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DESIGNING OF GENERALIZED TWO-PLAN SYSTEM WITH REFERENCE SAMPLING PLAN

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ABSTRACT: *This paper proposes a generalized two-plan sampling system with multiple repetitive group sampling plan as the reference plan, designated as GTPMRGSS (n; c_N, c_T), is introduced. The efficiency of GTPMRGSS (n; c_N, c_T) with respect to smaller sample sizes have been established over the attributes scheme. The sampling inspection scheme will be useful when testing is costly and destructive. The advantages of the sampling inspection scheme over attributes single, double and various reference plans are discussed. Tables are constructed considering various combinations of acceptable and limiting quality levels and with their operating ratios.*

KEYWORDS: Acceptable Quality Level, Limiting Quality Level, Markov model, Operating ratio, Two-Plan sampling system.

INTRODUCTION

A sampling system consists of two or more sampling plans and rules for switching between them to achieve the advantageous features of each. Govindaraju and Subramani (1992) have proposed a two plan system which is an acceptance sampling plan involving only normal and tightened inspection. A normal inspection is carried out when there are good quality products and it is switched to tightened inspection when there is deterioration in quality of products. The two plan system using different switching criteria to achieve the desired discrimination on operating characteristic (OC) curve was earlier investigated by Dodge (1965), Hald and Thyregod (1966) and Stephens and Larson (1967). Calvin in (1977) has proposed the zero acceptance number tightened-normal-tightened which is a special case of the two plan system towards application of attributes characteristics. Vijayaraghavan and Soundararajan in (1996) has investigated the performance of another type of two-plan system designated as TNT (n; c_1, c_2) scheme. The advantage of the two plan systems is that it gives desired protection with minimum sample size.

The sampling system utilizes two zero acceptance number single sampling plans of different sample sizes, together with switching rules to build up the shoulder of the operating characteristic (OC) curve. Assuming acceptance number, c, to take values other than zero, the generalized two plan sampling system can be designated as GTPS (n, kn; c), which refers to a GTPS scheme where the normal and tightened single sampling plans have the acceptance number, c, but, on tightened inspection, the sample size is n(< kn). Another way of defining the Two Plan sampling system is to say that the normal and tightened plans utilize the same sample size n but with different acceptance numbers. This type of generalized two plan system can be designated as GTPS (n; c_N,c_T) which refers to a GTPS where the normal and tightened plans have fixed sample size n but different acceptance numbers, say c_N and c_T (< c_N) respectively.

In this paper, a Generalized Two Plan System with Multiple Repetitive Group Sampling plan as the reference plan, designated as GTPMRGSS (n; c_N,c_T) is introduced, where n is the sample size under the reference plan, and c_N and c_T are the acceptance constants corresponding to normal and tightened plans respectively. The efficiency of GTPMRGSS (n; c_N,c_T) with respect to smaller sample sizes has been established over the attribute sampling scheme. The proposed plan may more economical and cost effective one which is used for destructive items to find the non-defective item using the various sampling plan. Switching rules based plans are generally advantageous than classical sampling plan in terms of sample size efficiency and shoulder effect on OC curve.

GENERALIZED TWO PLAN SYSTEM

Dodge (1969) has proposed a sampling inspection involving normal and tightened inspection plans which are usually referred as a generalized two-plan system. This system is largely incorporated in MIL-STD-105E (1989), which forms an integrated sampling inspection system guaranteeing the consumer that the outgoing quality will be better than the specified AQL and at the same time assuring the producer that the risk for rejection will be smaller for products of AQL quality or better.

Kuralmani (1992) has designed two-plan switching system involving acceptable and limiting quality levels. The procedure with a pair of plans gives an overall OC curve that generally lies in between the OC curve of the normal and tightened plans in a Two-Plan switching system. Balamurali and Chi-Hyuck Jun (2009) have made contributions to designing of a variables two-plan system by minimizing the average sample number (ASN). Suresh (1993) has proposed procedures to select certain reference plans indexed through producer and consumer quality levels considering filter and incentive effects. The concept of Repetitive Group Sampling (RGS) plan was introduced by Sherman (1965) which acceptance or rejection of a lot is based on the repeated sample result of the same lot. The operation of the plan is similar to that of sequential sampling plan. According to Sherman, the RGS plan gives minimum sample size as well as desired protection. Ramasamy (1983) made contributions towards the construction of RGS plans.

Gauri Shankar and Joseph (1993) have developed another new RGS plan as extension of the Conditional RGS plan in which the acceptance or rejection of a lot on the basis of repeated sample results is dependent on the outcome of inspection under RGS inspection system of the preceding lots. In this paper, the proposed plan will be designated as Multiple Repetitive Group Sampling plan.

Selection of Sampling Plan:

Conditions for Application

1. The production is steady and the results on current, preceding and succeeding lots are broadly indicative of a continuing process.
2. Lots are submitted substantially in the order of production.
3. The product comes from a source in which the consumer has confidence.

Operating Procedure

Switching rules for generalized Two-plan Systems are:

Normal to Tightened

When normal inspection is in effect, tightened inspection shall be instituted (when‘s’ out of ‘m’ consecutive lots or batches are rejected on original inspection (s<m)).

Tightened to Normal

When tightened inspection is in effect, normal inspection shall be reinstated (when‘d’ consecutive lots or batches are accepted on original inspection).

A diagrammatic representation for the switching rules to a generalized two-plan system is shown in Figure1.

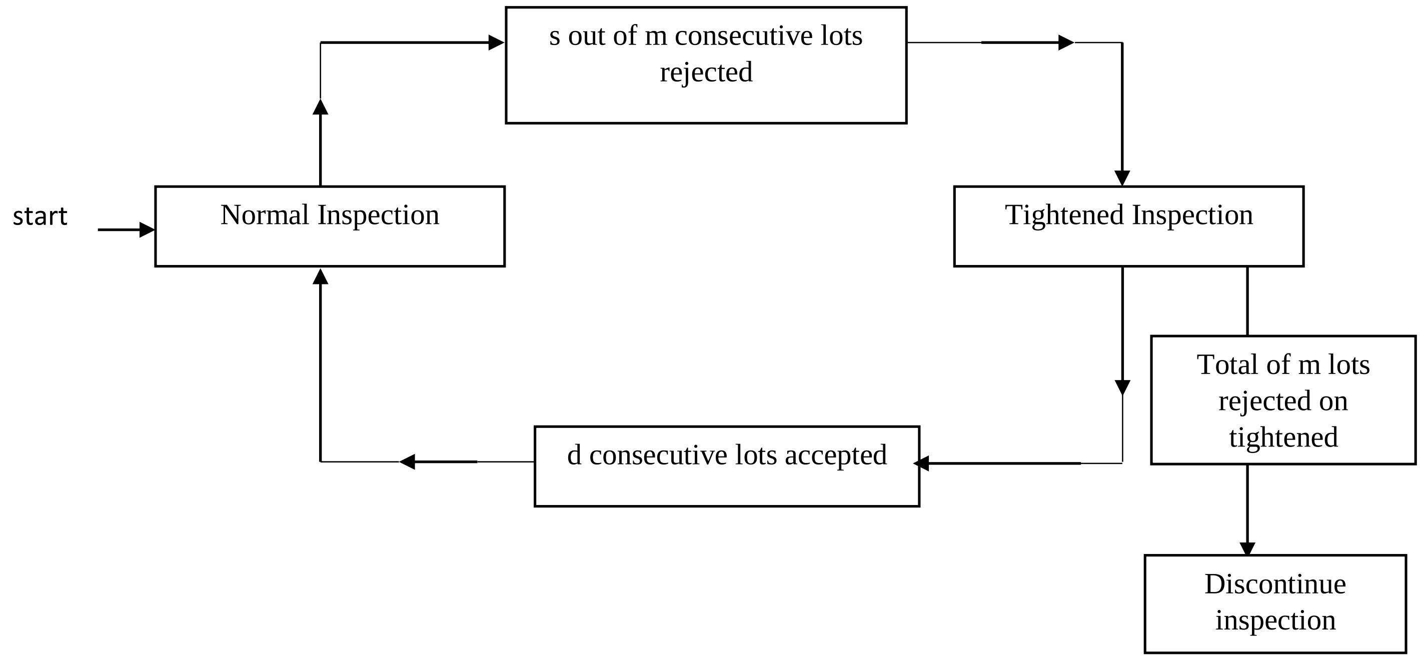

Figure 1. Switching rules for a Generalized Two-Plan System

A number of important measures of performance are to be determined and used in the evaluation of OC function which will be discussed.

P_N = the proportion of lots expected to be accepted under normal inspection.

P_T = the proportion of lots expected to be accepted under tightened inspection.

I_N = the expected proportion of lots inspected on normal inspection.

I_T = the expected proportion of lots inspected on tightened inspection.

Using above measures, the composite operating characteristic function can be determined as,

$$P_a(p) = I_N P_N + I_T P_T \qquad (1)$$

Where

n_N = the (average) sample size for the normal inspection plan.

n_T = the (average) sample size for the tightened inspection plan.

The method for deriving various measures of performance for the Generalized Two- Plan System are also studied.

MARKOV CHAIN AND SAMPLING SYSTEM

In order to represent the generalized two-plan system as a Markov chain, a set of states (events) is defined so as to completely describe the operation of the system. These events are mutually exclusive since at any trial, the state of the system is described by one and only one event. Moreover, these events have Markov property in the sense that at any trial the probability of being in a particular state depends only on the state occupied at the previous trial.

The events and definitions are given below:

Ni = the event that the normal inspection is in effect (i=1, 2,…, m)

Ti = the event that the tightened inspection is in effect (i=1, 2,…, d)

P_{Ni} = the probability of the system being in state Ni (i=1,2, …, m)

P_{Ti} = the probability of the system being in state Ti (i=1,2, …, d)

For the sake of convenience, let us denote P_N and P_T as ‘a’ and ‘b’ respectively for evaluating the above measures. The transition probabilities of the generalized two-plan system are shown in Figure 2.

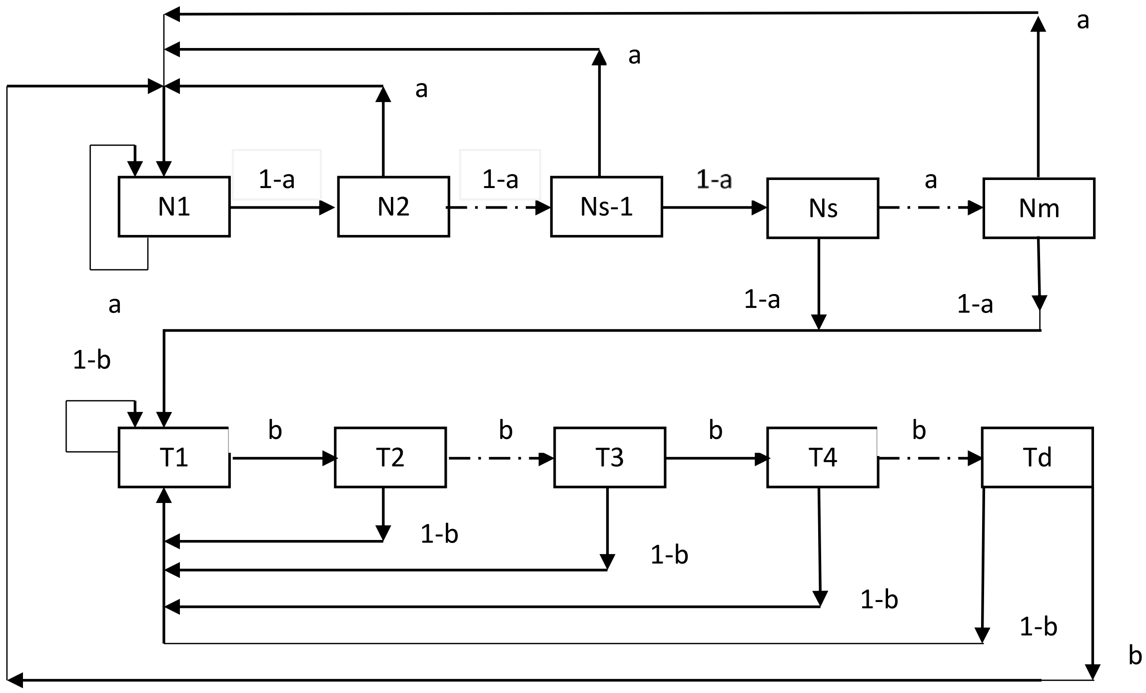

Figure 2. Transition Probabilities for a Generalized Two-Plan System

Under the assumption of constant product quality p, it is not difficult to see that the Markov chain defined by the transition matrix in Table 1 is irreducible and aperiodic, and thus sufficient conditions are satisfied for the Markov chain to possess a non-zero stationary distribution. Thus the Markov chain is ergodic, ie., the limiting distribution is the same as the stationary distribution. These conditions are discussed for the derivation of the following necessary formulas.

It can be seen that normal inspection is derived by the union of the events, Ni, i = 1,2,..,m. Thus the expected proportion of time that normal inspection is in effect is given as,

$$I_N = \sum_{i=1}^{m} P_{Ni} \tag{2}$$

Similarly, the expected proportion of time that tightened inspection is in effect is given as,

$$I_T = \sum_{i=1}^{d} P_{Ti} \tag{3}$$

All the probabilities in these expressions are limiting state probabilities. As discussed above, due to the ergodicity of the Markov chain, the limiting probabilities are same as the stationary of well known equations,

$$P_{sj} = \sum_{i=1}^{\infty} P_{Si} P_{ij} \quad ; j = 1,2,\ldots.$$

and $$\sum_{j=1}^{\infty} P_{Sj} = 1 \quad (4)$$

where P_{ij} are the one step transition probabilities from states Si to Sj. P_{si} is stationary and hence limiting probability of the system being in state S_i .

DERIVATION OF THE OC AND ASN FUNCTIONS

The OC and ASN functions can be derived using the transition probabilities given in Table 1, and equations (4) yield equations (5) through (12),

State i

Table 1. Transition probability matrix for a Generalized Two plan system

$$P_{Ni} = (1-a)^{i-1}P_{N1} \text{ ; } i = 1,2,3,\ldots,s \quad (5)$$

$$= a^{i-s}(1-a)^{s-1} P_{N1}\text{; } i = s+1,s+2,\ldots,m$$

$$P_{Ti} = b^{i-1} P_{T1} \text{ ; } i = 1,2,\ldots,d \quad (6)$$

All probabilities can now be evaluated using the condition that the sum of all probabilities equals to one,

ie, $$I_N + I_T = 1 \quad (7)$$

one can get,

$$I_N = \sum_{i=1}^{s}(1-a)^{i-1}P_{N1} + \sum_{i=s+1}^{m} a^{i-s}(1-a)^{s-1}P_{N1}$$

on simplification,

$$I_N = \frac{P_{N1}[1+(1-a)^{s-2}(2a-a^{m-s+2}-1)]}{a} \quad (8)$$

$$I_T = \frac{P_{N1}(1-a^{m-s+1})(1-b^d)(1-a)^{s-1}}{(1-b)b^d} \quad (9)$$

Substituting equations (8) and (9) in (7), we have

$$P_{N1} = \frac{a(1-b)b^d}{A+B} \quad (10)$$

where $A = (1-b)b^d[1+(1-a)^{s-2}(2a-a^{m-s+2}-1)]$

$B = a(1-a^{m-s+1})(1-b^d)(1-a)^{s-1}$

Again substituting equation (10) in (8) and (9), we have

State i-1

	N1	N2	N3	…	Ns	Ns+1	Ns+2	…	Nm	T1	T2	T3	…	Td-1	Td
N1	a	1-a	…	…	…	…	…	…	…	…	…	…	…	…	…
N2	a	…	1-a	…	…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
Ns-1	a	…	…	…	1-a	…	…	…	…	…	…	…	…	…	…
Ns	…	…	…	…	…	a	…	…	…	1-a	…	…	…	…	…
Ns+1	…	…	…	…	…	…	a	…	…	1-a	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
Nm-1	…	…	…	…	…	…	…	…	a	1-a	…	…	…	…	…
Nm	a	…	…	…	…	…	…	…	…	1-a	…	…	…	…	…
T1	…	…	…	…	…	…	…	…	…	1-b	b	…	…	…	…
T2	…	…	…	…	…	…	…	…	…	1-b	…	b	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
Td-1	…	…	…	…	…	…	…	…	…	1-b	…	…	…	…	b
Td	…	…	…	…	…	…	…	…	…	1-b	…	…	…	…	…

$$I_N = \frac{\mu}{\mu + \tau} \tag{11}$$

$$I_T = \frac{\tau}{\mu + \tau} \tag{12}$$

where,

$$\mu = \frac{1 + (1-a)^{s-2}(2a - a^{m-s+2} - 1)}{a(1 - a^{m-s+2})(1-a)^{s-1}}$$

$$\tau = \frac{1 - b^d}{(1-b)b^d}$$

Substituting equations (11) and (12) in (1) with a as P_N and b as P_T, the composite OC function obtained as

$$P_a(p) = \frac{\mu P_N + \tau P_T}{\mu + \tau} \tag{13}$$

Where,

P_N = Probability of acceptance under the normal inspection.

$P_N = p(d \le c_N / n,p)$

P_T = Probability of acceptance under the tightened inspection.

$P_T = p(d \le c_T / n,p)$

Note that where μ and τ are the average number of lots inspected using normal inspection before going to tightened inspection and average number of lots inspected using tightened inspection before going to normal inspection respectively.

Selection of Gtpmrgss

The MRGS plan is an extension of Conditional Repetitive Group Sampling plan in which acceptance or rejection of a lot on the basis of repeated sample results is dependent on the outcome of inspection under a Repetitive Group Sampling inspection system of the preceding lots. Further they derived the formulae for OC and ASN functions. An attempt has been made to model and analyse the dynamics of the proposed inspection system through GERT approach.

Operating Procedure

Following the notations similar to those of Sherman, the Multiple Repetitive Group Sampling plan is carried out through the following steps;

Step 1: Draw a random sample of size n and determine the number of defectives d found there in.

Step 2: Accept the lot, if $d \le c_1$.

Reject the lot, if $d > c_2$.

Step 3: If $c_1 < d \le c_2$, repeat the steps 1, 2 and 3 provided i successive previous lots are accepted, under RGS inspection system, otherwise reject the lot.

Both the plans are characterized through four parameters, namely n, c_1, c_2 and acceptance criteria i. Here, it may be noted when $c_1=c_2$ the resulting plan is single sampling plan. Also when i=0, this plan becomes RGS plan due to Sherman. Further, it may be noted that the conditions for the applications of the proposed plan is same as Sherman RGS plan.

OPERATING CHARACTERISTICS FUNCTION

The operating characteristic function $P_a(p)$ of Multiple Repetitive Group Sampling plan is derived by poisson model as,

$$P_a(p) = \frac{p_a(1-p_c)^i}{(1-p_c)^i - p_c p_a{}^i}$$

where, $$p_a = p[d \le c_1] = \sum_{r=0}^{c_1} \frac{e^{-x}(x)^r}{r!}$$

$$p_c = p[c_1 < d < c_2] = \sum_{r=0}^{c_2} \frac{e^{-x}(x)^r}{r!} - \sum_{r=0}^{c_1} \frac{e^{-x}(x)^r}{r!} \text{ and } x = np.$$

Designation

GTPMRGSS (n; c_{1N}, c_{2N}; c_{1T},c_{2T}) and i refers to Generalized Two Plan System of type I (n;c_N,c_T) where the normal MRGS plan has a sample size n and acceptance number c_{1N}, c_{2N} ($c_{1N}<c_{2N}$) and the tightened MRGS plan has a sample size n and acceptance number c_{1T}, c_{2T}($c_{1T}<c_{2T}$, $c_{1T}\le c_{1N}$ and $c_{2T}\le c_{2N}$).

Designing Gtpmrgss with Different Parameters

The OC curve for a GTPMRGSS can be constructed using table 2. This can be done by dividing each entry for the given values of c_{1N}, c_{2N}, c_{1T}, c_{2T} and i by the corresponding value of sample size n. The result of each division is the number of non-conformities per unit for which the $P_a(p)$ is shown below.

For example, when n=25, c_{1N}=1, c_{2N}=4, c_{1T}=1, c_{2T}=2, when i=1 division of each of the entries in the c_{1N}=1, c_{2N}=4, c_{1T}=1, c_{2T}=2, when i=1 row of table 2 by 25 leads to the following values,

$P_a(p)$	0.99	0.95	0.90	0.50	0.10	0.05	0.01
P	0.0152	0.0293	0.0376	0.0777	0.1563	0.1896	0.2629

For plotting OC curve the Generalized Two Plan Multiple Repetitive Group Sampling System (25; 1,4; 1,2) when i=1.

The OC curve has been obtained using SAS program for the values generated (Figure. 3)

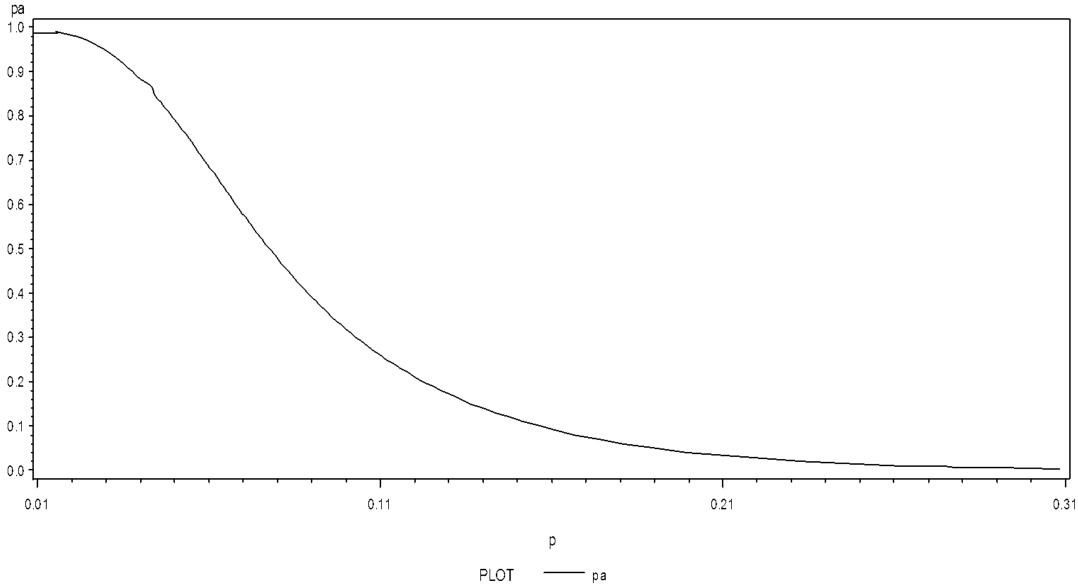

Figure 3. OC curves of Generalized Two Plan Multiple Repetitive Group Sampling System

Designing Systems for given p_1, α, p_2 and β

Table 3 can be used to design Generalized Two Plan Multiple Repetitive Group Sampling System (GTPMRGSS), when two points on the OC curve (p_1, 1-α) and (p_2,β) are given. To design a GTMRGSS calculate the Operating Ratio (OR) = p_2/p_1. From table 3 one can determine the value of OR which is nearest to the desired ratio. Corresponding to the selected OR values of c_{1N}, c_{2N}, c_{1T}, c_{2T}, i and np_1 when i=1. The sample size is determined thus dividing np_1 by p_1.

For example, let p_1=0.06, α=0.05, p_2=0.16, and β=0.05, calculate the Operating Ratio (OR) = p_2/p_1 = 0.16/0.06 = 2.6666. From the table 3 the value of OR for α= 0.05, β=0.05 which is nearest to the desired ratio is 2.668. Corresponding to this selected OR values is c_{1N}=1, c_{2N}=2, c_{1T}=0, c_{2T}=1, i=1 and np_1=1.131. The sample size is obtained as n= np_1/p_1=1.131/0.06=18.85≈19. The desired system is GTMRGSS (19; 1,2 ; 0,1) when i=1.

Construction of Tables

The expression for probability of acceptance of Generalized Two Plan Multiple Repetitive Group Sampling System (GTMRGSS), under the assumption of Poisson model, the composite OC function equation (13) is given as,

$$P_a(p) = \frac{\mu P_N + \tau P_T}{\mu + \tau}$$

$$P_N = \frac{p_a(1-p_c)^i}{(1-p_c)^i - p_c p_a{}^i} \qquad (14)$$

where $p_a = \sum_{r=0}^{c_{1N}} \frac{e^{-x}(x)^r}{r!}$ $\quad p_c = \sum_{r=0}^{c_{2N}} \frac{e^{-x}(x)^r}{r!} - \sum_{r=0}^{c_{1N}} \frac{e^{-x}(x)^r}{r!}$ and x = np.

$$P_T = \frac{p_a(1-p_c)^i}{(1-p_c)^i - p_c p_a{}^i} \qquad (15)$$

where $p_a = \sum_{r=0}^{c_{1T}} \frac{e^{-x}(x)^r}{r!}$ $\quad p_c = \sum_{r=0}^{c_{2T}} \frac{e^{-x}(x)^r}{r!} - \sum_{r=0}^{c_{1T}} \frac{e^{-x}(x)^r}{r!}$ and x = np.

For various assumed values of c_{1N}, c_{2N}, c_{1T}, c_{2T}, s, m, d, i and $P_a(p)$ the equation (13) is solved with equation (14) and (15) for np using iteration techniques for different values of c_{1N}, c_{2N}, c_{1T}, c_{2T}, s, m, d and i. From table 2 various incoming quality levels, outgoing quality levels and operating ratio values are calculated for different α and β values which are given in table 3.

CONCLUSION

An attempt is made towards the concept of Two Plan Multiple Repetitive Group Sampling System (TPMRGSS) in which disposal of a lot is on the basis of normal and tightened plans. Poisson unity values have been tabulated for a wide range of plan parameters. Whenever one finds the OC curve for attribute plan to be unsatisfactory, then its shape can be improved by using two-plan system provided here, which allows switching between two kinds of sampling system similar to the case of attributes normal and tightened inspection schemes. The design parameters such as number of acceptance numbers are determined by satisfying the producer, consumer, and engineers at various incoming and outgoing quality levels. On comparing the performance of the sampling procedures the techniques are proved to be better than existing ones. Further studies may consider involving variable sampling plan and reliability of sampling plans.

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s	m	d	c_N	c_T	c_{1N}	c_{2N}	c_{1T}	c_{2T}	Probability of Acceptance						
									0.99	0.95	0.90	0.50	0.10	0.05	0.01
2	2	0	1	0	0	3	0	2	0.127	1.230	1.661	1.217	2.354	3.005	4.651
1	1	0	1	0	0	4	0	3	0.132	0.832	0.965	1.494	2.399	3.014	4.708
2	2	0	1	0	1	2	0	1	0.986	1.131	0.957	1.624	2.327	3.017	4.559
1	2	0	2	0	1	3	0	1	0.084	0.811	1.172	1.628	2.331	3.013	4.573
1	2	0	1	0	1	3	0	2	0.105	1.097	1.455	1.217	2.344	3.034	4.524
1	1	0	1	0	1	3	0	2	0.101	1.079	1.198	1.217	2.352	3.007	4.650
2	2	0	3	0	1	4	1	3	0.497	1.925	1.833	2.226	3.932	4.757	6.549
1	2	0	1	0	1	4	1	2	0.380	0.732	0.940	1.942	3.908	4.740	6.572
2	2	0	1	0	1	4	1	3	0.500	2.199	1.324	2.226	3.922	4.757	6.598
1	2	0	2	0	2	5	1	4	0.497	1.472	1.687	2.521	3.961	4.784	6.713
2	2	0	1	0	2	5	1	2	0.422	1.229	1.389	1.942	3.910	4.744	6.752
2	2	0	1	0	2	6	1	3	0.494	1.107	1.326	2.226	3.926	4.757	6.671
2	2	0	3	0	2	6	1	3	0.496	2.584	2.562	3.186	3.939	4.742	6.705
1	1	0	1	0	2	7	1	3	0.497	1.107	1.324	2.226	3.938	4.769	6.710
1	1	0	1	0	2	7	2	4	0.997	1.716	2.015	3.226	5.357	6.301	8.330
1	2	0	1	0	2	5	2	3	1.066	2.139	2.428	3.527	5.385	6.298	8.366
2	2	0	1	0	2	6	2	4	1.092	2.535	2.824	3.835	5.420	6.334	8.433
2	2	0	2	0	2	5	2	3	0.777	16.687	1.581	2.939	5.343	6.291	8.462
2	2	0	1	0	3	6	2	4	0.999	1.716	2.015	3.226	5.363	6.314	8.520
2	2	0	1	0	3	7	3	5	3.034	2.375	2.741	4.222	6.715	7.763	10.017
2	2	0	1	0	3	8	3	6	3.264	2.826	3.182	4.526	6.741	7.758	10.028
2	2	0	1	0	3	8	3	3	0.826	1.572	1.749	3.671	6.676	7.770	10.070
2	2	0	3	0	3	4	3	4	1.273	1.882	2.264	3.937	6.699	7.752	10.095
2	2	5	2	0	4	5	4	5	1.553	2.523	2.981	4.892	8.009	9.154	11.581
2	2	5	1	0	4	6	4	5	1.749	2.477	2.918	4.935	8.007	9.157	11.586
2	2	5	1	0	4	7	3	6	1.689	2.826	3.182	4.526	6.756	7.778	10.024
2	2	5	3	0	4	8	4	6	2.114	3.039	3.481	5.220	8.029	9.151	11.627

Table 2. Unity values for Generalized Two Plan System ($n;c_N,c_T$) with MRGS plan when i=1

Table 3. Operating Ratio values for Two Plan System ($n; c_N, c_T$) with MRGS Plan when i=1

s	m	d	c_N	c_T	c_{1N}	c_{2N}	c_{1T}	c_{2T}	p_2/p_1 for α =0.05			p_2/p_1 for α =0.01		
									α = 0.05 β = 0.10	α = 0.05 β = 0.05	α = 0.05 β = 0.01	α = 0.01 β = 0.10	α = 0.01 β = 0.05	α = 0.01 β = 0.01
2	2	0	1	0	0	3	0	2	1.914	2.443	3.781	18.535	23.661	36.622
1	1	0	1	0	0	4	0	3	2.883	3.623	5.659	18.174	22.833	35.667
2	2	0	1	0	1	2	0	1	2.057	2.668	4.031	2.360	3.060	4.624
2	2	0	3	0	1	4	1	3	2.043	2.471	3.402	7.911	9.571	13.177
1	2	0	1	0	1	4	1	2	5.339	6.475	8.978	10.284	12.474	17.295
2	2	0	1	0	1	4	1	3	1.784	2.163	3.000	7.844	9.514	13.196
1	2	0	2	0	2	5	1	4	2.691	3.250	4.560	7.970	9.626	13.507
2	2	0	1	0	2	5	1	2	3.181	3.860	5.494	9.265	11.242	16.000
2	2	0	1	0	2	6	1	3	3.547	4.297	6.026	7.947	9.630	13.504
2	2	0	3	0	2	6	1	3	1.524	1.835	2.595	7.942	9.560	13.518
1	1	0	1	0	2	7	1	3	3.557	4.308	6.061	7.924	9.596	13.501
1	1	0	1	0	2	7	2	4	3.122	3.672	4.854	5.373	6.320	8.355
1	2	0	1	0	2	5	2	3	2.518	2.944	3.911	5.052	5.908	7.848
2	2	0	1	0	2	6	2	4	2.138	2.499	3.327	4.963	5.800	7.723
2	2	0	2	0	2	5	2	3	0.320	0.377	0.507	6.876	8.097	10.891
2	2	0	1	0	3	6	2	4	3.125	3.679	4.965	5.368	6.320	8.529
2	2	0	1	0	3	7	3	5	2.827	3.269	4.218	2.213	2.559	3.302
2	2	0	1	0	3	8	3	6	2.385	2.745	3.548	2.065	2.377	3.072
2	2	0	1	0	3	8	3	3	4.247	4.943	6.406	8.082	9.407	12.191
2	2	0	3	0	3	4	3	4	3.560	4.119	5.364	5.262	6.090	7.930
2	2	0	2	0	4	5	4	5	3.174	3.628	4.590	5.157	5.894	7.457
2	2	0	1	0	4	6	4	5	3.233	3.697	4.677	4.578	5.236	6.624
2	2	0	1	0	4	7	3	6	2.391	2.752	3.547	4.000	4.605	5.935