ABSTRACT: Continuum and discontinuum problems were considered in the solid-liquid medium under study. The necessitated the use of combined finite-discrete element method to generate model expressions from contact force and seepage force considered to be the major forces contributing to the flow of fluid through soil mass and boiling or quicksand effect when seepage force becomes more in effect under critical hydraulic gradient and / or critical hydraulic head. The equilibrium model has deduced an expression for the safe hydraulic head during well pumping as $H(x) = 0.0065814 \cdot r^4X_5N_x^2$ and this has been verified using a laboratory check; prototype well failure test. It has been established that there is strong agreement between model result and the laboratory check at 1.8m flow distance and the correlation analysis carried out has also shown a perfect correlation of 0.989879999701. Note, a perfect correlation lies between -1 (perfect negative) and 1 (perfect positive) (Agunwamba, 2007; Inyama and Iheagwam, 1995). For safe pumping and corresponding yield in the borehole system, inter-granular force between granular particles should equal the seepage force and this is achieved by ensuring that the deduced model expression is used to determine the safe hydraulic head. For the system under study, the safe hydraulic head must be maintained. Finally, as long as the model hydraulic head expression deduced is used under the above conditions, safe pumping can be achieved at any voltage between 150volts and 240volts.

KEYWORDS: Equilibrium, Contact Force, Seepage Force, Modelling, Borehole.

INTRODUCTION

Identification and establishment of the factors that cause the failure of boreholes is one of the main targets of this research work. The medium under study is a solid-liquid medium with the liquid (fluid) migrating through the voids of the solid (granular soil) to where it is pumped for use. During this process, whereby the fluid moves from areas of high potential to that of low potentials there is the introduction of forces acting both on the fluid and the granular material causing dislodgment and displacement of the particles to be collected at the walls of the well casing. These collected particles also block the well casing perforations or screens making the well casing inefficient to transmitting the collected fluid into the well for pumping (Durlofsky and Aziz, 2004).

Two critical factors have been identified for study in the present research work as those that cause the failure of water boreholes by acting on both the fluid and solid phases of the well operation thus;

1. Interaction force between the soil particles (restoring force).
2. Motion force due to mass of particle causing phenomenal displacement of the particles as a result of seepage force.

The above factors are to be extensively studied to arrive at an equilibrium model and solution to the problem under study. Nigeria has a total land mass of 932,768Sq.Km falling between latitude 4°1 and 13°9N and longitudes 2°21 and 14°31W and a population, currently of about 120million people (Eduvie, 2006). The total replenishable water resource in Nigeria is estimated at 319 billion cubic meters, while the ground water component is estimated at 52 billion cubic meters. Water shortages are acute in some major centers and in numerous rural communities due to a variety of factors including variation in climatic conditions, drought increasing demands, distribution system losses and breakdown of works and facilities (Eduvie, 2006). Ground water is the water stored in an aquifer in pore spaces or fractures in rocks or sediments. Groundwater is generally a readily available source of water throughout populated Africa but the construction costs for sustainable supplies are high. The reason why groundwater is preferred to surface water includes:

- Its relative low costs compared to surface water
- Availability in most areas
- Potable without treatment
- Employs low cost technologies
- The frequent drought problems enforce the use of groundwater source as many small intermittent rivers and streams dry out during the dry seasons.

LITERATURE

Groundwater development in Nigeria

The establishment of the Nigerian geological survey in 1817 has as one of its major objectives to search for groundwater in the semiarid areas of the former northern Nigeria. These activities of the authorities of the Nigerian geological survey culminated in the commencement in 1928 of systematic investigations of towns and villages for the digging of hand dug wells. In 1938, a water drilling section of the geological survey was setup and by 1947; the engineering aspects of the water supply section were handed over to the public works Department, which is the forerunner of Nigeria’s today’s ministry of works while the geological survey maintained the exploration department. The aim of studying borehole failures is to identify the factors responsible for borehole engineering solutions. According to (Eduvie, 2006), the most plausible causes of these borehole failures can be attributed to

(i) Design and construction
(ii) Groundwater potential/hydro geological consideration and
(iii) Operational and maintenance failures.

With the foregoing, Eduvie has failed to recognize the purely engineering factors that could cause the failure of boreholes and this has stimulated the present research work to establish
those factors that cause failure or operational inefficiency of water boreholes. Consequently, seepage and contact force (inter-granular force) are the two major opposing physical factors that fell within the scope of the present work for study.

**The Combined Finite-Distinct Element Method**

The combined FDEM is aimed at problems involving transient dynamics of systems comprising a large number of deformable bodies that interact with each other, and that may in general fracture and fragment, thus increasing the total number of discrete (distinct) elements even further. Each individual distinct element is of a general shape and size, and is modeled by a single distinct element. Each distinct element is discretized into finite elements to analyze deformability, fracture and fragmentation. A typical combined FDEM system comprises a few thousand to a few million separate interacting solids, each associated with separated finite element meshes (Munjiza, 2004; Sitharam, 2003; Cheng et al, 2004). In this work, one of the key issues in the development of the combined FDEM is the treatment of contact between the elements, fluid flow through the voids between the elements and the displacement of the elements. The only numerical tool currently available to a scientist or engineer that can properly take systems comprising millions of deformable distinct elements that simultaneously fracture and fragment under both fluid and solid phase is the combined FDEM. The combined FDEM merges finite element tools and techniques with distinct element algorithms (Cheng et al, 2004; Mahabadi et al, 2012; Frederic et al, 2008; Bell et al, 2005). Finite element based analysis of continua is merged with distinct element-based transient hydrodynamics, contact detection and contact interaction solutions. Thus, transient dynamic analysis of systems comprising a large number from a few thousands to more than a million of deformable bodies which interact with each other and in through seepage process can break fracture or fragment, becomes possible (Munjiza, 2004).

**The Combined Continua-Discontinua Problem**

Consider a container problem with particles being made of a very soft rubber or jelly so that in addition to interacting with each other, they deform as well. In addition, the walls of the container also deform. This problem is called the “Flexible Container Problem”. Even in the case of less deformable particles, the deformation of the container and that of individual particles significantly influences the way particles move inside the container. Thus, the total mass of the particles deposited into the container is also influenced by deformability (elastic properties) of both particles and container (Munjiza, 2004; Bell et al, 2005; Sitharam and Dinesh, 2003).

Each individual particle deforms under external forces and interaction with other particles already in the container and also under interaction with container walls. Changes in the shape and size of individual particles are in essence a problem of finite strain elasticity since finite rotations at least are present. The deformability of the individual particles is therefore well represented by a hypothetical continuum-based model. Interaction among individual particles, interaction between particles and container problem and fluid flow through the voids of the particles as seen in the case of the present research work is best represented by discontinua-based model. The flexible container problem involves aspects of continua and discontinua, (Munjiza, 2004; Sukumaram and Ashmawy, 2008; Lu et al, 2007).

Problems such as the analysis of the failure of the walls of the water borehole/well faced with dislodgment and deformation of individual particles and the displacement of the particles under
Fluid velocity are a combination of continua and discontinua, and are therefore termed combined continua-discontinua problems. The deformability of individual particles is best described using the hypothetical continua formulation. Interaction and motion of individual particles and the fluid flow through the voids or pores of the material medium is best described using discontinua formulation. The set of governing equations obtained at the end of the operation describes both deformability of individual particles and interaction between particles and the fluid flow through the pores of the particles. The number of equations is a function of the total number of particles under study. Analytical solutions of the governing equations obtained are rarely available, and numerical approaches have to be employed. These include discontinuous deformation analysis, DDA and DEM with added features to capture deformability (Munjiza, 2004). Meanwhile, the most advanced approach is to use the state of the art method, FEM to model continuum-based phenomena (in this case, deformability and discretisation of distinct element into finite elements) and the state of the art method, the DEM to model discontinuum-based phenomena (in this case interaction and motion of individual particles and fluid flow through voids). The new method is therefore a combination of both the FEM and DEM, and is termed combined FDEM.

In the combined FDEM, each particle (body) is represented by a single distinct element that interacts with distinct elements that are close to it. In addition, each distinct element is discretized into finite elements. Each distinct element has its own finite element mesh. The total number of finite element meshes employed is a function of the total number of distinct elements. Each finite element mesh employed captures the deformability of a single distinct element (Particle, body).

**METHODOLOGY AND FORMULATION**

Contact force (inter-granular force) and seepage force are two fundamental physical phenomena under serious study in the present work because of their pronounced effect on the failure of the walls of water boreholes. As they will be investigated, they are two opposing forces i.e. disturbing and restoring forces and therefore deserve our keen attention and study. The basic principle involved in the formulation is the combined FDEM because of the continuum and discontinuum nature of the studied region. From the foregoing, the problem of contact force (intergranular force) existing within the region of the soil mass or volume is a discontinuum problem, therefore employs discrete element method in the formulation of the matrix contact force equation where every particle that make up the soil mass is considered a discrete element. Similarly, the problem of volume force or seepage force is a continuum problem and employs the finite element method in its formulation.

**Contact Force Model**

Contact interaction between neighbouring distinct elements occurs through solid surfaces as illustrated in Figure 1 which are generally irregular and as a consequence, the contact pressure between two solids is actually transferred through a set of points, and with increasing normal pressure, surfaces only touch at a few points, and with increasing normal pressure, elastic and plastic deformation of individual surface asperities occurs, resulting in an increase in the real contact area (Munjiza, 2004).

Problems of contact interaction in the context of the combined FDEM are even more important, due to the fact that in this method, the problem of contact interaction and handling of conext
also defines the constitutive behaviour of the system, because of the presence of large numbers of separate bodies. Thus, algorithms employed must pay special attention to contact kinematic in terms of the realistic distribution of contact forces, energy balance and robustness (Munjiza, 2004).

The present research on contact interaction algorithm makes use of finite element discretizations of discrete elements, and combines this with the so-called potential (pressure/stress) contact force concept. This algorithm assume discretization of individual discrete elements into finite elements, thus imposing no additional database requirements in handling the geometry of individual discrete elements. They also yield realistic distribution of contact for use over finite contact area resulting from the overlap of discrete elements that are in contact.

![Figure 1: Particle contact geometry](image1)

The distributed contact force is adopted for two discrete elements in contact, shown in Figure 1, one of which is denoted as the contactor C and the other as the target, t. When in contact, the contactor and target discrete elements overlap each other over area S, bounded by boundary \( \varepsilon \) (Figure 2).

It is assumed that penetration of any elementary area \( dA \) of the contactor into the target results in an infinitesimal contact force, given by

\[
dF = [\nabla \psi_c(P_c) + \nabla \psi_t(P_t)] \, dA
\]

(1)

![Figure 2: Contact force due to an infinitesimal overlap around points P_c and P_t](image2)
\[ dF = \text{Infinitesimal contact force} \]
\[ dA = \text{Infinitesimal area} \]
\[ \psi(p) = \text{Potential function} \]
\[ \sigma_c, \sigma_t = \text{Contactor and target stresses} \]

Equation 1 can be written as
\[ dF = dF_t + dF_c \] (2)

Where
\[ dF_c = \text{grad}\psi(t) dA_c, dA_c = dA \] (3)
\[ dF_t = \text{grad}\psi_c(P_c) dA_t, dA_t = dA \] (4)

Figure 3: Discretisation of contactor, target and support discrete elements contact zone to finite elements

Considering a third discrete element known as supporter discrete element S and consider its effects on the contact force, Equation 2 will become,
\[
\begin{bmatrix}
F_c \\
F_t \\
F_s
\end{bmatrix} =
\begin{bmatrix}
\sigma_{c_1} & \sigma_{c_2} & \ldots & \ldots & \sigma_{c_n} \\
\sigma_{t_1} & \sigma_{t_2} & \ldots & \ldots & \sigma_{t_n} \\
\sigma_{s_1} & \sigma_{s_2} & \ldots & \ldots & \sigma_{s_n}
\end{bmatrix}
\begin{bmatrix}
A_1 \\
A_2 \\
\vdots \\
A_n
\end{bmatrix}
\] (5)

Seepage Force Model

Soils are permeable to fluids (water) because the voids between soil particles are interconnected. The degree of permeability is characterized by the permeability coefficient \( K \), also referred to as hydraulic conductivity. The basic concepts of seepage and flow through
Granular soil materials viz fluid velocity, seepage quantity, discharge velocity, hydraulic gradient etc. obey Darcy’s law thus

$$ q = KiA $$  

(6)

Where,

- \( q \) = discharge in m\(^3\)/s
- \( K \) = hydraulic conductivity or permeability constant
- \( i \) = hydraulic gradient
- \( A \) = cross section area of flow region

The seepage quantity \( q \) is the volume of water passing through the pores voids of a soil cross-section area during a unit interval. \( q \) is the flux of water:

$$ q = \int_A v_f \, dA $$  

(7)

Where \( v_f \) = fluid velocity and

- \( A \) = total cross section area of medium

Three discrete particles; target, contactor and support particles and the fluid flow through the contact zone were considered as in Fig.3 below;

![Figure 4: Elements and nodal points of the contact zone](image)

![Figure 5: Soil volume subjected to three force components](image)
In strict agreement with (Fox et al, 2010); seepage force (Fig.5) as a volume force is given by the expression (Sivakugan, 2005),

\[ S_F = i \gamma_w \]  

Where

\[ i = \text{hydraulic gradient} \]

\[ \gamma_w = \text{unit weight of water KN/m}^3 \]

Consider the elemental area under study, the elemental hydraulic head \( dH \) that causes flow of water in the soil mass or volume is given as

\[ dH = S_F dx \gamma_w^{-1} \]  

Equilibrium Condition of Studied Region

Under equilibrium conditions, there is fluid flow without its attendant particle dislodgement and displacement. This emplies that at this state, the disturbing force and the restoring force are equal or the algebraic sum of the fundamental forces equals zero. Thus;

Contact force = Seepage force

That is to say that,

\[ \Sigma dA - i \gamma_w = 0 \]  

Where

\[ \Sigma = \text{contact stress of the region} \]

\[ dA = \text{elemental surface area of granular particles} \]

\[ i = \text{hydraulic gradient} \]

\[ \gamma_w = \text{unit weight of water} \]

\[ \Sigma dA = i \gamma_w \]  

\[ \Sigma dA = \frac{dH}{dx} \gamma_w \]  

\[ \Sigma dA dx = dH \gamma_w \]  

\[ dH = \frac{1}{\gamma_w} \Sigma dA dx \]  

Where;

\[ \frac{dA}{\gamma_w} \] is a constant

\[ dH = \frac{dA}{\gamma_w} \Sigma dx \]  

The stress between particles as they come in contact and are held together by contact force varies node to node and from particle to particle in the direction of flow. However, within the three directions of flow \( x, y \) and \( z \), the head at which the borehole is to be operated to forestall
failure of the soil medium by dislodgement of the particles or grains that make the soil volume or mass is calculated as,

\[ dH = Hx + Hy + Hz \]  \hspace{1cm} (16)

The stress of the domain \( \mathbb{N} \) in three directional are:

\[
\begin{align*}
\mathbb{N}_x & = \mathbb{N}_{x1} + \mathbb{N}_{x2} + \mathbb{N}_{x3} + \ldots \ldots \mathbb{N}_{xn} \\
\mathbb{N}_y & = \mathbb{N}_{y1} + \mathbb{N}_{y2} + \mathbb{N}_{y3} + \ldots \ldots \mathbb{N}_{yn} \\
\mathbb{N}_z & = \mathbb{N}_{z1} + \mathbb{N}_{z2} + \mathbb{N}_{z3} + \ldots \ldots \mathbb{N}_{zn}
\end{align*}
\]  \hspace{1cm} (17)

Equation 15 becomes

\[ H = \sum_{x=1}^{n} \mathbb{N}_x \sum_{y=1}^{n} \mathbb{N}_y \sum_{z=1}^{n} \mathbb{N}_z \int_{x \cap y \cap z} \frac{dA}{\gamma_w} \mathbb{N} \cdot dx \]  \hspace{1cm} (18)

The matrix transformation of Equation 18 becomes

\[
\begin{bmatrix}
H_x \\
H_y \\
H_z
\end{bmatrix}
= \frac{A}{\gamma_w}
\begin{bmatrix}
\mathbb{N}_{x1} & \mathbb{N}_{x2} & \mathbb{N}_{x3} & \ldots & \mathbb{N}_{xn} \\
\mathbb{N}_{y1} & \mathbb{N}_{y2} & \mathbb{N}_{y3} & \ldots & \mathbb{N}_{yn} \\
\mathbb{N}_{z1} & \mathbb{N}_{z2} & \mathbb{N}_{z3} & \ldots & \mathbb{N}_{zn}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2 \\
X_3 \\
\vdots \\
X_n
\end{bmatrix}
\]  \hspace{1cm} (19)

Equation 19 above is the general equation of the system in equilibrium applied to all the nodes of the contact flow region in Fig.4 to formulate the global matrix equation which solution deduced the restoring hydraulic head equation as shown in Equation 20 below;
\[
\begin{bmatrix}
X_2(-N_{x2}) + X_5 \\
X_2(N_{y2}) - X_5 \\
X_2(-N_{x2}) + X_5 \\
X_4(-N_{x4}) + X_5 \\
X_4(N_{y4}) - X_5 \\
X_4(-N_{x4}) + X_5 \\
X_4(N_{x4}) - X_5 \\
X_4(N_{x4}) - X_5 \\
X_5 + X_5(-N_{y8}) \\
-X_5 + X_5(-N_{y8}) \\
X_5 + X_8(N_{x8}) \\
X_5 + X_8(-N_{y8}) \\
X_5 + X_6(-N_{x6}) \\
X_6(-N_{y6}) \\
X_5 + X_6(-N_{x6}) \\
X_5 + X_6(-N_{x6}) \\
X_5 + X_6(-N_{x6}) \\
X_5 + X_5(N_{x2}) + X_5 \\
X_5 + X_5(-N_{x2}) - X_5 \\
X_5 + X_5(N_{x2}) + X_5
\end{bmatrix}
\]

\[
\frac{A^2}{\gamma_w l} = f(x) \quad (20)
\]

Furthermore,

\[
H_{(x)} = \frac{A^2}{\gamma_w l} \left\{ [X_2(-N_{x2}) + X_5] + [X_2(N_{y2}) - X_5] + [X_2(-N_{x2}) + X_5] + [X_4(-N_{x4}) + X_5] + [X_4(N_{y4}) - X_5] + [X_4(-N_{x4}) + X_5] + [X_4(N_{y4}) - X_5] + [X_4(-N_{x4}) + X_5] + [X_5 + X_8(N_{x8})] + [X_5 + X_8(-N_{y8})] + [X_5 + X_8(N_{x8})] + [X_5 + X_8(-N_{y8})] + [-X_5 + X_6(-N_{x6})] + [-X_5 + X_6(-N_{x6})] + [-X_5 + X_6(-N_{x6})] + [-X_5 + X_6(-N_{x6})] + [X_5 + X_5(N_{x2}) + X_5] + [X_2(-N_{x2}) - X_5] + [X_2(N_{x2}) + X_5] \right\}
\]

Collecting like terms from Equation 21 and solving same would give;

\[
H_{(x)} = \frac{4X_5N_{x2}A^2}{\gamma_w l} = \frac{4\pi r x X_5N_{x2}}{\gamma_w l} \quad (22)
\]

\(r\) = average radius of the discrete soil particles = 0.002857m

\(X_5\) = flow radius and this varies between 0.6, 1.2, 1.8, 6.0

\(N_{x2}\) = equilibrium stress of the system which factors vary between 0.1, 0.2, 0.3, 1.0 (Munjiza, 2004)

\(\gamma_w\) = unit weight of water = 1000kg/m³
L= cross sectional length of the flow medium = 6m.

Substituting for values in Equation 22, we would have the model equation for the head restoring equilibrium at well pumping thus;

\[ H(x) = 0.0065814 \cdot r^4X^2N_x \]

(23)

**Laboratory Check**

The geophysical laboratory investigation was carried out on the sample collected from borehole sites located within Umuahia (BS 1377, 1995) where there have been records of failed boreholes and more yet fail, at the aquifer depth of 50 to 68 meters located on latitude North 5° 3' 32.80" and longitude East 7° 29' 46" with average rainfall of between 2000mm to 2500mm (Google.com, 2013). And finally, prototype well failure test was conducted as shown in Figure 6 below.

At the same time a power regulator of 10 voltage speeds was fabricated to power the submersible pump at 10 different voltages supplied between 150 volts and 240 volts.

![Figure 6: Prototype well failure test setup](image-url)
RESULTS AND DISCUSSION

The result of the geophysical examination carried out on the sample under study is as tabulated in Table 1 below;

**Table 1: Geophysical properties of soil sample under study (Onyelowe, 2013; Alaneme, 2014)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Result</th>
<th>Parameter</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Limit</td>
<td>14.00</td>
<td>OMC</td>
<td>7.075%</td>
</tr>
<tr>
<td>Plastic Limit</td>
<td>6.67</td>
<td>Specific Gravity G</td>
<td>2.857</td>
</tr>
<tr>
<td>Plasticity Index</td>
<td>7.33</td>
<td>Proven Ring Factor k</td>
<td>0.004105KN/div</td>
</tr>
<tr>
<td>Cu</td>
<td>6.79</td>
<td>Area of Shear Box</td>
<td>0.01m²</td>
</tr>
<tr>
<td>Cc</td>
<td>1.52</td>
<td>Normal Stress σ</td>
<td>10.275KN/m²</td>
</tr>
<tr>
<td>Classification(AASHTO)</td>
<td>A-2-4</td>
<td>Frictional angle</td>
<td>480</td>
</tr>
<tr>
<td>Grading</td>
<td>Well graded</td>
<td>Cohesion</td>
<td>40KN</td>
</tr>
<tr>
<td>MDD</td>
<td>1.84mg/m³</td>
<td>Soil Type</td>
<td>Gravel and sand</td>
</tr>
<tr>
<td>( Y_{\text{sat}} )</td>
<td>19.26KN/m³</td>
<td>( Y_{\text{w}} )</td>
<td>9.8KN/m³</td>
</tr>
<tr>
<td>( Y_{\text{b}} )</td>
<td>9.46KN/m³</td>
<td>( i_{\text{c}} )</td>
<td>0.9653</td>
</tr>
<tr>
<td>K</td>
<td>1.794E-5cm/s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 below shows the results of the prototype well failure test;

**Table 2: Prototype well failure test result and critical hydraulic head**

<table>
<thead>
<tr>
<th>Voltage</th>
<th>Pump discharge, ( q ) (m³/s)</th>
<th>Critical hydraulic head, ( h_c )</th>
<th>Generated pump power, ( P_o ) (hp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>0.03313</td>
<td>0.02041</td>
<td>1.95</td>
</tr>
<tr>
<td>160</td>
<td>0.03536</td>
<td>0.08052</td>
<td>2.08</td>
</tr>
<tr>
<td>170</td>
<td>0.03757</td>
<td>0.10561</td>
<td>2.21</td>
</tr>
<tr>
<td>180</td>
<td>0.03978</td>
<td>0.35565</td>
<td>2.34</td>
</tr>
<tr>
<td>190</td>
<td>0.04199</td>
<td>0.56687</td>
<td>2.47</td>
</tr>
<tr>
<td>200</td>
<td>0.04420</td>
<td>0.77234</td>
<td>2.60</td>
</tr>
<tr>
<td>210</td>
<td>0.04641</td>
<td>0.89934</td>
<td>2.73</td>
</tr>
<tr>
<td>220</td>
<td>0.04862</td>
<td>0.91123</td>
<td>2.86</td>
</tr>
<tr>
<td>230</td>
<td>0.05083</td>
<td>1.01023</td>
<td>2.99</td>
</tr>
<tr>
<td>240</td>
<td>0.05304</td>
<td>1.25599</td>
<td>3.12</td>
</tr>
</tbody>
</table>

Recall that the restoring hydraulic head of the system was deduced from the mathematical model as;

\[ H(x) = 0.0065814 \cdot r^4 X_5 N_{x_2} \], where;

\( r \) =average radius of the discrete soil particles =0.002857m

\( X_5 \) =flow distance and this varies between 0.6, 1.2, 1.8......, and 6.0

\( N_{x_2} \)=equilibrium stress of the system which factors vary between 0.1, 0.2, 0.3...., 1.0 and the matlab solution of the above equation is as shown below in Table 3 and Fig. 7;
### Table 3: Restoring hydraulic head and equilibrium stress model

<table>
<thead>
<tr>
<th>( N_{X_2} )</th>
<th>Restoring hydraulic head ( H_{(X)} = 0.0065814 \cdot r^4X_5N_{X_2} ) @ ( X_5 ) equals</th>
<th>Lab. Restoring head, ( h_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( N_{X_2} \) \( X_5 \) equals (column 2 to 11) \( H_R \) (last column)
2. Plot heads (H(x) and hR) versus stress factors (N_(x_2 )) from Table 3

%2. Plot heads (H(x) and hR) versus stress factors (N_(x_2 )) from Table 3
Nx2 = table3(2:end,1);
figure(2);
for n=1:11
    Head = table3(2:end,n+1);
    if n <= 10
        plot(Nx2,Head,'*--');
        gtext(['X_5 = ' num2str(table2(1,n+1))]);
    else
        h2=plot(Nx2,Head,'--ro','LineWidth',2,...
                'MarkerEdgeColor','k',...
                'MarkerFaceColor','g',...
                'MarkerSize',5);
        gtext('Lab H_R');
    end
    hold on;
end
legend(h2,'Lab. Restoring head','Location','Best')
grid on; xlabel('Stress Factor, \aleph_{x2} (in metres)');
ylabel('Restoring Hydraulic Head, H_{(x)} and Lab. H_R');
title('Graph of Restoring Hydraulic Head against Stress Factor');
hold off

Figure 7: Model of equilibrium condition of the restoring hydraulic head
From the foregoing, it can be established the;

1. The hydraulic head restoring equilibrium between contact force (inter-granular force) and seepage force is deduced as $H(x)=0.0065814 \cdot r^4 X_5 X_2$ from the mathematical model.
2. The head restoring equilibrium in the aquifer medium is seen to be less than the heads causing boiling, Tables 2 and 3 refers, which proves the effect of inter-granular force in the system positive.
3. There is strong agreement between the mathematical model hydraulic heads and the laboratory model head in that they both increased progressively and relatively with equilibrium stress of the system as shown in Table 3 and Figures 7 and 8 with the closest agreement at the flow distance of 1.8m and the correlation analysis carried out has also shown a perfect correlation of 0.989879999701. Note, a perfect correlation lies between -1 (perfect negative) and 1 (perfect positive) (Agunwamba, 2007; Inyama and Iheagwam, 1995).
4. It can also be deduced that hydraulic head increased with increase in flow distance for all the stress factors.

CONCLUSION

The following could be concluded from the present research work;

1. For safe pumping and corresponding yield in the borehole system, inter-granular force between granular particles should equal the seepage force and this is achieved by ensuring that the deduced model expression is used to determine the safe hydraulic head.
2. Finally, as long as the model hydraulic head expression deduced is used under the above conditions, safe pumping can be achieved at any voltage between 150volts and 240volts.
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