ESTIMATING NON-LINEAR REGRESSION PARAMETERS USING DENOISED VARIABLES

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ABSTRACT: *The observed data from various fields are frequently characterized by measurement error and this has been challenging problem to construct consistent estimators of the parameters in a nonlinear regression model.This study uses simulated data under three (3) sample sizes (i.e 32,256 and 1024) applying Kernel, Wavelet and Polynomial Spline on noisy data in two approaches (i.e denoising only the explanatory variables and denoising both dependent and explanatory variables). The study reveals the performance of denoised nonlinear estimators under different sample sizes for each denoising approach and comparison was made using the mean squared error criterion. The result of the studies shows that the denoised nonlinear least squares estimator (DNLS) is the best under each sample size considered.*

KEYWORDS: Production model, Denoising, Smoothers, Measurement Error, Monte-Carlo Simulation, Non-linear regression

INTRODUCTION

The Statistical estimation can be regarded as a subfield of statistics, and lies at the core of a number of areas of science and engineering, including data mining, and signal processing. Each of these disciplines provides some information on how to model data and how best to exploit the hidden structure of the model of interest. In this work, we are interested in estimating nonlinear regression model (Nonlinear Cobb- Douglas production model).

In nonlinear regression, observational data are modeled by a function which contains parameters that are not linear in nature. The data consist of independent variables (explanatory variables) and their associated observed dependent variables (response variables) which may contain measurement error or noise.

Variables are said to be noisy if they are not exactly equal to the variable of interest because the generating system of the measurement may not be perfectly measured. In statistics, an error is not a mistake because variability is an inherent part of things being measured and of the measurement process. Error-in-variables (EIV) model are regression models that account for measurement errors in independent variables. Many economic data sets are contaminated by the mismeasured variables. The problem of measurement errors is one of the most fundamental problems in empirical economics. The presence of measurement errors causes biased and inconsistent parameter estimates and leads to erroneous conclusions to various degrees in economic analysis. A measurement error is called classical if it is independent of the latent true values; otherwise it is called non-classical. There have been many studies on the identification and estimation of linear, nonlinear, and even non parametric model with classical measurement errors, see, Cheng and Van Ness (1999) and Carroll, et al. (2006) for detail reviews.

A natural approach to overcome this problem is to apply the smoothing techniques to handle the data for proper removal of the noisy observation (i.e denoise the data). In statistics and image processing, to smoothen a data set is to create an approximating function that attempt to capture important patterns in the data, while leaving out noise or other fine scale structure or rapid phenomena. Smoothing extract more information from the data as long as the assumption of smoothing is reasonable and provides flexible and robust analysis. There are several methods of smoothing techniques which can be used to screen out noise, such as: wavelets, developed by Donoho and Johnstone, (1994, 1995a and 1995b). Other methods are kernel, polynomial spline etc. These appear often in applied fields such as marketing (Blattberg and Neslin (1990)), Medicine and Biology (Aldroubi and Unser (1996)), and Image Processing (Prasad and Lyengar (1997)).

There have been many studies on denoising. So far, denoising has been extended to least squares estimator, least absolute deviation estimator and M-estimator using kernel, wavelet and polynomial spline as smoothers. The study carried out by Cai et al. (2000) denoised both the dependent and explanatory variables, while Cui et al. (2002) suggested denoising only the explanatory variables. Furthermore, a series of papers (You and Zhou, 2007; You et al., 2009; Zhou and Liang, 2009) adopted the approach of only denoising explanatory variables. Cui et al. (2011) denoised only the explanatory variables and showed that the denoised nonlinear least squares estimator is not robust to outliers. The study carried out by Fasoranbaku and Soyonbo (in press) showed that the denoised nonlinear least square estimator under the several smoothers (Epanechnikov, Gaussian, wavelet and polynomial spline) considered outperforms both the denoised nonlinear least absolute deviation estimator and nonlinear M-estimator. Soyombo and Fasoranbaku (2015) also used the known Epanechnikov Kernel smoother,to perform the denoising procedures, carry out simulation studies under some settings to determine the performance of the denoised non-linear estimators when the parameter values are varied. The results show that the DNLS outperforms both the DNLAD and DNM. Therefore, parameters of non-linear model are not sensitive and thus have no effect on the performance of denoised non-linear estimators.

This study estimating non-linear regression parameters using denoised data from investigating well known Cobb Douglas Production model in economics. The model with additive error is written as

$$P_t = \beta_1 L_t^{\beta_2} K_t^{\beta_3} + u_t \quad \text{.......................................} \quad (1)$$

(β_1>0), (0< β_2<1), (0< β_3 <1)

where P_t is output at time t, L_t is the labour input, K_t is the capital input , β_1 is a constant(β_1>0),

β_2 and β_3 are the output elasticity of labour (0< β_2<1), and capital (0< β_3 <1) and u_t is the stochastic disturbance term.

Suppose that $\{(L_t, K_t, P_t): 1 \le t \le n\}$ are unobservable "true" variables satisfying a nonlinear relationship, measurements of (L_t, K_t, P_t) are collected to yield an observable data set of $\{(x_{t1}, x_{t2}, y_t): 1 \le t \le n\}$ i.e. the true variables plus additive measurement errors such that

$$x_{t1} = L_t + \delta_t, \quad x_{t2} = K_t + \varepsilon_t \text{ and } y_t = P_t + u_t \quad \text{………....……………} \quad (2)$$

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where δ_t and ε_t are measurement errors. To be in line with the usual nonlinear model, the model (3.1) becomes:

$$y_t = \beta_1 x_{t1}{}^{\beta_2} x_{t2}{}^{\beta_3} + u_t \quad \ldots\ldots (3)$$

The study apply four (4) different smoothers (i.e Epanechnikov Kernel, Guassian Kernel, Wavelet and Polynomial Spline) to first denoise only the explanatory variables and later denoise both the dependent and explanatory variables. The regression to the denoised data is fitted and then applied to the estimators one after the other to provide information on the performance of denoised nonlinear estimators under three (3) different sample sizes.

Denoising procedure

The basic idea behind smoothing a data set is the creation of an approximating function that attempts to capture important patterns in the data while leaving out the noise, and is also referred to as "denoising". There are various methods to help restore a data set from measurement noise. In this study, the following smoothing method are used

1) Kernel denoising: Given a random sample $X_1 \ldots X_n$ with a continuous, univariate density function $f(.)$,The kernel density estimator is:

$$\hat{f}(x,h) = \frac{1}{nh}\sum_{i=1}^{n} k\left(\frac{x - X_i}{h}\right) \quad \ldots\ldots (4)$$

where x is the value of the scalar variable for which one seeks an estimate while X_i are the values of that variable in the data. K is a function of a single variable called the *kernel*. The kernel determines the *shape* of the function. The parameter *h* is called the *bandwidth* or *smoothing constant*. It controls the degree of smoothing and adjusts the size and form of the function.

$$u = \left(\frac{x - X_i}{h}\right) \quad \ldots\ldots (5)$$

For the purpose of this study, the two most commonly used Kernels are utilized:

a) Epanechnikov Kernel denoising:

$$K(u) = 0.75(1 - u^2) I_{(|x| \le 1)} \; on \quad u \in (-1,1) \quad \ldots\ldots (6)$$

b) Gaussian Kernel denoising:

$$k(u) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) \quad \ldots\ldots (7)$$

2) Wavelet denoising: they are generated from dilations and translations of a "father" wavelet ϕ

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$$\varphi_{j_0,k}(x) = 2^{\frac{j_0}{2}}\phi(2^{j_0}x - k); k = 0,1,...,2^{j_0} - 1 \quad (8)$$

and a “mother” wavelet ψ .

$$\psi_{j,k}(x) = 2^{\frac{j}{2}}\psi(2^{j}x - k);\ j = j_0,..., j; k = 0,1,...,2^{j} - 1 \quad (9)$$

3) Polynomial spline denoising: A smoothing spline is a method of smoothing (fitting a smooth curve to a set of noisy observations) using a spline function which minimizes:

$$p(y,x) = \sum [y_i - \hat{u}(x_i)]^2 \lambda \int_{x_1}^{x_n} \hat{u}''(x)^2 dx \quad (10)$$

where λ is positive smoothing parameter which controls the amount of smoothing of the data, it is defined between 0 and 1. $\lambda = 0$ Produces least squares straight line fit to the data, while $\lambda = 1$ produces a piecewise cubic polynomial fit that passes through the data points.

Nonlinear Regression Solved by Successive Linear Approximation Using Newton Raphson Method.

$$g(\beta) \approx g(\beta^t) + G(\beta^t)(\beta - \beta^t) + \frac{1}{2}(\beta - \beta^t)' H(\beta^t)(\beta - \beta^t)$$

Where, $G(\beta^t) = \left[\frac{\partial g}{\partial \beta_i}\right]_{\beta^t}$ is the score vector and ……(11)

$H(\beta^t) = \left[\frac{\partial^2 g}{\partial \beta_i \partial \beta_k}\right]_{\beta^t}$ is the Hessian matrix. ……(12)

This Hessian matrix is positive definite, the maximum of the approximation $g(\beta)$ occurs when its derivative is zero

$$G(\beta^t) + H(\beta^t)(\beta - \beta^t) = 0 \quad (13)$$

$$\beta = \beta^t - [H(\beta^t)]^{-1} G(\beta^t) \quad (14)$$

This gives a way to compute β^{t+1}, the next value in iterations which is

$$\beta^{t+1} = \beta^t - [H(\beta^t)]^{-1} G(\beta^t) \quad (15)$$

The iteration procedures continue until convergence is achieved. Near the maximum the rate of convergence is quadratic as defined by $\left|\beta^{t+1} - \hat{\beta}_t\right| \le c\left|\beta^t - \hat{\beta}_t\right|^2$ for some $c \ge 0$ when $\beta_i^{\,t}$ is near $\hat{\beta}_t$ for all i. Thus we get estimates $\beta_i^{\,t}$ by Newton Raphson methods.

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Let us consider (1), a nonlinear production model:

Let $f(L_t, K_t, \beta_1, \beta_2, \beta_3)$ represents the function, then the nonlinear production model becomes:

$$P_t = f(L_t, K_t, \beta_1, \beta_2, \beta_3) \qquad (19)$$

Where we know the form of the equation, we have observed $P_t, L_t,\ K_t$ and we must estimate $\beta_1, \beta_2, \beta_3$.

For brevity, henceforth we suppress L_t and K_t in our notation, but we retain $(\beta_1, \beta_2, \beta_3)$ so that we may write (3.9) more briefly as

$$P_t = f(\beta_1, \beta_2, \beta_3) + u_t \qquad (20)$$

To estimate the parameters in (20) for nonlinear model we use the score vector and the Hessian matrix from (11) and (12)

$$\text{Let } \sum_{t=1}^{n} u^2 = [p_t - f(\beta_1, \beta_2, \beta_3)]^2 = S(\beta) \qquad (21)$$

$$G(\beta\ ) = \left[\frac{\partial S(\beta)}{\partial \beta_1}, \frac{\partial S(\beta)}{\partial \beta_2}, \frac{\partial S(\beta)}{\partial \beta_3}\right]'$$

$$H(\beta\ ) = \begin{bmatrix} \frac{\partial^2 S(\beta)}{\partial \beta^2{}_1}, \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_2}, \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_3} \\ \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_2}, \frac{\partial^2 S(\beta)}{\partial \beta_2{}^2}, \frac{\partial^2 S(\beta)}{\partial \beta_2 \partial \beta_3} \\ \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_3}, \frac{\partial^2 S(\beta)}{\partial \beta_2 \partial \beta_3}, \frac{\partial^2 S(\beta)}{\partial \beta_3{}^2} \end{bmatrix}$$

From the linearization result in equation (14) we can obtain estimate of $\beta_1, \beta_2, \beta_3$ as follow:

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} \beta_1^0 \\ \beta_2^0 \\ \beta_3^0 \end{bmatrix} - \begin{bmatrix} \frac{\partial^2 S(\beta)}{\partial \beta^2{}_1}, \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_2}, \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_3} \\ \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_2}, \frac{\partial^2 S(\beta)}{\partial \beta_2{}^2}, \frac{\partial^2 S(\beta)}{\partial \beta_2 \partial \beta_3} \\ \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_3}, \frac{\partial^2 S(\beta)}{\partial \beta_2 \partial \beta_3}, \frac{\partial^2 S(\beta)}{\partial \beta_3{}^2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial S(\beta)}{\partial \beta_1} \\ \frac{\partial S(\beta)}{\partial \beta_2} \\ \frac{\partial S(\beta)}{\partial \beta_3} \end{bmatrix}$$

Once a parameter vector is obtained, the estimates are likely better than the old trial estimates, and so can be used in place of $(\beta_1^0, \beta_2^0, \beta_3^0)$ and the computation can be done

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again. The iteration can continue, obtaining new and better estimates until the difference between successive parameter vectors is small enough to assume convergence.

Denoised Non-Linear Regression Estimators.

When the regressors in a non-linear regression model are subject to measurement errors, it becomes a problem to construct consistent estimators of the parameters. It is possible, however, to construct consistent estimators in a non-linear model like (1) by first applying the denoising techniques discussed ealier to the variables, then estimators like the least squares, least absolute deviation and M-estimator will be applied to these denoised variables to yield consistent estimators which are called

i. Denoised nonlinear least squares (DNLS) of $(\beta_1, \beta_2, \beta_3)$ minimizes

$$D_n = \sum_{t=1}^{n} [P_t - f(\hat{L}_t, \hat{K}_t, \beta_i)]^2 \quad i = 1,2,3 \ldots\ldots\ldots\ldots \text{(22)}$$

ii. Denoised nonlinear least absolute deviation(DNLAD) of $(\beta_1, \beta_2, \beta_3)$ minimizes

$$L_n = \arg\min_{\beta_i} \sum_{t=1}^{n} \left| P_t - f(\hat{L}_t, \hat{K}_t, \beta_i) \right| \ldots\ldots\ldots\ldots \text{(23)}$$

where β_i is the solution of the parameters and

iii. Denoised Mestimators $M_n = \arg\min_{\beta_i} \sum_{t=1}^{n} \rho\left[P_t - f(\hat{L}_t, \hat{K}_t, \beta_i)\right]$ ....…….….….(24)

Where ρ is a loss function. The function ρ can be chosen in such a way to provide desirable properties of estimators (in terms of bias and efficiency) when the data are truly from the assumed distribution. Least-squares estimators are special M-estimators with $\rho(x) = x^2$, where $x = \left[P_t - f(\hat{L}_t, \hat{K}_t, \beta_i)\right]$

Simulation Studies

A Monte Carlo simulation is a problem solving techniques used to approximate the probability of certain outcomes by running multiple trials, using random variables.

In this work, an extensive Monte Carlo simulations is conducted to generate random data of sample sizes 32, 256 and 1024 to examine the performance of the denoised nonlinear estimators from the model

$$y_t = P_t + u_t \quad and \quad x_{t1} = L_t + \delta_t, \quad x_{t2} = K_t + \varepsilon_t \quad \ldots\ldots\ldots\ldots \text{(25)}$$

where $\hat{L}_t \sim U(1,30)$, $\hat{K}_t \sim U(10,200)$, $u_t \sim N(0,0.25)$, $\delta_t \sim N(0,0.16)$, $\varepsilon_t \sim N(0,0.16)$, $y_t = \beta_1 L_t^{\beta_2} K_t^{\beta_3} + u_t$ with standard parameter values $(\beta_1 = 1.01,\ \beta_2 = 0.75,\ \beta_3 = 0{,}25)$, which derived from the theory of production by Charles Cobb and Paul Douglass with the following assumption: $\beta_1 > 0,\ \ 0 < \beta_2 < 1,\ \ 0 < \beta_3 < 1$.

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Four (4) different smoothers (i.e Epanechnikov Kernel, Gaussian Kernel, Wavelet and Polynomial Spline) are used to denoise the data in two approaches, firstly, only the explanatory variables are denoised, later, both the dependent and explanatory variables are also denoised under three (3) different sample sizes (i.e 32, 256 and 1024). The choice of the smoothing parameter for the Kernels, Wavelet and Polynomial Spline smoothers is selected by Plug-in-method, Universal threshold and interesting range methods respectively. The regression to the denoised data is fitted and then applied to the estimators one after the other.

Sample sizes 32, 256, and 1024 are drawn repeatedly from the model (25). In each case, the MSE of the estimators are computed to compare the performance of the denoised nonlinear estimators, i.e. the MSE of the denoised nonlinear least squares (DNLS) estimator, denoised nonlinear least absolute deviation (DNLAD) estimator and denoised nonlinear M- estimator are computed from 1,000 Monte Carlo samples. The analysis is carried out using R statistical package and the simulation results are summarized in the numerical tables below.

Table 4.1: Mean Squared Errors of the denoised nonlinear estimators when Epanechnikov Kernel is used as a smoother.

Estimators	Parameters	Denoise explanatory varibles			Denoised both dependent and explanatory variables		
		32	256	1024	32	256	1024
DNLS	β_1	0.0007606	0.0001465	0.0000866	0.0007606	0.0000968	0.0000387
	β_2	0.0000393	0.0000088	0.0000058	0.0000404	0.0000063	0.0000033
	β_3	0.0000160	0.0000020	0.0000005	0.0000169	0.0000020	0.0000005
DNLAD	β_1	0.0015072	0.0002509	0.0001993	0.0013879	0.0001309	0.0001157
	β_2	0.0000883	0.0000203	0.0000193	0.0000841	0.0000163	0.0000139
	β_3	0.0000304	0.0000027	0.0000007	0.0000307	0.0000031	0.0000009
DNM	β_1	0.0009225	0.0001690	0.0000910	0.0008680	0.0001275	0.0000455
	β_2	0.0000458	0.0000097	0.0000063	0.0000435	0.0000079	0.0000037
	β_3	.0000203	0.0000026	0.0000006	0.0000203	0.0000026	0.0000007

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Table 4.2: Mean Squared Errors of the denoised nonlinear estimators when Gaussian Kernel is used as a smoother.

Estimators	Parameters	Denoise explanatory varibles			Denoised both dependent and explanatory variables		
		32	256	1024	32	256	1024
DNLS	β_1	0.0006835	0.0001164	0.0000670	0.0006622	0.0000775	0.0000176
	β_2	0.0000339	0.0000069	0.0000046	0.0000337	0.0000054	0.0000027
	β_3	0.0000160	0.0000020	0.0000005	0.0000160	0.0000020	0.0000005
DNLAD	β_1	0.0013939	0.0002083	0.0001719	0.0013600	0.0001309	0.0001059
	β_2	0.0000819	0.0000157	0.0000193	0.0000798	0.0000098	0.0000133
	β_3	0.0000322	0.0000026	0.0000007	0.0000305	0.0000027	0.0000009
DNM	β_1	0.0008562	0.0001407	0.0000733	0.0008331	0.0001178	0.0000393
	β_2	0.0000442	0.0000075	0.0000063	0.0000411	0.0000065	0.0000029
	β_3	0.0000194	0.0000026	0.0000006	0.0000203	0.0000026	0.0000007

Table 4.3: Mean Squared Errors of the denoised nonlinear estimators when Wavelet is used as a smoother

Estimators	Parameters	Denoise explanatory varibles			Denoised both dependent and explanatory variables		
		32	256	1024	32	256	1024
DNLS	β_1	0.0006827	0.0000797	0.0000202	0.0006622	0.0000775	0.0000202
	β_2	0.0000353	0.0000040	0.0000011	0.0000337	0.0000040	0.0000011
	β_3	0.0000168	0.0000020	0.0000005	0.0000160	0.0000020	0.0000005
DNLAD	β_1	0.0013525	0.0001371	0.0000663	0.0013637	0.0001497	0.0000628
	β_2	0.0000730	0.0000093	0.0000085	0.0000745	0.0000104	0.0000133
	β_3	0.0000304	0.0000031	0.0000007	0.0000303	0.0000034	0.0000007

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DNM	β1	0.0008127	0.0001069	0.0000271	0.0008468	0.0001066	0.0000277
	β2	0.0000410	0.0000070	0.0000015	0.0000423	0.0000053	0.0000029
	β3	0.0000194	0.0000026	0.0000006	0.0000203	0.0000026	0.0000006

Table 4.4: Mean Squared Errors of the denoised nonlinear estimators when Polynomial Spline is used as smoother

Estimators	Parameters	Denoise explanatory varibles			Denoised both dependent and explanatory variables		
		32	256	1024	32	256	1024
DNLS	β1	0.0006665	0.0000781	0.0000204	0.0006665	0.0000781	0.0000204
	β2	0.0000337	0.0000041	0.0000011	0.0000337	0.0000041	0.0000011
	β3	0.0000160	0.0000020	0.0000005	0.0000160	0.0000020	0.0000005
DNLAD	β1	0.0013335	0.0001212	0.0000664	0.0013335	0.0001212	0.0000664
	β2	0.0000742	0.0000102	0.0000139	0.0000742	0.0000102	0.0000139
	β3	0.0000315	0.0000026	0.0000007	0.0000315	0.0000026	0.0000007
DNM	β1	0.0008311	0.0001066	0.0000279	0.0008311	0.0001066	0.0000279
	β2	0.0000411	0.0000053	0.0000015	0.0000411	0.0000053	0.0000015
	β3	0.0000194	0.0000026	0.0000006	0.0000194	0.0000026	0.0000006

From the result of the analysis, it can be seen that the average estimated value of the parameters from the three (3) denoised nonlinear estimators (i.e DNLS, DNLAD and DNM) under the three (3) different sample sizes considered are close to the true parameter values. Therefore, the denoised nonlinear estimators are almost unbiased.

Tables 4.1, 4.2, 4.3 and 4.4 show the estimated mean squared errors (MSE) of the denoised nonlinear estimators (i.e MSE of DNLS, DNLAD and DNM) under the three (3) sample sizes (i.e 32,256 and 1024).

Comparing the parameters β_1, β_2, β_3 of each denoised nonlinear estimator for each smoother based on their mean squared error, it can be observed that the denoised nonlinear least square estimator is the most efficient followed by denoised nonlinear M-estimator and lastly denoised nonlinear least absolute deviation (DNLAD) estimator. Therefore, the denoised nonlinear least squared (DNLS) estimator which has the smallest MSE outperforms both the denoised nonlinear M-estimator and the denoised nonlinear least absolute deviation (DNLAD) estimator under the three (3) sample sizes. Equally, the denoised nonlinear estimators reveal smallest mean squares error under large sample size (1024), compared to medium sample size (256) and small sample size (32). Therefore, the denoised nonlinear estimators are more efficient under large sample size (1024), but the denoised nonlinear least squared estimator is the most efficient among the three (3) nonlinear estimators. Also, it is obvious from the estimated mean squared error (MSE) that each of the nonlinear estimators considered performed better under the Wavelet and Polynomial Spline denoising than the Kernels, and it can be seen that the denoised nonlinear estimators have smaller mean square error under kernels when both the dependent and explanatory variables are denoised than when only the explanatory variables are denoised while there is little or no difference in mean squares error under the Wavelet and Polynomial spline for the two denoising approaches.

CONCLUSION

This study presents an application of smoothing techniques to denoise nonlinear regression estimators under different sample sizes. The Epanechnikov Kernel, Gaussian Kernel, Wavelet and Polynomial Spline smoothers are firstly used to denoise only the explanatory variables and later denoise both dependent and explanatory variables under the three (3) different sample sizes (i.e 32, 256, and 1024). The performance of the denoised nonlinear estimators is compared based on the mean squared error criterion to determine their efficiency. The simulation studies carried out for sample sizes 32, 256,and 1024 with 1,000 Monte Carlo samples, show that the denoised nonlinear least squares (DNLS) estimator which has the smallest MSE is the best (most efficient) estimator among all the three (3) denoised nonlinear estimators under the four smoothers considered. However, the idea of denoising both the dependent and explanatory variables gives room for more efficiency of the nonlinear estimators when Kernels are used as smoother but Wavelet and Polynomial Spline smoothers are effective than Kernels smoothers. Besides, the denoised nonlinear estimators (i.e DNLS, DNLAD and DNM) performed better under the large sample size 1024 than the rest of the sample sizes (i.e medium and small) considered.

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