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ESTIMATION OF EXTREME VALUE AT RISK IN RWANDA EXCHANGE RATE

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ABSTRACT: *Estimating the probability of rare and extreme events is a crucial issue in the risk estimation of exchange rate returns. Extreme Value Theory (EVT) is a well-developed theory in the field of probability that studies the distribution of extreme realizations of a given distribution function, or of a stochastic process, satisfying certain assumptions. This work has fitted the Generalized Pareto Distribution (GPD) to the excess returns assuming the residuals are independent and identically distributed. The results are used to estimate extreme Value at Risk (VaR) in Rwanda exchange rate process.*

KEYWORDS: Exchange rate**,** EVT approach, Generalized Pareto Distribution, Value at Risk, Maximum Likelihood Estimation, Confidence intervals

INTRODUCTION

The last decades have been characterized by significant instabilities in financial markets worldwide which require for tools to assess the probabilities of rare financial extreme events. In different period financial markets have been affected by several crises such as; the stock market crash (1987), the financial crisis (1997-1998) and Global financial crisis (2007-2008). Estimating and managing foreign-exchange rate risk exposure is crucial for cutting down financial markets' exposure from major exchange rate fluctuations which could affect negatively the value of portfolios. This prompted the research for more appropriate methodologies able to deal with rare event that have big effects on financial markets. Thereafter, various models have been developed to measure and estimate extreme risk in financial markets. A common used risk measures are value at risk and expected shortfall. Value at risk summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence φ. In other words, Value at Risk answers the question about, how much one can lose over a certain time horizon with a given probability level. It also summarizes in a single number the overall market risk faced by a financial institution, according to Jorian (2007). Expected Shortfall (coherent risk measure) is an expected value of the loss, given that a VaR violation occurred. In other word, Expected shortfall estimates the potential size of the loss exceeding VaR at φ probability level, Freddy D. (2002).

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Manfred and Evis (2006) applied extreme value theory for measuring financial risk and came up with the typical question one would like to answer which is" when things go wrong, how wrong they go?". Extreme Value Theory approach is developed theory in the field of probability that studies the distribution of extreme realizations of a given distribution function, or of a stochastic process, satisfying certain assumptions.

Extreme Value Theory provides possibility to concentrate on each one of the two tails of the distribution independently, thus allowing a flexible approach which can take skewness of the underlying distribution into account. It is primarily focused on avoiding big unexpected losses and sudden crashes rather than long sequences of medium-sized losses (this is mainly a consequence of the market crisis experienced in the recent past and the empirical observation that the final position of a portfolio is more affected by a few extreme movements in the market rather than by the sum of many small movements).

There are two main approaches when identifying extremes in real data. The first approach is *Block Maxima* which considers the maximum (minimum) values that returns take over successive periods of same length (blocks). The selected values are considered extreme events that constitute block maxima (minima) method and this approach is assumed to follow a Generalized Extreme Value distribution. The second approach is known as *Peaks over Threshold* (POT) method which focuses on the realizations exceeding a given (high) Threshold and is considered to follow a Generalized Pareto Distribution (GPD). The threshold method uses data more efficiently and, for that reason, it seems to become the method of choice in recent applications. For financial time series, *the peak over threshold* (POT) method is employed to modeling extreme events.

The EVT approach is a very significant theorem since it explains the limiting distribution of sample extrema. This guides us to estimate the asymptotic distribution of extreme without making any assumptions about an unknown parent distribution. In this work we fitted Generalized Pareto Distribution (GPD) approach proposed by EVT to the excesses returns over the threshold to estimate extreme VaR assuming that residuals are independent and identically distributed.

LITERATURE REVIEW

Morgan (1990) started the Value at Risk method as a project in order to assist practitioners to summarize their risk in a single number. He showed that typical Value at Risk approach would add a figure on a probability of losing on more than a certain amount of money over a time horizon. Value at Risk may be estimated using various methods depending on the assumptions made and methodologies used.Extreme Value Theory is a well developed theory in the field of probability that studies the distribution of extreme realizations of a given distribution function, or of a stochastic process, satisfying suitable assumptions. The foundations of the theory were set by Fisher and Tippett (1928) and Gnedenko (1943), who proved that the distribution of the extreme values of an independent and identically distributed (iid) sample, can converge to one out of only three possible distributions (Frechet, Weibull or Gumbel), Acerbi at el (2001). Many researchers have oriented their work towards more efficient tail-oriented models of risk, namely EVT

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approach. The superiority of EVT has been extensively demonstrated by many researchers, in fields like insurance or financial risk management.

Embrechts et al (1997) applied EVT approach to assess fat tails of different time series, like hydrologic, insurance and financial data, supported by a very detailed and complex mathematical framework. Similar work is found in Resnick (2007), who studies extreme events in data networks, finance and insurance. Ana-Maria and Moisa (2009) applied exchange rate returns of four currencies against the Euro to analyze the relative performance of several VaR models and Extreme Value Theory. They revealed that in extreme market conditions, extreme measures are needed. McNeil and Frey (2000) suggested EVT to estimate the tail related risk measures for heteroscedastic financial time series and they compared this approach to the various other methods for extreme value at risk estimation for financial time series data. They found that the EVT outperform other models for extreme value at risk estimation.

A considerable amount of research has also been dedicated to more specific issues of EVT, for example, tail index and graphical tools of the framework, like mean excess function plot, Hill plot, QQ plots etc. Tail index estimation is yet a very widely debated problem of EVT. Starting with the work of Hill (1975) and Pickands (1975), many studies have tried to establish a measure of the tail thickness of fat-tailed distributions. Dekkers et al. (1989) improved the Hill estimator and prove consistency as well as asymptotic normality.

MATERIALS AND METHODS

Our aim is to estimate the extreme value at risk in Rwanda exchange rate returns for the following pair currencies: Rwanda francs (Frw) against Kenya Shillings (Ksh), US dollars (USD), Euro (EUR) and Sterling Pound (GBP) respectively. The choice of these currencies was based on their relative proportions, in the Bank's foreign exchange investment portfolio and also on their currency composition of imports. The data used was the 2758 daily observations of average of buying and selling of the Rwanda Exchange Rate covering the period from January 1st 2002 to December 31st 2012. The daily data were obtained from National Bank of Rwanda.

In our previously study, we have estimated risk in Rwanda exchange rates with assumption that the innovations of the process follow normal distribution and the results revealed that neither the exchange rate returns series nor the residuals series can be considered to be normally distributed since both the series have a leptokurtic distributions. These imply that even if we have assumed that innovations are iid standard normal we still get returns have kurtosis great than 3. Therefore, we came up with the evidence that the assumption of conditional normality is not realistic for real data, Ntawihebasenga et al. (2014).

We model the Exchange rate returns as Autoregressive model with conditionally heteroskedastic financial time series as follows:

$$r_t = \mu(y_t, z_t) + \sigma(y_t, z_t)e_t, t = 1,2, \dots, T \quad (1)$$

$y_t = (r_{t-1}, \dots, r_{t-\pi})$,where y_t represents endogenous variables in the model, z_t represents explanatory variables consisting of information other than the past of the returns. μ represents

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conditional expected return which may be arbitrary, σ represents conditional volatility of daily exchange rate returns, π represents the order of Autoregressive, e_t represents standardized returns. The randomness in the model comes through the random variables e_t, which are referred to as noise variables or the innovations of the process and assumed to be independent and identically distributed with unknown distribution function $F(e)$. Conventionally, the distribution of e_t, is assumed to have mean zero and unit variance, $e_t \sim iidN(0,1)$, so that σ_t is directly interpreted as volatility of exchange rate return r_t.

We begin this stage of estimation of extreme value at risk by plotting standardized residuals

$$\hat{e}_t = {}^{r_t}/_{\hat{\sigma}_t}, t = 1,2, \dots, T \qquad (2)$$

where r_t is returns series, $\hat{\sigma}_t$ is estimated volatility in the returns and T is a total number of observations. The ACF and PACF for residuals were plotted in Figures 2 and 3 below to show that standardized residuals are free from autocorrelation.

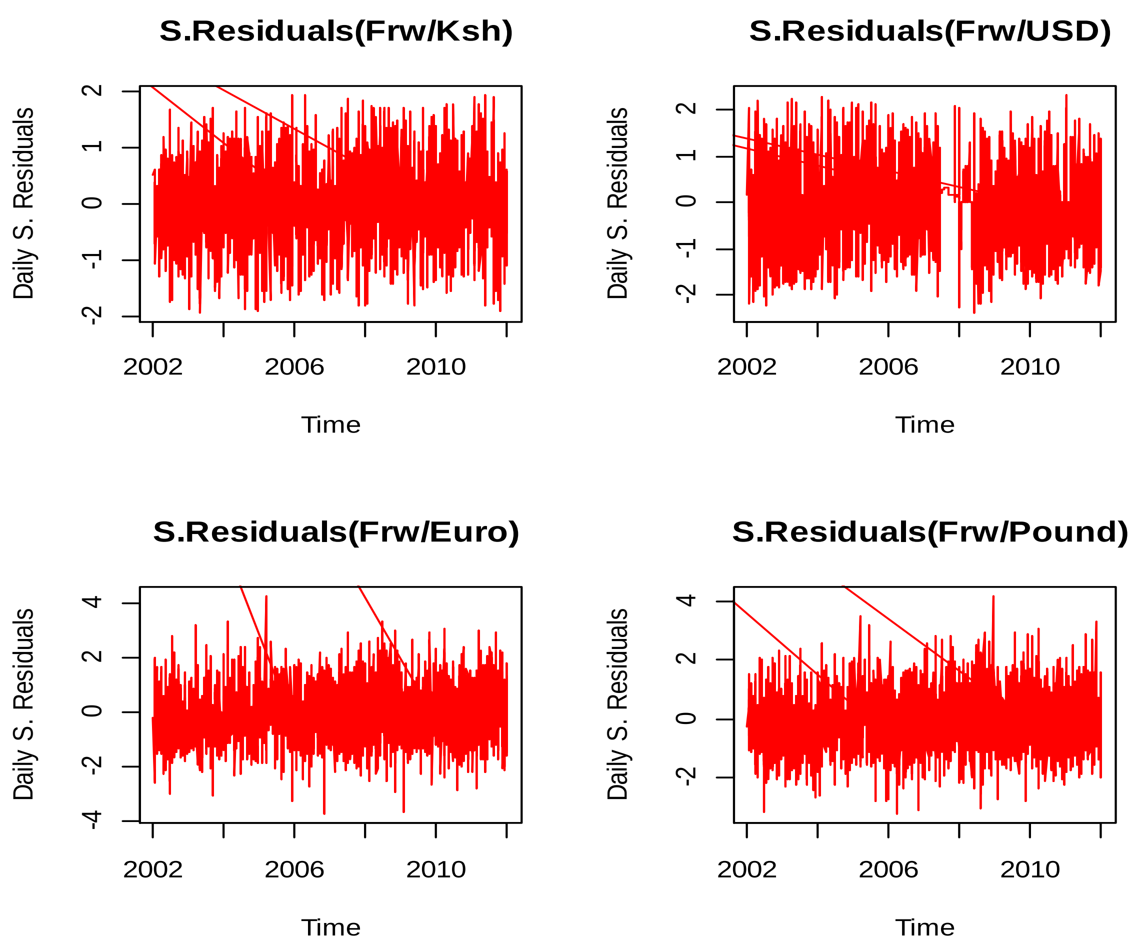

Figure 1: Daily standardized residuals

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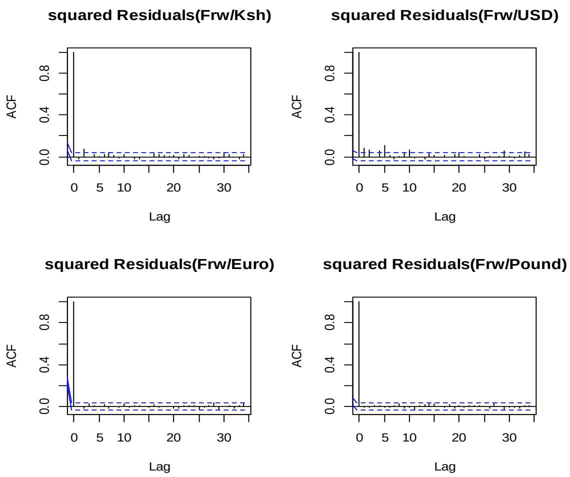

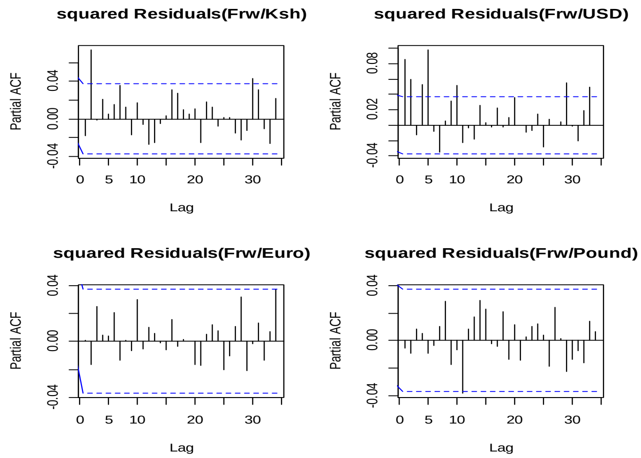

Figure 2: ACF of squared residuals **Figure 3: PACF of squared Residuals**

The plots in Figures 2 and 3 reveal that the squared residuals exhibit no autocorrelation in all lags for Frw/Euro as well as Frw/Pound but exhibit low autocorrelation in Frw/Ksh for only first lag and for Frw/USD up to 4th lag.

The results presented in Table1 show that the squared residuals is not normally distributed since Jarque Bera test with 2 degree of freedom rejects null hypothesis of normal distribution hypothesis to indicate that the squared residuals are not normal. Lagrange Multiplier test fail to reject null hypothesis of no ARCH effect in squared standardized residuals of Frw versus Kshs, Euros and Pounds respectively. This implies that there are no ARCH effects since the volatility was removed. Augmented Dickey Fuller test for stationarity reveals that squared residuals for all currencies are stationary in mean. The summary statistics of squared residuals are presented in Table1below.

Table 1: Summary statistics of squared standardized residuals

Tests	Statistics	Frw/Ksh	Frw/USD	Frw/Euro	Frw/Pound
JB	$\chi^2(2)$	7428.669	5415.314	51183.32	40820.44
	$p-value$	< 2.2e-16	<2.2e-16	< 2.2e-16	<2.2e-16
LM	$\chi^2(12)$	16.8744	31.5687	2.2033	3.8016
	$p-value$	0.1544	0.001612	0.999	0.9868
ADF	$\chi^2(14)$	-13.6569	-12.2571	-13.1807	-12.9404
	$p-value$	0.01	0.01	0.01	0.01

Threshold selection techniques

The threshold level selection is not a simple task since for a low threshold value estimator would be biased. By choosing a low threshold the risk is to introduce some central observations in the series of extremes. The tail index (shape) is in this case more accurate (less variance but biased). For setting a too high threshold would lead to a reduction of the number of extreme observations and hence to overestimation of the variance. The high threshold implies a less biased but less robust tail index. Here the major problem is to find the optimal threshold for Generalized Pareto Distribution.

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The QQ-plots known as Quantile-Quantile plots of residuals against normal distribution are needed for two reasons, first, to support an assumption of conditional normal that is not realistic and reveals that squared residuals are fat tailed. This is the reason why we have applied EVT to estimate the innovations. The second reason is that QQ-plot is also useful to check if the data points satisfy the generalized Pareto Distribution. According to Picklands (1975), Balkema and De Haan (1974) theorem said that if the empirical plots seem to follow a reasonably straight line with a positive gradient above a certain threshold, therefore these indicate that the data follow a Generalized Pareto Distribution (GPD). It is possible to choose the threshold where an approximation by the GPD is reasonable by detecting an area with a linear shape on the plot.

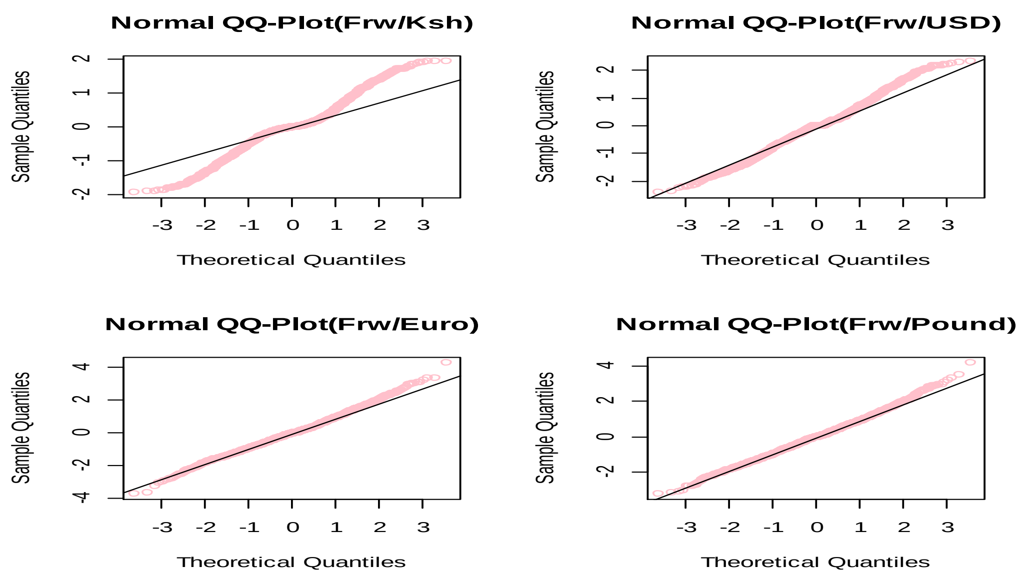

Figure 4: Quantile-Quantile plots of residuals against the normal distribution

As it can be seen in plots from Figure4, Q-Q plots of residuals against normal distribution confirm that the innovations process for all currencies have a fat tail since each plot curve down the left and up the right (concave curve). Hence the assumption of conditional normality is unrealistic.

Estimated parameters of Generalized Pareto Distribution

After identifying the best threshold for each currency, we use the observations in excess of the thresholds to determine the GPD parameters which are ζ and ψ shape and scale parameters respectively. Mathematically we can define GPD as follow;

Let x_t be $i.i.d$ random variables and $v_1, v_2, \dots, v_\tau$ be a series of exceedances over threshold τ. The generalized Pareto distribution function $G_{\zeta,\psi_\tau}(v)$ can be defined as follows:

$$G_{\zeta,\psi_\tau}(v) = \begin{cases} 1-(1+\ \zeta v/\psi_\tau)^{-1/\zeta}, if\ \zeta \neq 0 \\ 1-\exp\left(-\frac{v}{\psi_\tau}\right), if\ \zeta = 0 \end{cases} \tag{3}$$

With $x_t - \tau \geq v \geq 0$ and ζ the shape parameter is independent of τ and is the same as for GEV distribution and ψ the scale parameter, $\psi_\tau = \psi(\tau) > 0$. The distribution is said to be generalized because it contains other distributions under common parametric form.

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RESULTS AND DISCUSSIONS

We estimated the parameters using Maximum likelihood estimation which is a general method for estimating the parameters of an econometric model. The Maximum likelihood procedure used has parametric estimators that are consistent and asymptotically normal. The statistic results of the estimated parameters of GPD are presented in Table2 below;

Table 2: Summary on the use of Generalized Pareto Distribution

Frw/	**Threshold(τ)**	$\hat{\zeta}$	$\hat{\psi}$	**LLV**
Ksh	0.769	0.297208 (0.06817)	0.622267 (0.05524)	32
USD	0.896	0.124434 (0.06858)	0.691002 (0.06255)	10
Euro	1.224	0.016719 (0.05636)	0.583135 (0.04823)	31
Pound	1.170	0.046124 (0.06151)	0.579125 (0.04964)	20

$\hat{\zeta}$ represents the estimated shape parameter which determines the type of the distribution, since it is positive for all currencies. These indicate that the distributions of selected currencies belong to maximum domain of attraction of Frechet distribution which is heavy tailed. The values into brackets are the shape and scale parameters of standard deviation. $\hat{\psi}$ represents the estimated scale parameter of underlying distribution.

Extreme Value at Risk Estimates

Given risk horizon and confidence level $\varphi \geq 0.95$, therefore we can obtain the unconditional Value at Risk estimate which is equal to the quantile at confidence interval $\varphi \geq 0.95$. Using the shape and scale parameters estimates obtained above, we can obtain the Value at Risk at extreme for independent and identically distributed standardized residuals. Let $\widehat{VaR}_{\varphi}$ be the VaR estimate of innovations at φ probability level. Typically, the probability φ is such that $0.95 \leq \varphi < 1$. Mathematically the extreme value at risk estimate $\widehat{VaR}_{\varphi}$ can be defined as follows.

$$\widehat{VaR}_{\varphi} = \tau - \hat{\psi}\ln(1-\varphi) \quad (4)$$

In this work we choose only three values for $\varphi = 0.95, 0.99$ and 0.995 the results are presented in Table3 below

Table 3: summary statistics of extreme quantiles estimates at different probability level

Frw/	$\widehat{VaR}_{0.95}$	$\widehat{VaR}_{0.99}$	$\widehat{VaR}_{0.995}$
Ksh	1.659356	2.137693	2.343702
USD	2.966057	4.078182	4.557148
Euro	2.970916	3.909436	4.313634
Pound	2.904903	3.836969	4.238388

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SIGNIFICANCE TO THE RESEARCH AND PRACTICE

The estimation of extreme value at risk in Rwanda exchange rate will assist risk managers, portfolio managers, traders, investors and market makers in different manners. For instance, it can help the risk practitioners to monitor the exposure of market risk, therefore, protecting their institution from collapsing. They can also be used to ensure that financial institutions can still be in business after a catastrophic event. In general, results of this work will contribute a lot to understanding of how changes in exchange rate affect the prices of goods and services in international trade. The Rwandans academicians and researchers in the same or related fields of studies will provide knowledge and basis for their further studies.

CONCLUSIONS AND RECOMMENDATIONS

This study has estimated extreme value at risk in Rwanda exchange rate series using extreme value theory. The data used were daily exchange rate series for the period from January 2002 to December 2012 provided by National Bank of Rwanda. GPD proposed by EVT was fitted to the standardized residuals to estimate extreme quantiles (VaR) at probability levels $0.95 \leq \varphi < 1$. MLE used has estimators which are consistent and asymptotically normal. Estimating the uncertainty of VaR is significant since it allows policy makers risk managers to make good decisions about direction of portfolio. When we compare EVT approach to the GARCH model it is clear that GARCH model with normal distribution assumption provides a good estimates as well as the EVT with independent and identically distribution residuals assumption. However, both tend to be violated more often because they do not take into account the leptokurtosis of the residuals. We recommend for further study to combine these two models in order to perform both the centre and the tails of the Rwanda exchange rate process.

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