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DEVELOPMENT OF A DYNAMIC PROGRAMMING MODEL FOR OPTIMIZING PRODUCTION PLANNING

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ABSTRACT: *Production planning is the backbone of any manufacturing operation, and its main objective is to determine the quantity of products to be produced and inventory level to be carried from one period to the other, with the objective of minimizing the total costs of production and the annual inventory, while at the same time meeting the customers' demand. A mathematical model was developed for a multi-product problem using Dynamic Programming approach and the solution procedure proposed by Wagner and Whitin was adopted. The model is very useful in solving a problem with multi-stage problem, a particular situation in which there is appreciable variation in average periodic demand and availability of raw materials among the different periods. It also stipulates the minimum quantities of the product to produce per period and the corresponding inventory levels such that total production cost is minimized over the planning periods.*

KEYWORDS: Cost, Dynamic, Inventory, Minimum, Model, Production.

INTRODUCTION

Operations research methods are supposed to develop and analyze mathematical models of systems that incorporate factors, such as chance and risk, to predict and compare the outcomes of alternative decisions [1]. In operations research there is modelling of complex systems, analysis of system models using mathematical and statistical techniques, and application of the techniques to engineering problem paradigms. The resulting models help decision makers determine policy, allocations, and the best courses of action in the control of complex systems and makes planning and scheduling easier.

Production planning and scheduling are decision making processes that are used on a regular basis in many manufacturing and service industries, these forms of decision making play an important role in procurement and production, in transportation and distribution, and in information processing and communication [2]. The planning and scheduling functions in a company rely on mathematical techniques and heuristic methods to allocate limited resources to the activities that have to be done. This allocation of resources has to be done in such a way that the company optimizes its objectives and achieves its goals. Monitoring feedback and control is the final and most crucial aspect of production planning and control activities. At this stage, from the system's inbuilt feedback arrangement, it is basically determined if the system in implementation maintains standard output, quality and cost, so as to help achieve the firm's objective of minimizing both production cost and resource wastages and finally maximizing production output and profit.

One of the problem often encountered in production planning in industries with large product demand, is production planning requirement. The problem is that of determining the quantity to be

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produced and the inventory quantity to be carried, such that the demand of each period will be met at minimum total production cost. The above problem has characteristics of dynamic programming problem [3].

A general dynamic programming model can be easily formulated for a single dimension process from the principle of optimality. The programming situation involves a certain quantity of economic resources (space, finance, people, and equipment) which can be allocated to a number of different activities [2]. Dynamic programming is handy in solving a problem with multi-stage problem, a particular situation in which there is appreciable variation in average monthly demand and availability of raw materials among the different periods under consideration [4]. A general Dynamic Programming Algorithm; is applicable in a situation in which there is absence of shortage, the inventory model is based on minimizing the sum of production and holding cost for all periods and it is assumed that the holding cost for these periods is based on end of period inventory [4].

An inventory is the quantity of commodity that a business must maintain to ensure smooth operation, with the goal of minimizing the total cost of inventory [5]. Inadequate inventory can lead to undue costs, production delays and inefficiencies including lost orders or even loss of customers. More than adequate inventories results into excessive inventory holding cost and the control of production activities rest firmly upon the control of inventories, quality and cost. Typically, holding costs are estimated to cost approximately 15-35% of the material's actual value per year [6]. The primary factors that drive this up include additional rent needed, great insurance premiums to protect inventory, opportunity costs, and the cost of capital to finance inventory. The standard "rule of thumb" for inventory carrying cost is 25% of inventory value on hand and the cost of capital is the leading factor in determining the percentage of carrying cost.

Specifically Wagner and Whitin dynamic programming inventory model solution procedure was adopted in this study and it is characterised by three types of equations, namely; Initial conditions, a recursive relation and an optimal value function [2]. They used dynamic economic lot size model as a guide to formulating a model that will handle both the production rate and inventory levels simultaneously [7]. The decision variables are the production rate and inventory levels. Because the number of combinations in general can be as large as the product of the number of possible values of the respective variables, the number of required calculations tends to "blow up" rapidly when additional state variables are introduced (Hillier and Lieberman, 2001), this phenomenon is known as "curse of dimensionality".

METHODOLOGY

Model Assumption.

The following assumptions were set to construct the mathematical model of the production planning problem.

1. The average periodic demand varies appreciably among the different periods.
2. Raw materials are available, but there is periodic change in their prices.
3. The model will handle both the production rate and inventory level simultaneously.
4. Multiple products are produced.
5. Only conservative of material constrained will be considered.
6. Single objective i.e minimizing total cost.
7. The model is deterministic.
8. Shortages are not allowed.

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9. Unit production cost vary from period to period
10. Unit holding cost is unchanged for all period.

Model Notations.

G	-	Total cost (Objective function)
z_{ij}	-	Quantity of product j produced in period i (Kg).
c_{ij}	-	Cost of producing one unit of product j for period i (N/Kg).
h_{ij}	-	Unit cost of storage of product j for period i (N/KG).
x_{ij}	-	Inventory of product j at the start of period i (KG).
$x_{j,\,i+1}$	-	Inventory of product j at the end of period i (KG).
d_{ij}	-	Demand of product j at period i (KG).
K_{ij}	-	Setup cost in period i for all j.

Model - 1

A production plan required which stated the quantities of each product *j* produced per period *i* so as to meet the demand for the period at a minimal total cost. The cost function was made up of two components (production and inventory costs).

The Production cost for product *j* in period *i* given by;

Production cost for product *j* = $c_{ij} z_{ij} + k_{ij}$ (1)

The cost of carrying *x* units of product *j* from period *i* to period *i + 1* given by;

Inventory cost of product *j* = $h_{ij} x_{j,i+1}$ (2)

The cost function for product *j* for period *i* given by;

$$(c_{ij} z_{ij} + k_{ij}) + h_{ij} x_{j,i+1} \quad (3)$$

The total cost for all the products over the planning horizon given by;

$$\sum_{j=1}^{J} \sum_{i=1}^{I} \left[(c_{ij} z_{ij} + k_{ij}) + h_{ij} x_{j,i+1} \right] \quad (4)$$

Model - 2

The only constraint of the problem is the material balance constraint. From fig.1, the sum of inventory brought into period *i*, and production at period *i* must be equal to the demand of period *i* plus inventory carried from period *i* to period *i* + 1. That is, sum of materials entering period *i* must be equal to the sum of materials leaving period *i*.

$$x_{ij} + z_{ij} = di_j + x_{j,i+1} \quad (5)$$

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$$\therefore\ x_{j,i+1} = x_{ij} + z_{ij} - d_{ij} \qquad (6)$$

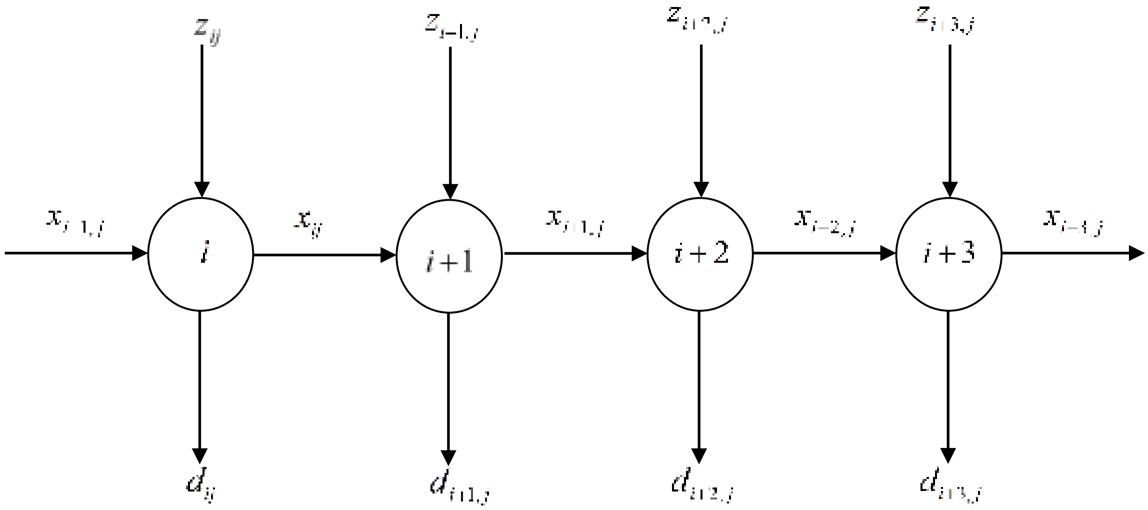

Fig 1: Stage diagram for the constraint.

The Proposed Mathematical Model

The planning problem may be stated as:

$$\text{Minimize (G)} = \sum_{j=1}^{J}\sum_{i=1}^{I}\left[\left(c_{ij}z_{ij} + k_{ij}\right) + h_{ij}x_{j,i+1}\right]$$

$$S.t,$$

$$x_{j,i+1} = x_{ij} + z_{ij} - d_{ij} \qquad (7)$$

for i = 1, 2,..........., I.

j = 1, 2,..........., J.

$$x_{ij}, z_{ij} \geq 0, \forall i, j$$

Solution Procedure

Wagner and Whitin solution procedure was used and under the given conditions it can be proved that:

1. Given the initial inventory $x_i = 0$, then at any period i of the I periods model, it is optimal to have a positive production quantity z_i^* or positive entering inventory x_i^* but not both; that is $z_i^* x_i^* = 0$.
2. The amount produced z_i at any period i is optimal only if it is zero or if it satisfies the exact demand of one or more succeeding periods. ($z_i = 0$, d_i, $d_i + d_{i+1}$, $d_i + d_{i+1} + d_{i+2}$ e.t.c). These succeeding periods are such that if the demand in period $i + m$ (< I) is satisfied by z_i^* then the demands of period i, i+1, i+2,....i+m-1, must also be satisfied.

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The solution procedure begins by finding the optimal policy for the first stage. The optimal policy for the first stage prescribes the optimal policy decision for each of the possible states at that stage. There was a recursive relationship that identifies the optimal policy for period *i*, given the optimal policy for period *i+1* is available. This recursive relationship is;

$$G_i(x_{j,i+1}) = \text{Minimize } ((c_{ij}z_{ij} + k_{ij}) + h_{ij}x_{j,i+1}) \qquad (8)$$

Therefore, finding the optimal policy decision at period *i* requires finding the minimizing value of x_i and the corresponding minimum cost is achieved by using this value of x_i and then following the optimal policy when you start at period $i + 1$. The precise form of the recursive relationship differs somewhat among dynamic programming problems.

The recursive relationship keeps repeating as we move from period to period. When the current period *i* is increased by 1, the new function is derived by using the $G_{i+1}(x_{j,i+1})$ function that was just derived during the preceding iteration, and then this process keeps repeating, until it finds the optimal policy starting at the final period. This optimal policy immediately yields an optimal solution for the entire problem.

Model Application

In Planning production of an item and considering four periods with periodic demands at d_1, d_2, d_3, and d_4 respective unit production cost per period taken as c_1, c_2, c_3, and c_4. Setup cost k_1, k_2, k_3, and k_4. Inventory holding cost h_1, h_2, h_3 and h_4. Hence, the production and inventory model as stated in equation (7) can be thus be expressed as;

$$\begin{aligned}\text{Minimize } (G) = & ((c_{11}z_{11} + k_{11}) + h_{11}x_{21}) + ((c_{21}z_{21} + k_{21}) + h_{12}x_{31}) + ((c_{31}z_{31} + k_{31}) + h_{13}x_{41}) \\ & + ((c_{41}z_{41} + k_{41}) + h_{14}x_{51}) + ((c_{12}z_{12} + k_{12}) + h_{21}x_{22}) + ((c_{22}z_{22} + k_{22}) + h_{22}x_{32}) \\ & + ((c_{32}z_{32} + k_{32}) + h_{23}x_{42}) + ((c_{42}z_{42} + k_{42}) + h_{24}x_{52}) \qquad (8)\end{aligned}$$

S.t;

$x_{21} = x_{11} + z_{11} - d_{11}$
$x_{31} = x_{21} + z_{21} - d_{21}$
$x_{41} = x_{31} + z_{31} - d_{31}$
$x_{51} = x_{41} + z_{41} - d_{41}$
$x_{22} = x_{12} + z_{12} - d_{12}$
$x_{32} = x_{22} + z_{22} - d_{22}$
$x_{42} = x_{32} + z_{32} - d_{32}$
$x_{52} = x_{42} + z_{42} - d_{42}$

Non negativity constraint:
$z_{11}, z_{21}, z_{31}, z_{41}, z_{12}, z_{22}, z_{32}, z_{42}, x_{21}, x_{31}, x_{41}, x_{51}, x_{22}, x_{32}, x_{42}, x_{52} \geq 0$

Model Validation

The model was applied to production planning in an animal feed mill. Two products were considered, the layers feed and the growers feed, over four (4) production periods in a year. Data for the period under consideration were collected and analyzed. Aggregate demand for four (4)

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quarters of the year, setup cost, unit production cost (labour cost, machine cost and cost of raw materials) and inventory holding cost was obtained from the company studied. The results show that the minimum total cost was achieved, with the productions in period 1, 2 and 4. While demand for period 3 were satisfied with inventory from period 2. The minimum total cost of this plan was ₦ 6,155,755.00 less than the existing plan. Excel spreadsheet was used in solving this particular problem.

CONCLUSIONS

Problem of optimizing production planning can be tackled, using a dynamic programming approach to make production and inventory level decisions with the objective of minimizing the total cost of production and the annual inventory cost, satisfying customers' demand. The use of Wagner and Whitin inventory model was adopted, because it stipulates the minimum quantities of the product to produce per period and the corresponding inventory levels such that total production cost is minimized over the planning period. The model can be applied for different period of planning horizon.

It is advisable that this type of model be adopted when dealing with making decisions on production and inventory levels for varying period. It helps, without exhaustive enumeration to determine the minimum quantities of product to produce to meet demand at the same time not incurring excessive storage cost by way of inventory in an attempt to meet all demand.

Operations research or management science techniques as used in this study are very useful tools in providing mathematically feasible solution to the problem of production planning. However, the management must still play a major role of reconciling the scientific solution with the environmental conditions to arrive at wise decisions.

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