been provided.*

KEYWORDS: Mathematics Education, Knowledge for teaching Algebra, Assessment of teacher knowledge.

INTRODUCTION AND BACKGROUND TO THE STUDY

In U.S. junior high and high schools, separate courses in algebra (e.g., Algebra I, Algebra II etc.) could be offered to students. However, in Ghana only one integrated mathematics course is offered at the junior high school level, JHS, (the equivalent of seventh to ninth grade) to all students. This mathematics course is a national curriculum, and is therefore, offered to all students in the public school system for the entire three years of the JHS education. The major content areas covered in the Ghana JHS mathematics curriculum comprise Number, Investigations with Numbers, Shape and Space "Geometry", Estimation and Measurement, Introduction to the Set Theory, Algebra, Collecting and Handling Data.

In addition to these content areas, problem solving which does not appear as a topic in itself is emphasized throughout the syllabus. Furthermore, these topics are not covered in succession. They have been broken down into smaller content pieces, called units (and sub-units) and have been sequenced in a spiral manner. The various units are arranged in such a manner that the topics taught in the early grades are not covered in complete detail but are returned to repeatedly throughout the years and developed further, with increasing detail and depth, as students progress through the grade levels. In *The Process of Education*, Bruner (1960) made a case for this type of sequencing when he said, "A curriculum as it develops should revisit the basic ideas repeatedly, building upon them until the student has grasped the full formal apparatus that goes with them" (p. 13).

At the time of this study, there were two types of mathematics programs offered in Ghana at the Senior High School (SHS) level (the equivalent of grades 10 to 12 in the US). These are Core Mathematics and Elective Mathematics. In Ghana's the public school system, every SHS student took Core Mathematics for the entire three years of SSS education. Elective Mathematics, on the other hand, was selected by students who require further mathematics content preparation beyond the core mathematics coverage. For instance, Elective Mathematics was an automatic elective course for students in the Science and Technical programs. Other students in the General Arts and Business programs could also select Elective Mathematics. Like the mathematics course at the JHS level, both of the mathematics courses in SHS were, at the time of this dissertation study, also integrated mathematics programs with their content sequenced in a spiral manner. In addition, like all other school subjects, the Ghana Education Service centrally controlled the syllabi for both Core Mathematics and Elective Mathematics. Being a national curriculum, the content of each of these mathematics courses is also the same for all public schools in Ghana.

The major content areas to be covered in Core Mathematics comprise the following: Number and Numeration, Algebra, Mensuration, Plane Geometry, Trigonometry, Statistics and Probability, Vectors and Transformation in a Plane, Investigations and Problem-solving, Use of Calculators and Computers. Investigations and problem solving together with the use of calculators are not topics by themselves in the syllabus but nearly all topics include activities involving them. Core Mathematics is offered for ten class periods of 40 minutes a week for the two terms of the first year. Thereafter, Core Mathematics is supposed to be offered six periods a week for the third term of the first year and the remaining two years of SSS. In contrast, the syllabus for Elective Mathematics contains the following content: Algebra, Logic, Coordinate Geometry, Trigonometry, Calculus, Linear Transformation, Vectors, Mechanics, Statistics, and Probability.

In spite of the two distinct approaches to offering algebra to all students in both Ghana and the U.S., there continues to be national outcry over the performance of students in mathematics. In both countries, among other issues raised is the underperformance of students in the Trends in International Mathematics and Science Study (TIMSS). While American mathematics education lament over students in Easter Asian and some European countries outperforming their counterparts in the US on the TIMSS assessment, mathematics educators in Ghana, have constantly refer to the same assessment and lament over Ghanaian eighth graders performing either as the worst or among the worst in the TIMSS assessment. For instance, in analyzing the terrible performance of Ghana's eighth graders in the 2003 TIMSS, Anamuah and Mereku (2005), drew attention to the fact that candidates performed poorest in Algebra and Geometry where they made correct responses of only 13.6, and 13.4 percent respectively to the released items. Anamuah and Mereku (2005) pointed to the nature of the Ghanaian mathematics curriculum and assessment system as the reasons for such abysmal performances of eighth graders in Ghana. standard deviations below the international average of 500 (the international average has been 500 and the standard deviation has been 100). For instance, in 2003 Ghana's eighth graders obtained average score of 274, placing last but one among the 45 participating countries. Though there was a slight improvement in 2007, their average score was still around two standard deviations below the international average of 500. Not only that, according to Mereku and Anumel (2011), the TIMSS results indicate that Ghana's eighth graders remain

among the lowest in Africa and the world. As they put it, “The poor performance is therefore largely a reflection of the nature of school Mathematics curriculum and assessment system that students have experienced in this country in the last three decades” (p.1).

In contrast, however, US eighth graders’ average scores of 504 and 529 on the 2005 and 2007 TIMSS for instance, departs remarkably from those of their Ghanaian counterparts and such has been the trend throughout the assessment periods which the two countries have both participated. However, as already mentioned, Americans have lamented, not because their students perform among the poorest in the world, but over the inability of their eighth graders to perform well compared to countries in several East Asian countries and some European nations in mathematics.

This study is premised on the fact that since literature is replete with the fact that teachers’ knowledge is related to students’ performance (see for instance, Begle, 1972, 1979; Harbison & Hanushek, 1992; Mullens, Murnane & Willett, 1996; Hill, Rowan & Ball, 2005), one factor to examine carefully to understand poor performance in mathematics is the knowledge base of teachers. In addition, since mathematics is a broad subject, the study was set up to focus on algebra because of the foundation it provides for other mathematics courses and its application to other fields (Usiskin, 1995; Senk and Thompson, 2003). In the light of the aforementioned and the fact that US students have consistently outperformed their counterparts in Ghana, this study was set up to compare the knowledge base of Ghana’s senior high School mathematics teachers for teaching algebra to their US counterparts. The purpose was to investigate whether any differences exist between teachers’ knowledge base in algebra in these two countries.

THEORETICAL FRAMEWORK

The study relied on from the Knowledge of Algebra for Teaching (KAT) project’s framework for conceptualizing teacher knowledge and adapted the instrument developed on this framework for the fieldwork. In Wilmot (2008 and 2013) this framework has been discussed. It is being reproduced here for the benefit of those who may not have had access to these two versions.

Through analyses of research literature, recommendations by professional organizations and videos of teaching, researchers in the KAT project have hypothesized that the knowledge used by teachers in teaching school algebra consists of three types. These are “knowledge of school algebra” (referred to in short as “school knowledge”), “advanced knowledge of mathematics” (also referred to as “advanced knowledge”), and “teaching knowledge”. These three types of knowledge, discussed below, constitute the theoretical frame of algebra knowledge for teaching that guided this study.

The KAT project defines “Knowledge of School Algebra” (or simply “School Knowledge”) as the knowledge of mathematics in the intended curriculum of middle school and high school. This is the content of school algebra that teachers are expected to help students discover or learn in their algebra classes. In the US, the big ideas of this type of knowledge are described in documents such as the National Council of Teachers of Mathematics (NCTM)’s Principles and Standards for School Mathematics (NCTM, 2000) while the specific grade-level algebra content

is described in the various states' standards, textbooks and other instructional materials used in the schools. In their work, researchers in the KAT project delimited this type of knowledge by reviewing content standards of ten different states in the US. At the senior high school level in Ghana, the content of this type of knowledge is included in both the Core and Elective Mathematics Syllabuses. This type of knowledge is considered important by the KAT project because unless teachers understand the grade-level algebra content they are to teach, they would find it difficult to influence student learning. Since students are expected to learn their school algebra, it sounds reasonable to hypothesize that for teachers to influence students learning, they (teachers) need to understand the content of school algebra themselves.

The second type of knowledge conceptualized by the KAT project is the advanced knowledge of mathematics or simply "Advanced Knowledge". According to the KAT project, this type of knowledge "includes other mathematical knowledge, in particular college level mathematics, which gives a teacher perspective on the trajectory and growth of mathematical ideas beyond school algebra" (Ferrini-Mundy, Senk, & McCrory, 2005, p.1). The KAT project lists areas such as calculus, linear algebra, number theory, abstract algebra, complex numbers and mathematical modeling as some of these general content areas included in this type of knowledge (see Ferrini-Mundy et al., 2005). In addition, in the conceptualization of advanced knowledge, members of the KAT project acknowledge that "knowing alternate definitions, extensions and generalizations of familiar theorems, and a wide variety of applications of high school mathematics are also characteristics of an advanced perspective of mathematics" (Ferrini-Mundy et al., 2005, p. 1). Thus, it can be argued that having an advanced perspective of mathematics affords teachers with a deep or profound understanding of school algebra. It is the possession of this type of knowledge that could make it possible for a teacher to make connections across topics while unpacking the complexity of a mathematics content to make that content more comprehensible. As already indicated, the KAT project considers "advanced knowledge" important because possessing it affords teachers with a deep or profound understanding of school algebra. In addition, it is hoped that teachers who possess this type of knowledge would have a good knowledge of the trajectory of the content of school mathematics. This knowledge in turn could help teachers to engage in bridging (making connections across topics), trimming (removing complexity while retaining integrity and decompressing (unpacking complexity to make content more comprehensible) of the content of school algebra to students; processes that could be vital to effective teaching.

The third category of knowledge in the KAT framework is the "Teaching Knowledge". In the KAT framework, this knowledge is described as "knowledge specific to teaching algebra that may not be taught in advanced mathematics courses. It includes such things as what makes a particular concept difficult to learn and what misconceptions lead to specific mathematical errors. It also includes knowledge needed to identify mathematical goals, within and across lessons, to choose among algebraic tasks or texts, to select what to emphasize with curricular trajectories in mind and to enact other tasks of teaching" (Ferrini-Mundy, McCrory, Senk & Marcus, 2005, p.2). Thus, this is the type of knowledge that teachers have and which they use in teaching the subject matter of school algebra. This point is made by the KAT researchers when they say that, "the knowledge referred to here may fall into the category of pedagogical content knowledge or it may be pure mathematical content applied to teaching" (Ferrini-Mundy et al.,

2005, p.1). In addition, since this type of knowledge may not be taught in advanced mathematics courses, it may not necessarily be available to mathematicians. Consequently, this is the knowledge that could differentiate an engineer or a mathematician from an algebra teacher.

The KAT project conceptualizes that these three hypothesized types of algebra knowledge for teaching are not hierarchical in nature. Neither do they exist in a continuum with well-definable boundaries. Rather, their boundaries are blurry in the sense that they are interwoven in many ways. A schematic diagram of this conceptualization is presented in Figure1 below.

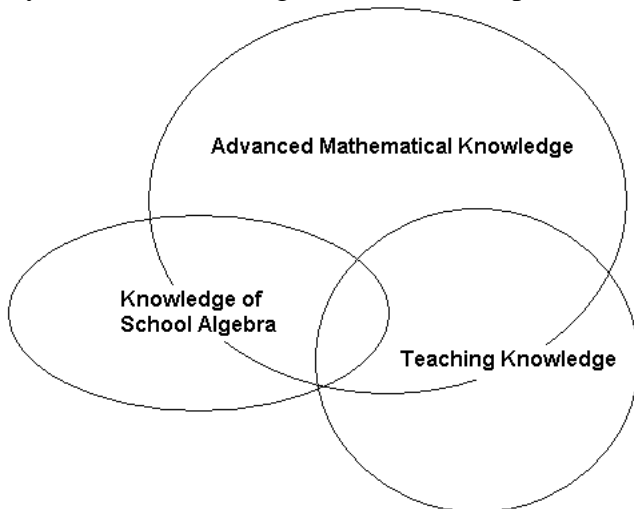


Figure 1. Conceptual Representation of the Three Types of Knowledge.

It was from this framework that the KAT framework developed and validated two versions of instruments, Form 1 and Form 2, which were adapted for this study. Each form comprised 20 items; 17 multiple choice and three open-ended items. The items were based in part on the content of high school algebra (i.e., school knowledge items), related advanced mathematics items, and items based on the tasks of teaching (i.e., teaching knowledge items). Form 1 comprised 6 school knowledge items, 7 advanced knowledge items and 7 teaching knowledge items. Form 2, on the other hand, comprised 7 school knowledge items, 6 advanced knowledge items and 7 teaching knowledge items.

PROCEDURE

As already mentioned, this study adapted both versions of the assessment instrument developed by the Knowledge of Algebra for Teaching (KAT) project at Michigan State University in the US. Through their validation study, the KAT project had administered the instrument to a number of prospective and in-service teachers in the US. Data used from the US, therefore, came, with permission, from a section of the KAT validation study data. With permission from the project's Principal Investigators permission was also granted for the adaptation and administration of the KAT instrument in Ghana. The adaptation involved changing the contexts and wording of questions in the KAT instrument to reflect Ghanaian contexts. For example an item that originally read,

“At a storewide sale, shirts cost \$8 each and pants cost \$12 each. If S is the number of shirts and P is the number of pants bought, which of the following is a meaning for the expression $8S + 12P$?”

was adapted into:

“At a storewide sale, shirts cost GH¢40 each and a pair of trousers cost GH¢60 each. If S is the number of shirts and P is the number of trousers bought, which of the following is a meaning for the expression $40S + 60P$?”

In this way, not only was the US currency changed into the Ghanaian currency, the prices of the items were also changed to reflect market values in Ghana at the time of the study. In addition, variations in names commonly used for the commodities used in the item were also changed to reflect the right contexts in Ghana. For example, in the adaptation made in the item used above, “pants” was changed into “trousers” as is commonly called in Ghana. In all, 3,449 participants from the US and 150 from Ghana completed Form 1 while Form 2 was completed by 392 and 189 participants respectively in the KAT and Ghana studies.

In scoring the responses to the items, multiple-choice items (items 1 to 17) on each of the two forms were given a score of 1 or 0 for right or wrong responses respectively. The open-ended items (items 18 to 20) were scored on a 4-point scale (i.e., scores from 0 to 4) but later rescaled to a 2-point scale (i.e., scores from 0 to 2) to avoid giving too much weight to the open-ended items in the analyses.

FINDINGS

To examine the extent to which participants from Ghana and the US performed on the items on each version of the instrument, Table 1 was used. In this table the difficulty levels of each of the items is presented as well as a ranking of the items in the order of increasing level of difficulty. As already mentioned, the item difficulty levels presented in Table 1 are the proportions of responses to the item that were correct. For instance, a difficulty level of .584 for an item means 58.4% of the participants in the study answered that item correctly. In this way, a difficulty level measure for an item is indicative of the item being more difficult than another item with a higher difficulty measure.

Table 1. Difficulty Levels of Items on Forms 1 and 2

Item ID	Form 1		Form 2	
	Ghana Data	US Data	Ghana Data	US Data
1	.880	.771	.989	.684
2	.653	.760	.979	.620
3	.933	.889	.328	.625
4	.147	.314	.296	.635
5	.347	.394	.169	.235
6	.247	.510	.640	.758
7	.487	.731	.222	.490
8	.060	.356	.275	.513
9	.340	.379	.730	.847
10	.427	.751	.682	.745
11	.753	.708	.217	.783
12	.200	.368	.164	.306
13	.130	.560	.130	.560
14	.201	.584	.201	.584
15	.457	.339	.457	.339
16	.298	.546	.298	.546
17	.913	.748	.381	.281
18**	.270	.482	.270	.482
19**	.362	.451	.314	.529
20**	.067	.377	.302	.317

** Open-ended items

A cursory look at this table reveals that on Form 1, with the exception of six items (i.e., items 1, 2, 3, 11, 15 and 17), the items were generally more difficult for the Ghanaian participants than their counterparts from US. A similar pattern was observed on the performance of the participants on Form 2 (where the exceptions were items 1, 2, 6, 10, 15, and 17). In other words, the only items that appeared slightly less difficult to the Ghana participants than the US participants were items 1, 2, 3, 11, 15 and 17 on Form 1 and items 1, 2, 6, 10, 15, and 17 on Form 2. It was thus concluded that, majority of the items (i.e., 14 out of twenty, on each of Form 1 and Form 2) were more difficult for participants in Ghana than their counterparts in the US. Of these, only item 17 on Form 2 was an advanced knowledge item. The rest were either school knowledge (items 1 and 6) or teaching knowledge items (items 2, 10, 15). On Form 1, three of these items (items 1, 3 and 17) had been classified by the KAT project members as school knowledge and teaching knowledge items respectively.

Next, to examine whether, using the differences observed on the individual items, a case could be made about whether any significant difference or not existed between the general

performances of the two samples on the two versions of instrument administered in the study *t*-test was run on the performance of the Ghana participants and their counterparts in the US. The test was run using the item difficulty coefficients discussed in Table 1. Using these difficulty level coefficients ensured that on each of the forms, there were equal number of scores for the Ghana and US participants. Consequently, paired-sample *t*-test performed to compare the difficulty level of items on Forms 1 and 2 to the two samples Table 2 presents the results of this test.

Table 2. T-Test of the Difference in Difficulty of the Two Forms

Form	Mean Difficulty		Variance		df	<i>t</i> -stat	p-values
	Ghana	US	Ghana	US			
1	.4086	.5509	.0778	.0325	19	-3.62	.0018**
2	.4022	.5450	.0681	.0316	19	-2.71	.0138**

** Significant difference in difficulty level

From Table 2, the mean difficulty level of the items on each of the two forms, Form 1 and Form 2, were lower for the sample from Ghana. The smaller variances from the US data also point to more participants clustering around the relatively higher US mean than the case of Ghana. These two issues indicate that the items were more difficult to the sample from Ghana than their US counterparts. Furthermore the p-values in Table 2 show that the difference in performance between the two samples on each of the two Forms was significant. In other words, items on each of the forms were significantly more difficult to the sample from Ghana than the sample from the US. Thus, in general, the US participants performed significantly better on both Forms 1 and 2 than their Ghanaian counterparts.

DISCUSSIONS

One point that stood out from the foregoing analyses was that in general, the US participants performed significantly better than the Ghanaian participants in this study on both Forms 1 and 2. Three possible explanations could be given for this difference in performance. These could generally be classified as coming from; 1) differences in curricular emphasis, 2) possible affordances of handheld technological devices which the US participants use and which their Ghanaian participants did not have, and 3) familiarity to the nature of type of questions on the KAT instruments.

Differences in Curricular Emphasis

As already mentioned, one possible explanation of the differences in performance of the US participants and their Ghanaian counterparts on items such as those cited above would be the differences in the curricula emphasis between US schools and those in Ghana. To illustrate this point, let us begin by taking a look at the issue of proof. What constitutes a proof varies from course to course. For instance, in Ghana, informal proofs are accepted in the core mathematics

course at the high school level. Therefore, a Ghanaian teacher who teaches only core mathematics may take a good informal proof as a valid proof especially when the proof is presented to him as a students' work. This could possibly explain why only about 13% of the Ghanaian participants answered item number 13 (one of the common items in Forms 1 and 2) correctly versus 56% of their US counterparts. In this question different proofs of a statement by three high school students and participants were asked to determine which of the constituted valid proofs. Since this was a multiple-choice item, Ghanaian core mathematics teachers who conceptualize the students being referred to as similar to those who could be in their classes were likely to selected the option that included the informal proof as the correct answer. In another item (i.e., item 19 of Form 2), a mathematical statement was given and participants were asked to determine if it was true and justify their answers. Compared with about 53% of the U.S. participants, only 31% of Ghanaians answered this questions right because many of the Ghanaian participants gave a number of correct examples as their justification and earned only 1 out of the maximum 4 points.

Another area where differences in curricula emphasis could have caused the difference in performance is the approaches projected in textbooks or the mathematics books available to teachers in the two countries. Presently in the US textbooks promote graphical approaches to dealing with functions more than textbooks in Ghana. For instance, in their precalculus book, Stewart, Redlin and Watson (2006) use graphs of standard functions such as $f(x) = x^2$, $g(x) = \frac{1}{x}$ and so on, and the idea of transformations to lead students into drawing graphs of functions such as $p(x) = (x + 4)^2$ and $h(x) = \frac{2}{x + 2}$. On the other hand, because of the fact that national examinations tend to emphasize analytical methods more than graphical approaches in dealing with functions, Ghanaian textbooks also tend to emphasize analytical methods more than such graphical solutions. Consequently, Ghanaian students and teachers would most likely graph the functions $p(x)$ and $h(x)$ by finding the intercepts, turning points, asymptotes and the behavior of the curves at the critical points. Such an approach, though effective, is not economical especially for "timed tests" such as the KAT instruments. The point being made is that with such graphical emphasis in most of the U.S. books, the KAT participants may have developed the ability to sketch graphs and use them to answer questions faster than their Ghanaian counterparts. That could explain why only about 20% of the Ghanaian participants answered item 14 of Form 2 correctly while 58% of U.S. participants got it right. In that question, an equation involving two distinct expressions under radical signs on each side of the equal sign was given and participants were asked to determine how many solutions the equation has. Using the graphs of the requisite standard functions and appropriate transformations, U. S. participants can save more time answering this question than their Ghanaian counterparts who could lose on other questions because of too much time spent on this one.

Possible Affordances of Handheld Technological devices

To explain the graphical calculator affordance it will be good to take a look at two multiple-choice questions on the KAT form that both groups of participants answered. One of the

questions gave participants a logarithmic function and asked them to determine which of three other logarithmic functions have the same graph as the original function. Only about 15% of the Ghanaian participants answered this question correctly compared with about 31% of the US participants (refer to Table 1).

For this question, using known properties of logarithmic functions, it is easy to see that the function that one of three options was the same as the given function. The question therefore reduces to making a decision about the other two options. However, by simply graphing these other options on the calculator together with original function one can decide between these two faster than graphing them from first principles without the calculator. In this case, any of the U.S. participants who had access to graphing calculators and had the opportunity to use them in completing the KAT forms were more likely to solve problems such as this faster than their Ghanaian counterparts (at the time of the study, graphing calculators were not permitted in Ghanaian high schools. As a result, not only were such devices not available for teachers to use, teachers did not develop the competence to use them. Teachers in Ghana were expected to lead students to graph functions from first principles).

Typical Ghanaian teachers and students who do not use the graphing calculator are more likely to solve this second problem analytically by squaring both sides and grouping like terms, a process that could be more time consuming than the graphical solution. This way, the Ghanaian participants could spend much more time on questions such as these to the detriment of the other questions.

In a synthesis of peer-reviewed, published research on the impact of graphing calculators on student performance, Burrill, Allison, Breaux, Kastberg, Leatham, and Sanchez (2003), concluded that, “overall, ... the use of handheld technology [in the form of graphing calculators] had a positive impact on student performance” (p. 38). Such positive impacts, according to Burrill et al. (2003), are possible when “calculator-friendly tasks [are used than when] parallel tasks that removed the calculator advantage were presented” (p. 41). In completing the instruments, the KAT project did not discourage the use of handheld technology in completing the instruments. As a result, any of the US teachers in the KAT sample who had their graphing calculators available and used them would perform better on the calculator-friendly tasks such as the two items described than the Ghanaian teachers who, not only, do not have access to them but are also not exposed to using them even for instruction.

Even for the US teachers who may not have had graphing calculators available when completing the research instruments, it is possible that their sense of how the graphs might look could be more fully developed than their Ghanaian counterparts because of their experience with graphing calculators and other types of software that enhance graphing abilities. Ghanaian participants in this study, on the other hand, had not been exposed to the use of graphing calculators either in school or in their teaching practice. The affordances of the graphing calculator technology could contribute to the lower performance of the Ghanaian sample. The point being made is that given two people of identical knowledge, one in the US sample and the other in the Ghana sample, the US participant who uses the graphing calculator could work faster and have a higher chance of going through the items than the Ghana participants within the 60 minutes allowed. In addition,

for some of the questions, drawing a quick graph on the calculator could improve one's chances of getting it right. Under these conditions, therefore, higher scores from the US participants should not come as a surprise.

Differences in the Level of Familiarity to Types of Questions

A third possible explanation, as already mentioned, lies in the differences in the level of familiarity with the types of questions on the instruments used in the study. After subjecting the items on the KAT instruments to review by two mathematics educators in Ghana, they both agreed that the content being measured by the KAT instrument is covered in the mathematics curriculum of Ghanaian high schools. However, the items developed here in the US are not necessarily the type of items the sample in Ghana are used to in their curriculum. Some of the Ghanaian in-service teachers made anecdotal remarks to this effect (i.e., their unfamiliarity with some of the items on the instruments) during informal discussions after the administration of the instruments in Ghana. Those remarks confirmed for me similar remarks made by a section of mathematicians and mathematics educators with whom I shared a table with during one of the MSU item development workshops about an item I had formulated. One of the items I had formulated for our group's discussion did not see the light of the day because, as I was told, "it was not the typical question teachers in the US were exposed to". Therefore, the possibility of teachers in the US being familiar with some of the items could improve their chances of performing better than the teachers in Ghana.

IMPLICATION TO RESEARCH AND PRACTICE

The findings of this study have a number of implications for research and practice. Three of such implications are discussed in this section.

First, one thing that needs to be taken into consideration is that while the US participants all taught algebra, their counterparts taught a mathematics curriculum that had algebra integrated with other domains of mathematics such as calculus, geometry, probability, statistics, vectors etc. Sherin (2002) has argued that in the course of teaching new curriculum, especially reform oriented curricula, teachers adapt their knowledge and in the process develop new content and pedagogical content knowledge. Applying this argument to the Ghanaian and US situations, one could argue that after teaching algebra alone for some time, US teachers are more likely to have developed a deeper knowledge in algebra as against Ghanaian teachers who are likely to have developed a more lateral knowledge of the mathematics curriculum. Due to the foundation algebra provides for other mathematics courses and its application to other fields, instead of an integrated type of mathematics in Ghanaian schools, it is recommended that every student in Ghana be given the opportunity to study algebra as a course. Senk and Thompson (2003) have argued that such "algebra for all" calls were consistent with the call for increased mathematics requirement for all U.S. schools, highlighted in the "A Nation at Risk" report and this is what could increase Ghanaian teachers' knowledge base and probably their students' performance. The argument has been that, without the opportunity to study school algebra, it would be almost

impossible to raise the mathematics performance of many high school students in Ghana. In addition, without studying algebra, some Ghanaian students would be denied access to certain careers. If possible, a pilot study into the effect of making a couple separate courses in those domains algebra core and geometry (the areas they lack most on the TIMSS assessment), for instance, in Ghanaian high schools on students performance before going to scale would be necessary.

Second, as explained in the last but one section, it is possible that their sense of how the graphs of some algebraic functions might look could be more fully developed among the US participants than their Ghanaian counterparts. This in turn could have aided the US participants to use more effective approaches to solve some of the questions on the instrument in a more time saving manner. It is therefore recommended that senior high schools in Ghana be allowed to use handheld technological devices such as the graphing calculator in teaching and learning and for examination purposes in order for teachers in Ghana and their students to benefit from the affordances of devices. Such affordances have been well documented and Ghana cannot afford to be left out. However, to ensure positive results among students' calculator use (Wilmot & Yarkwah, In print), it is necessary for steps to be taken to ensure that the dominant factors that affect Ghanaian students' calculator use are first taken care of.

Third, the fact that some Ghanaian participants commented on the unfamiliarity if the type of items on the instruments used in the study calls for the need to expose teachers and students at all levels of Ghana's educational system to different types of questions than they are used to. The need for more exposure to higher order questions such as those assessing at the application, analysis, synthesis, and evaluation levels of Bloom's taxonomy may be useful in Ghana. That way, students in Ghana can perform creditably on international assessments such as the TIMSS.

CONCLUSION

From the foregoing, it can be concluded that high school teachers in the US outperformed their counterparts in Ghana. This could be seen from the fact that on both versions of the instruments used in this study, the items were generally more difficult for the Ghanaian participants than their counterparts from US.

In addition to this general performance, data from this study also revealed that the US participants demonstrated a deeper or more advanced knowledge than their Ghanaian counterparts. In each case, out of the 20 items on the instruments participants from Ghana outperformed their counterparts in the US on only 6 items (items 1, 2, 3, 11, 15 and 17 on Form 1 and items 1, 2, 6, 10, 15, and 17 on Form 2). The fact that, of these, only one item (i.e., item 17 on Form 2) was measuring advanced knowledge is an indication that with reference to school algebra, the US participants demonstrated a deeper or more advanced knowledge than their Ghanaian counterparts. This was against the backdrop of the fact that on Form 1 there were 7 advanced items and on Form 2 there were 6 of them.

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