

## DEVELOPMENT OF A SUITABLE MATHEMATICAL MODEL FOR PREDICTING YIELD STRENGTH (YS) OF STAINLESS STEEL WELD JOINTS

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**ABSTRACT:** *A mathematical model for predicting the yield strength (YS) of TIG welded austenitic stainless steel weld joint was developed. The validity and accuracy of the model was confirmed using deviational and statistical analyses as well as scatter diagram. The maximum deviation between the model-predicted YS values and those obtained from actual experiment was less than 6% in absolute terms and the  $R^2$  values were above 90%. Furthermore, the scatter diagram showed that the experimental values and the predicted values lie close to the 45° line.*

**KEYWORDS:** Yield Strength, Mathematical Model, Deviational Analysis, Statistical Analysis, Scatter Diagram.

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### INTRODUCTION

Yield strength refers to an indication of maximum stress that can be developed in a material without causing plastic deformation. It is the stress at which a material exhibits a specified permanent deformation and is a practical approximation of the elastic limit. Once the yield point is passed, some fraction of the deformation will be permanent and non-reversible. In engineering structural design, yield strength is very important. For example, when designing a component, it must support the force incurred during use, and the component must not deform plastically. Therefore, a material with sufficient yield strength should be selected.

In design applications, the yield strength is often used as an upper limit for the allowable stress that can be applied. It is especially important in material applications that require precise dimensional tolerances to be maintained in the presence of high stresses and loads. By altering dislocation density, impurity levels and grain size (in crystalline materials), the yield strength of the material can be fine-tuned. For materials without a clear distinct yield point, yield strength is usually stated as the stress at which a permanent deformation of 0.2% of the original dimension will result, known as the 0.2% yield stress.

The value of yield strength is important in the construction of structures, such that the structures are able to perform in the elastic region under normal servicing conditions. However, when faced with unexpected impact loads such as explosions, fires or natural disasters such as earthquakes, etc., the plastic region of the material becomes crucial, as a large portion of the energy being absorbed by the materials under such circumstances is mainly contributed by the plastic region.

Therefore the knowledge of the yield strength is very important in designing components, since it usually represents the upper limit of the load that can be applied. Yield strength is also very important for controlling many materials' production techniques, such as forging, rolling or pressing and welding.

Some researchers, Bang et al (2008) and Tewari, Gupta and Prakash (2010) have demonstrated that yield strength, like most mechanical properties, is generally influenced by welding parameters such as welding current, welding voltage and welding speed, which have been described as the most important factors affecting the quality, productivity and cost of weld joints (Srivastava, Tewari and Prakash, 2010),

### Effect of welding current on yield strength

Bahman and Alialhosseini (2010) in their study of Changes in hardness, yield strength and UTS of St37 steel grade welded joints produced with MAG welding process, Ghazvinloo and Honarbakhsh (2010) in their study on FCAW low carbon steel welds and Okonji, Nnuka and Odo (2015) in their work with GTAW stainless steel weld joints have shown that increase in welding current decreases yield strength. In welding practice welding current is the most influencing parameter which controls the depth of fusion, the electrode feed rate and depth of penetration. The amount of heat developed during welding depends upon the current used for given size of electrode and filler wires. It is therefore essential that a correct current is used to produce good quality of weld and reduce the distortion problems on the job.

Therefore this work is aimed at developing a suitable mathematical model for predicting the yield strength (YS) of low carbon austenitic stainless steel (AISI 304L) weld joints using the results of the research work of Okonji, Nnuka and Odo (2015) on the effect of welding current and filler metal types on the macrostructure and strength of GTAW austenitic stainless steel joints.

## DEVELOPMENT OF MATHEMATICAL MODEL

The data for the formulation of the model is contained in Table 1 (Okonji, Nnuka and Odo, 2015)

**Table1: Yield Strength**

Current (A)	91			92			93			94			95		
Filler Metal Type	308 L	309 L	316 L	308 L	309 L	316 L	308 L	309 L	316 L	308 L	309 L	316 L	308 L	309 L	316 L
YS (MPa)	796 .42 1	802 .15 7	771 .76 4	782 .94 0	786 .96 2	771 .64 4	769 .46 1	771 .76 4	771 .52 3	764 .34 5	750 .38 3	766 .66 4	756 .31 3	729 .00 2	756 .94 7

Source: Okonji, Nnuka and Odo (2015)

### Model Formulation

In developing the mathematical model for predicting the yield strength (YS), as a function of welding current (I), a procedure based on regression was employed and expressed as

$$Y = f(I) \quad \text{Where } Y = \text{YS}$$

The relationship selected is a first order response given as

$$Y = a + bI$$

The coefficient a, is the free term of the regression and the coefficient b, is the linear term.

The values of the coefficients were calculated by regression analysis with the help of the following equations

Calculation of regression coefficients a and b

$$Y = f(I) \quad (1)$$

$$Y = a + bI \quad (2)$$

Let actual response be  $\Phi$  and predicted be Y

$\therefore$  Standard error of prediction,  $\theta$  is given by

$$\theta^2 = \frac{\Sigma(Y-\Phi)^2}{n}$$

$$n\theta^2 = K = \Sigma(Y-\Phi)^2 = \Sigma(Y-a-bI)^2 \quad (3)$$

Differentiating with respect to a, we have

$$\frac{\delta K}{\delta a} = -2\Sigma(Y - a - bI) = 0 \text{ or } \Sigma Y - na - b\Sigma I = 0 \quad (4)$$

$$\therefore \Sigma Y = na + b\Sigma I \quad (5)$$

$$\therefore a = \frac{\Sigma Y}{n} - \frac{b\Sigma I}{n} = \bar{Y} - b\bar{I} \quad (6)$$

Differentiating with respect to b, we have

$$\frac{\delta K}{\delta b} = 2I\Sigma(Y - a - bI) = 0 \text{ or } \Sigma YI - a\Sigma I - b\Sigma I^2 = 0$$

$$\therefore \Sigma YI = a\Sigma I + b\Sigma I^2 \quad (7)$$

Multiplying (5) by  $\Sigma I$  gives

$$\Sigma Y\Sigma I = na\Sigma I + b(\Sigma I)^2 \quad (8)$$

Multiplying (7) by n gives

$$n\Sigma YI = na\Sigma I + nb\Sigma I^2 \quad (9)$$

(4.8) – (4.9) yields

$$\Sigma Y\Sigma I - n\Sigma YI = b[(\Sigma I)^2 - n\Sigma I^2]$$

$$\therefore b = \frac{\Sigma Y\Sigma I - n\Sigma YI}{(\Sigma I)^2 - n\Sigma I^2} \quad (10)$$

Where n = number of welding currents per filler metal.

The values of the coefficients for different responses were calculated with Texas Instrument, TI-84 plus using the data shown in Table 2. This resulted in:

**Table 2 Coefficients for various responses**

Filler metal type	308L	309L	316L
Coefficient a	1693	2469	1090
Value b	10	18	3

Introducing the values of the coefficients, the developed final mathematical equation is given below:

$$Y_{308L} = 1693 + 10I$$

$$Y_{309L} = 2469 + 18I$$

$$Y_{316L} = 1090 + 3I$$

### Boundary and Initial Conditions

The welding process was carried out under atmospheric condition. After welding, weldments were also maintained under atmospheric condition. The values of welding current and ultimate tensile strength (UTS) considered are as shown in Table 2. The grade of low carbon austenitic stainless steel used was 304L, grade of filler metal used was ER308L (2mm diameter), Electrode type: non-consumable 2% thoriated tungsten, (2mm), shielding gas, argon (99.99% purity level). Air was used as coolant for the welded spots. No pressure was applied to the HAZ during or after the welding process. No force due to compression or tension was applied in any way to the HAZ during or after the welding process. The sides and shapes of the samples were symmetries.

### Model Validation

The data for the formulation of the model was taken from Table 2 and the formulated model validated using Deviatonal Analysis, Statistical Analysis and Scatter Diagrams.

### **Deviational analysis**

This involves direct analysis and comparison of model-predicted values and those obtained from experiment for equality or near equality. Deviation (or error percent) of model-predicted values from the experimental values is given by

$$Dv = \frac{ExD - MoD}{MoD} \times 100 \quad (12)$$

Where ExD = Experimental results

MoD = Model-predicted values

The results are shown in Table 3.

### **Statistical analysis**

This was carried out to evaluate the correlations between process variables by calculating the coefficient of determination ( $R^2$ ). The results are given in Figures 1-6.

### **Scatter Diagrams**

The validity of the model was further tested by drawing scattered diagrams Figures 7-9. From the diagrams it can be observed that the values of the estimated and predicted values are scattered close to the 45° line, demonstrating an almost perfect fit of the developed models. This further establishes the validity and adequacy of the models.

## **RESULTS AND DISCUSSIONS**

### **Graphical Presentation**

The data for the formulation of the models were taken from Table 2 and the graphs shown in Figures 1-6

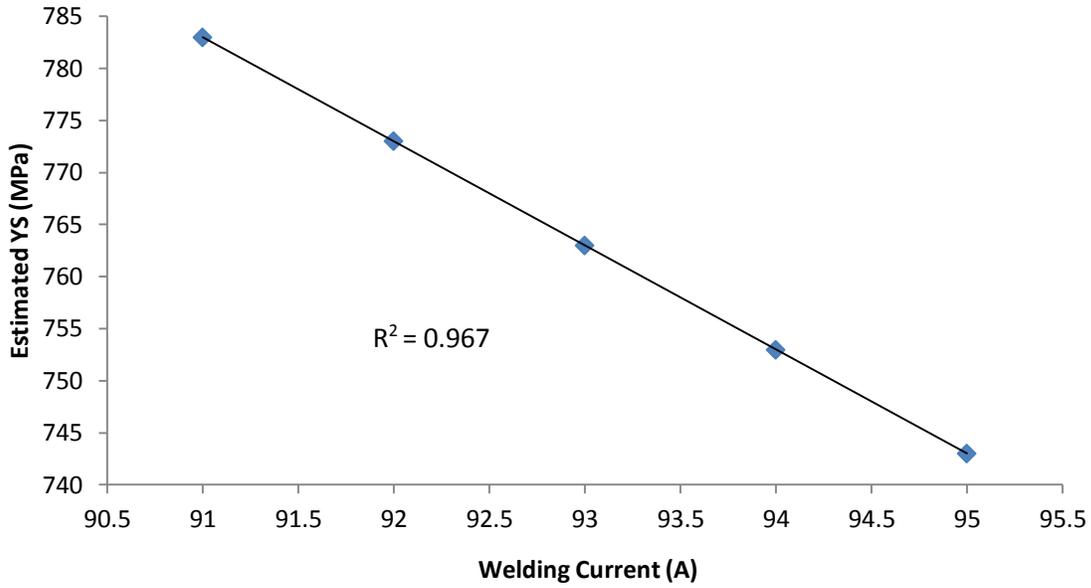


Figure 1 Interaction between estimated YS and welding current for 308L filler metal

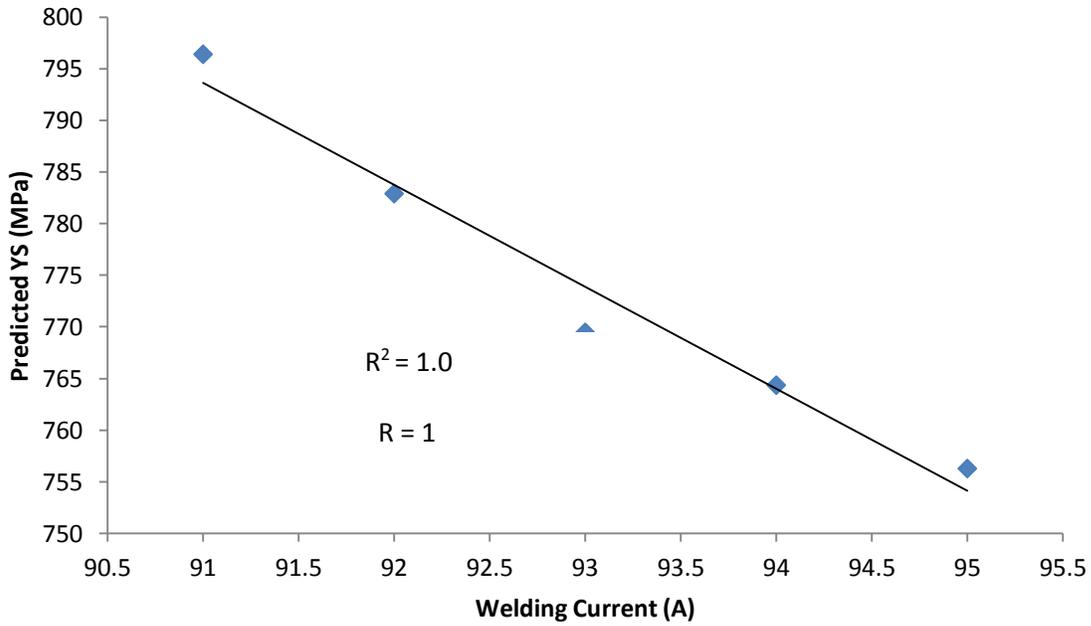


Figure 2 Interaction between predicted YS and welding current for 308L filler metal

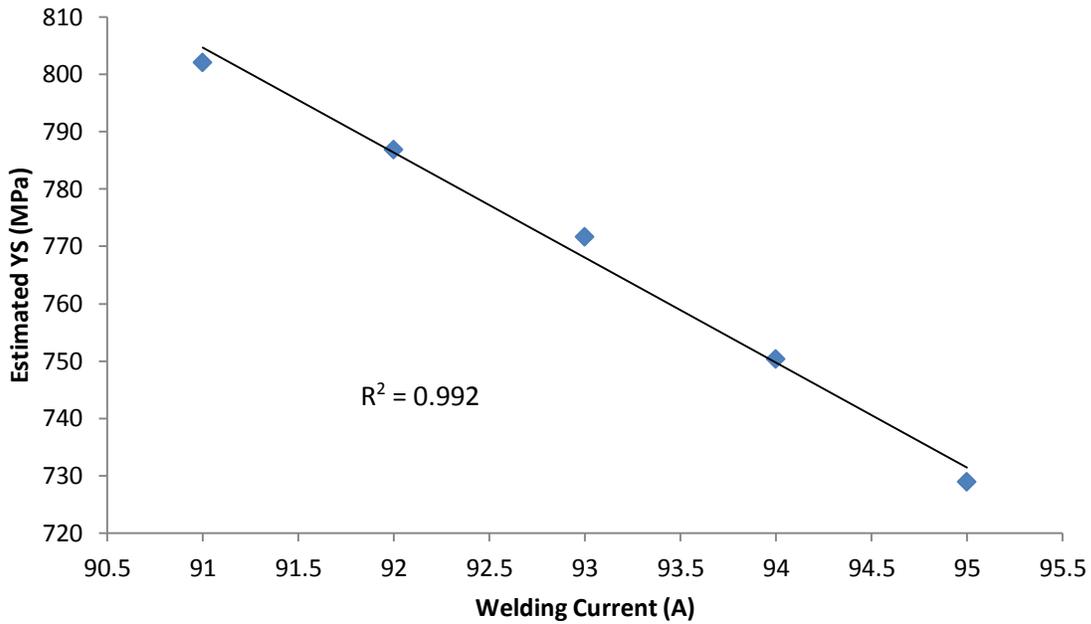


Figure 3 Interaction between estimated YS and welding current for 309L filler metal

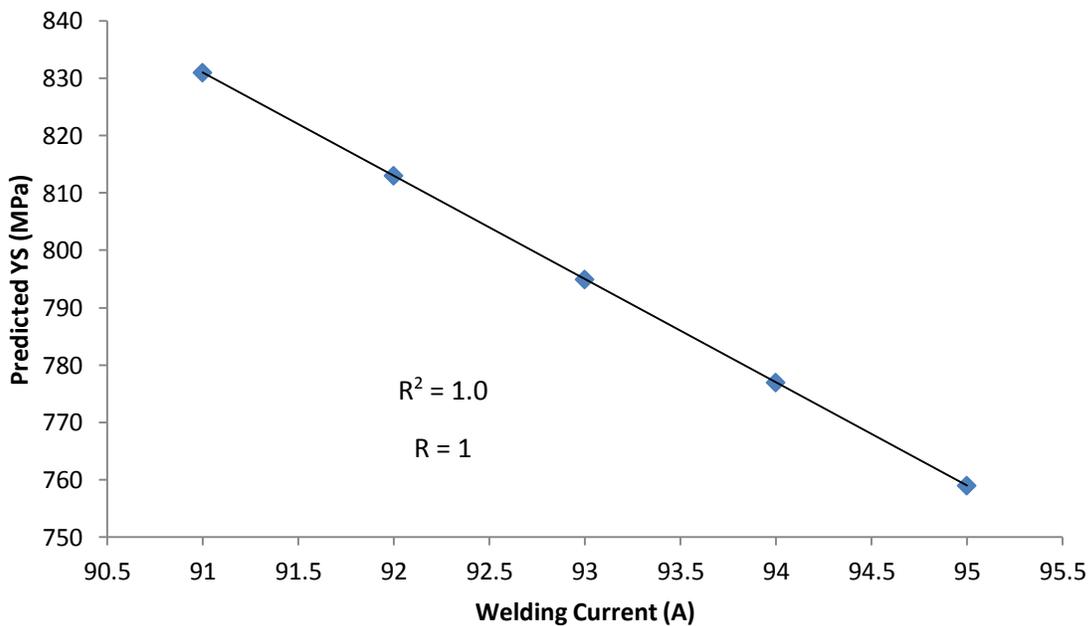


Figure 4 Interaction between predicted YS and welding current for 309L filler metal

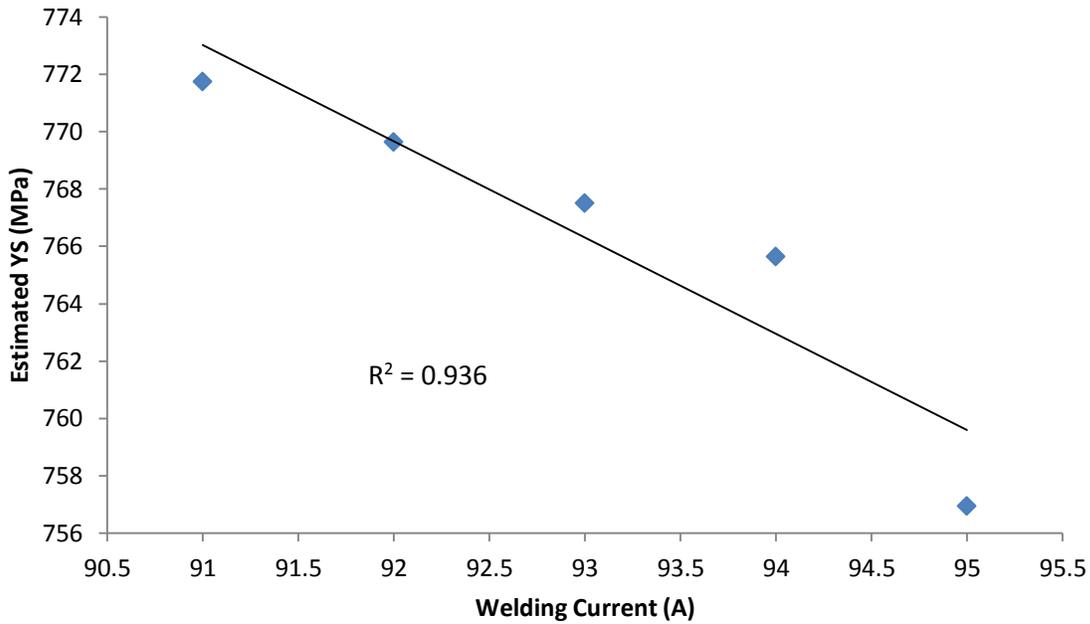


Figure 5 Interaction between estimated YS and welding current for 316L filler metal

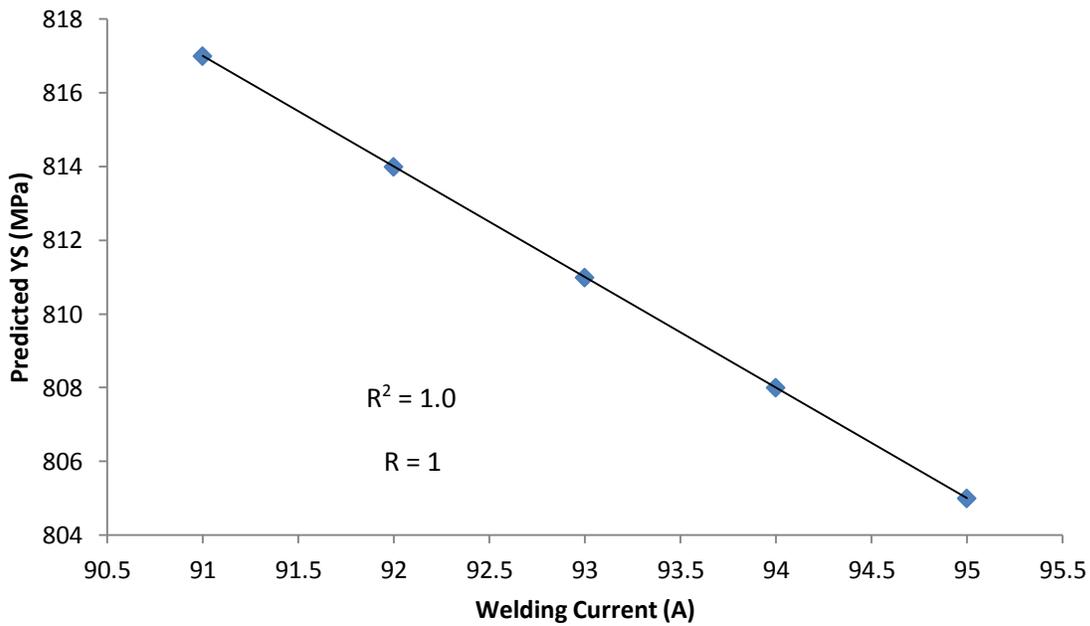


Figure 6 Interaction between predicted YS and welding current for 316L filler metal

### Deviation Analysis

From Table 3 the analysis and comparison between the predicted and experimental values reveal that the deviations between them were very low and quite within acceptable range. This demonstrates the validity of the models.

**Table 3 Comparison of model predicted and experimentally estimated values**

Welding current (A)	Filler Metal Type	YS (MPa)		Error (%)
		Estimated	Predicted	
91	308L	796.421	783.000	1.714
	309L	802.157	831.000	-3.471
	316L	771.764	817.000	-5.537
92	308L	782.940	773.000	1.286
	309L	786.962	813.000	-3.203
	316L	771.644	814.000	-5.203
93	308L	769.461	763.000	4.385
	309L	771.764	795.000	-2.923
	316L	771.523	811.000	-4.868
94	308L	764.345	753.000	1.507
	309L	750.383	777.000	-2.548
	316L	766.664	808.000	-5.116
95	308L	756.313	743.000	1.792
	309L	729.002	759.000	-3.952
	36L	756.947	805.000	-5.969

### Statistical analysis

The calculated  $R^2$  values of the developed models (Figures 1-6) were generally above 90% indicating that the regression models are quite adequate and hence the validity of the models. Furthermore the correlation coefficients (R) calculated from  $R^2$  above shows a better correlation (1.00) with model-predicted YS than that obtained by experiment. This suggests that the model predicts more accurate and reliable values than the actual experiment. This further strengthens the indication of the model validity.

### Scatter Diagrams

The validity of the model was further tested by drawing scattered diagrams Figures 7-9. From the diagrams it can be observed that the values of the estimated and predicted values are scattered close to the 45° line, demonstrating an almost perfect fit of the developed models. This agrees with

the views of Kim, Son, Yang and Yaragada, (2003) and Palanivel, Mathews and Murugan, (2011), thereby further establishes the adequacy and validity of the derived models.

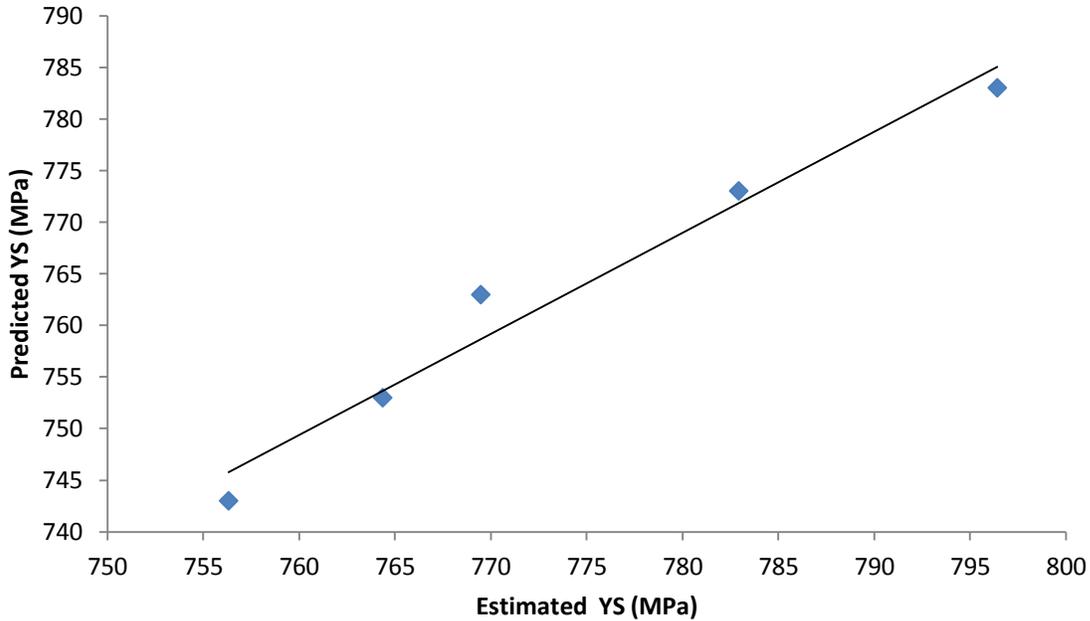


Figure 7 Scatter diagram of the YS for 308L filler metal

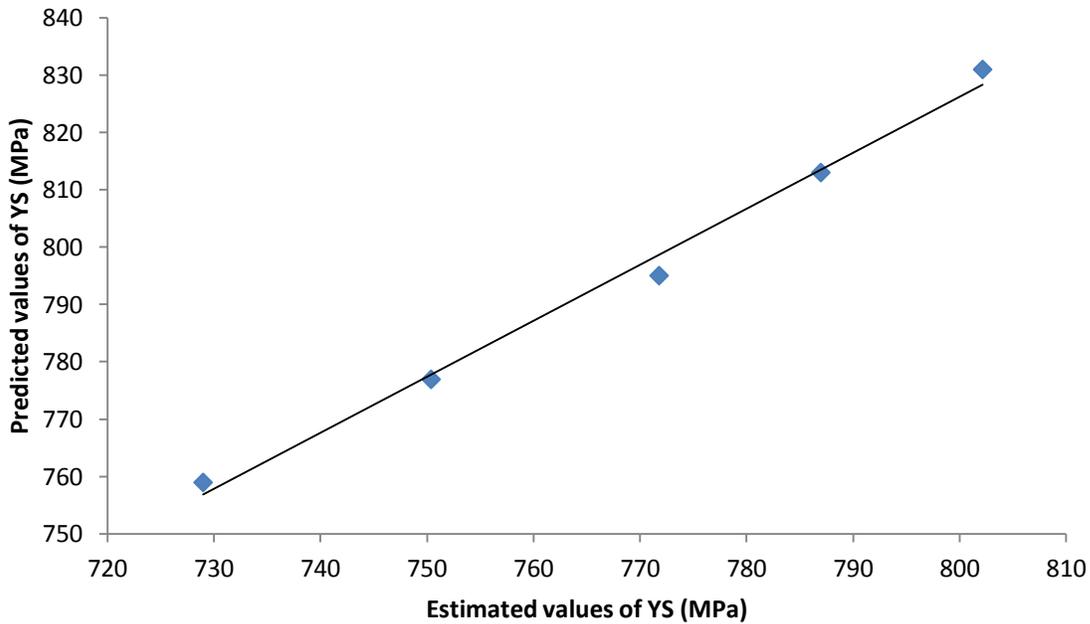


Figure 8 Scatter diagram of the YS for 309L filler metal

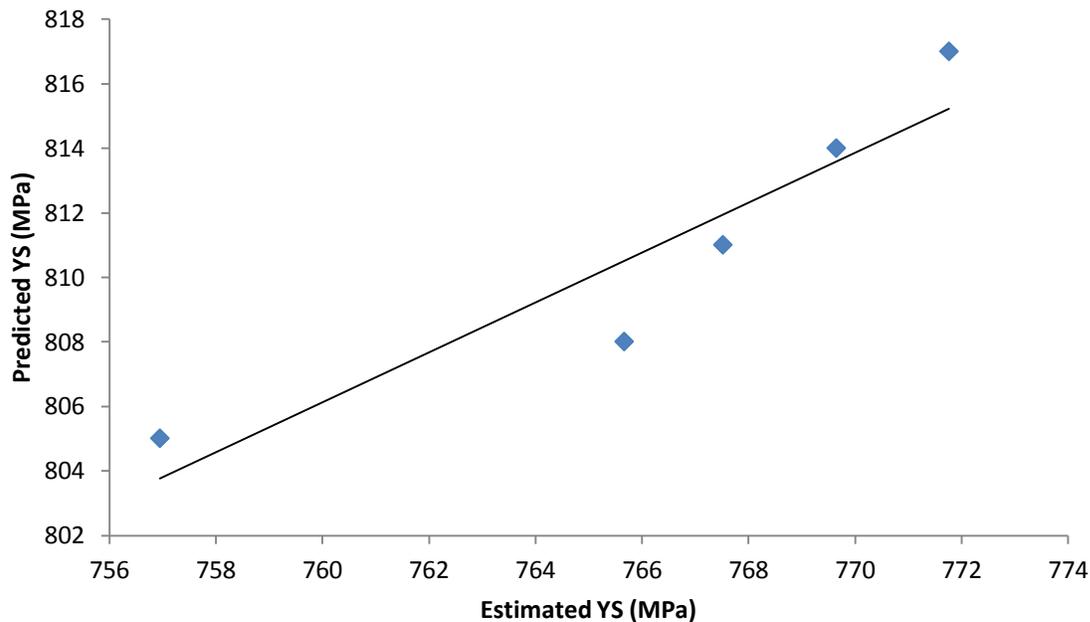


Figure 9 Scatter diagram of YS for 316L filler metal

## CONCLUSION

The results of tests of the validity and accuracy of the model showed that the developed model is reasonably accurate. The model-predicted YS values were in proximate agreement with the values obtained from the actual experiment. Therefore the developed model can be used to predict the YS values of TIG welded austenitic stainless steel weld joints.

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