

**DEPARTED TIME COMPENSATORS USING SMITH PREDICTOR****Deman Kosale<sup>1</sup> and Anita Khanna<sup>2</sup>**<sup>1</sup>Department of Electrical Engineering, Vishwavidhyalaya Engineering College Lakhanpur<sup>2</sup>Department of Electrical Engineering, Institute of Technology, Guru Ghasidas Central University, Bilaspur, Chhattisgarh (India)-495009.

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**ABSTRACT:** *Dead time is often present in control systems as computational or informational delay but in most cases it is very small and is neglected. Dead time is widely found in the process industries when transporting materials or energy. Generally stable processes are represented by first-order-plus-dead-time or second order-plus-dead-time models for analysis. The problem of control design for processes with dead time is quite crucial and long-standing. The advent of the Smith Predictor provided the industrial control community with another tool to tackle the control of processes where the presence of dead time was impairing closed-loop performance. In this paper analysis of stable processes with dead time is done. Here PI controller and Smith Predictor are used as dead time compensators. Also how to improve robustness and disturbance rejection points with respect to Smith Predictor has been also discussed.*

**KEYWORDS:** Process Dead Time, Smith Predictor (SP), Tuning.

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**INTRODUCTION**

All the feedback systems are generally represented by linear lumped parameters mathematical model. This is valid so long as the time taken for energy transmission is negligible i.e. the output begins to appear immediately on application of input. This is not quite true of transmission channel –lines, pipes, belts, conveyors etc. In such cases a definite time elapses after application of input before the output begins to appear. This type of pure time lag is known as transportation lag or dead time (I.J. Nagrath and M. Gopal, 1997). Dead times or time delays are found in many processes in industry. Dead times are mainly caused by the time required to transport mass, energy or information but they can also be caused by processing time or by the accumulation of time lags in a number of dynamic systems connected in series. Dead times produce a decrease in the system phase and also give rise to a non-rational transfer function of the system, making them more difficult to analyze and control (J.E. Normey-Rico and E.F. Camacho, 2007). A predictive PI controller is suitable for processes with long dead times. Compared to an ordinary PID controller it has advantage that it manages to predict the measurement signal even when the process has a long dead time and when the measurement signal is noisy (Tore Hagglund, 1992). Processes that contain a large transport lag ( $e^{-Ls}$ ) can be difficult to control because a disturbance in set point or load does not reach the output of the process until  $L$  units of time have elapsed. The control strategy is known as dead time compensating controller and is also referred to as a Smith Predictor. The control algorithm in a Smith Predictor is normally a PI controller (G.Saravanakumar, R.S.D.Wahidha Banu and V.I. George, 2006). The structure of Smith Predictor was devised to remove the delay effect from the closed loop design and is equivalent to IMC (Internal Model Control) in the sense that the

delayed behavior of the plant is cancelled by the plant model i.e. these methodologies lead substantially to a common structure for control systems with time delay (N.Abe and K.Yamanaka, 2003). The modified Smith Predictor with an integral mode has a simple structure which includes only three adjustable parameters that easily can be tuned manually (M.R. Matausek and A.D. Micic, 1996). It provides considerably faster load disturbance rejection than the modified Smith Predictor preserving the same set point response (Time -M. R. Matausek and A. D. Micic, 1999). The investigations of the control scheme with new virtual sensor have indicated that it can be used for elimination of the dead-time behavior in control systems with an integrator and a long dead-time with sufficient preciseness and low solution complexity (Alexander Dementjev, Denis Stein and Klaus Kabitzsch, 2009). Stable processes are those which possess pole (s) with  $\text{Re}(s) < 0$ . In this case two models are used, the first-order-plus-dead-time (FOPDT) model and second-order-plus-dead-time (SOPDT) model. The FOPDT model is represented by

$$P(s) = \frac{K_p}{1+Ts} e^{-Ls} \quad (1)$$

where  $K_p, T, L$  are real numbers.  $T > 0$  is the equivalent time constant of the plant and  $K_p$  is the static gain.  $L > 0$  is the equivalent dead time. When it is desirable to represent a smoother step response in the first part the transients or an oscillatory step response, a second-order process with a dead time is used

$$P(s) = \frac{K_p e^{-Ls}}{(1+T_1s)(1+T_2s)} = \frac{K_p e^{-Ls}}{1 + \frac{2\xi s}{\omega_n} + \frac{s^2}{\omega_n^2}} \quad (2)$$

where  $K_p, T_1, T_2, \xi, \omega_n$  and  $L$  are real numbers. As in the FOPDT model  $K_p$  is the static gain and  $L > 0$  the equivalent dead time.  $T_1 > 0$  and  $T_2 > 0$  are time constants of the plant in the case of a non-oscillatory response while the damping coefficient,  $\xi \in (0,1)$  and the natural frequency  $\omega_n > 0$  are used when the process exhibits an oscillatory step response.

## Dead Time Compensators

### PI Controller

When dead time is very small and for slow variations of the output signal PID control is a better choice but when dead time is long enough the control performance obtained with a proportional-integral-derivative (PID) controller is limited. Predictive control is required to control a process with a long dead time efficiently. Therefore, if a PID controller is applied on this kind of problems, the derivative part is mostly switched off and only a PI controller without prediction is used (Tore Hagglund, 1992). In an integral error compensation scheme, the output response depends in some manner upon the integral of the actuating signal. This type of compensation is introduced by using a controller which produces an output signal consisting of two terms, one proportional to the actuating signal and the other is proportional to its integral. Such a controller is called proportional plus integral controller. A PI controller is a special case of the PID controller in which the derivative (D) of the error is not used.

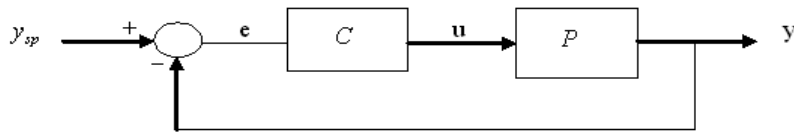


Figure 1: Block Diagram of PI control

In figure 1,  $y_{sp}$ ,  $e$ ,  $C$ ,  $P$  and  $y$  represent reference input, error signal, PI controller, process input, process model and process output respectively. The most famous tuning method for PI controllers is the Ziegler-Nicholas rule (ZN). It was developed using simulations with different systems where the equivalent dead time  $L$  and time constant satisfy the condition i.e.  $L/T < 1$  or called lag dominant systems. The ZN settings are benchmarks against which the performances of other controller settings are compared in many studies. This method starts by zeroing the integral gain and then raising the proportional gain until the system is unstable. The value of  $K_p$  at the point of instability is called  $K_{MAX}$  and the frequency of oscillation is  $f_0$ . This method then backs off the proportional gain a predetermined amount and sets the integral gain as a function of  $f_0$  (S.K. Singh, 2009).

Table 1: Ziegler-Nicholas settings for PI controller

Controller	$K_p$	$K_I$
PI controller	$0.45K_{MAX}$	$1.2f_0$

### The Smith Predictor

The most popular and very effective long dead- time compensator in use today is the Smith Predictor (O. J. Smith, 1959). Different modifications have been proposed to robustify the controllers based on the application of the Smith Predictor (C.C. Hang, K.W. Lim and B.W. Chong, 1989). This structure is shown in figure 2 and is known in literature as the “Smith predictor” (SP).

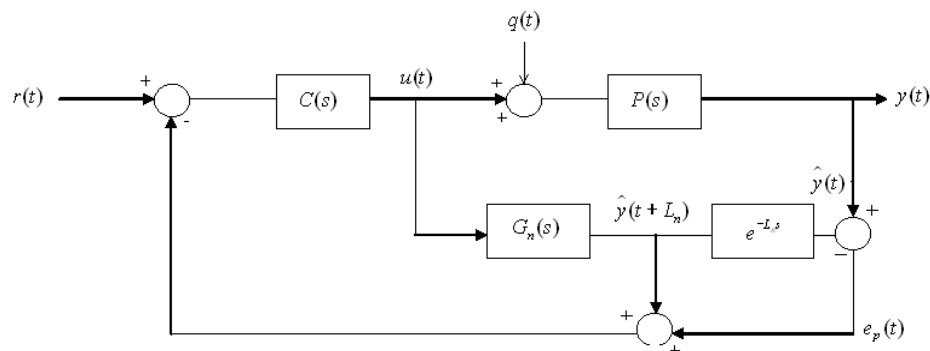


Figure 2: The Smith Predictor structure

In figure 2,  $P(s)$  is the real process given as  $P(s) = G(s)e^{-Ls}$ . A prediction model is formed in case of SP i.e.  $P_n(s)$ . The predicted model  $P_n(s)$  is generally equal to  $G_n(s)e^{-L_n s}$ . Now the difference of real process output and prediction model output is given by  $e_p(t)$ , where  $y(t)$  is the real process output and  $\hat{y}(t)$  is the predicted model output. With this structure, if there are no modeling errors or disturbances, the error between the real process output and the model output  $e_p(t)$  will be null and the controller can be tuned as if the plant had no dead time. This is the ideal case. But dead time errors can drive the SP to instability. The errors between the real and the predicted outputs are fed back to the controller in a periodic way. So when there is a change in set point is applied at  $t = t_0$ , the error between the real output and the predicted one  $e_p(t)$  will be zero until the instant  $t = t_0 + x$ , where  $x = \min(L_n, L)$ . This error signal is then fed back to the controller and its reaction will be perceived at  $e_p(t)$  only after  $x$  seconds. This error may cause closed loop instability but if this error is not fed back to the controller, the disturbances will not be rejected. Thus the effect of the dead time estimation error can be interpreted as the addition of the nominal response plus a periodic disturbance with a period approximately equal to  $\min(L_n, L)$ . A simple solution to this problem is to use a filter  $F_r(s)$  with unitary static gain  $F_r(0) = 1$  (J.E. Normey-Rico and E.F. Camacho, 2007). The filter should be designed to attenuate oscillations in the plant output especially at the frequency where the uncertainty errors are important. This can be done by low pass filter that increases the robustness of the controller. Therefore the modified Smith Predictor is shown in figure 3.

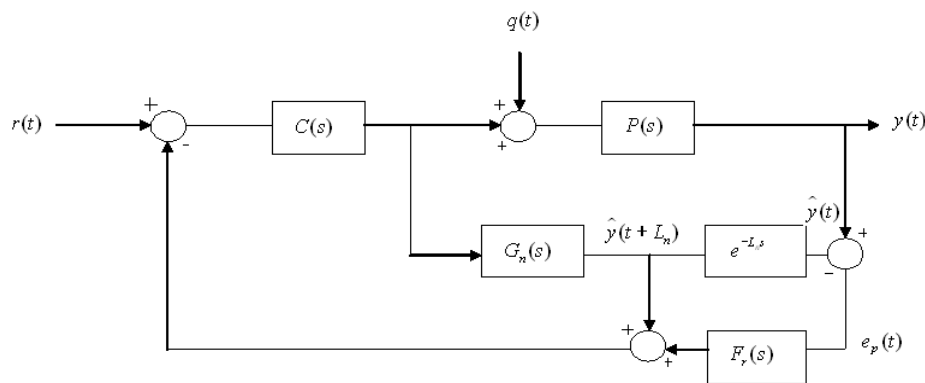


Figure 3: Structure of Filtered Smith Predictor (FSP)

When a dead time process is represented by FOPDT model Cohen and Coon (CC) has given an important method of tuning and is often used as an alternative to the Zeigler and Nicholas (Z-N) method (S.K. Singh, 2009). CC rule is an open-loop method in which the control action is removed from the controller by placing it in manual mode and an open loop transient is induced by a step change in the signal to the valve. Thus, this method is based on a single experimental test that is made with the controller in the manual mode. After inducing a small step change in the controller output, the process response is measured and recorded. This step response is also referred to as the process reaction curve. Figure 4, shows a typical S-shape process reaction curve showing graphical construction to determine first-order with transport

lag model. The S-shaped process reaction curve can be represented by a first-order with transport lag model and is given as

$$G_p(s) = \frac{K_s e^{-T_d s}}{Ts+1} \quad (3)$$

Using expression in equation 3, Cohen and Coon obtained the controller settings for PI controller is shown in table 2. Generally the controller which is used in the Smith Predictor  $C(s)$  is a PI controller.

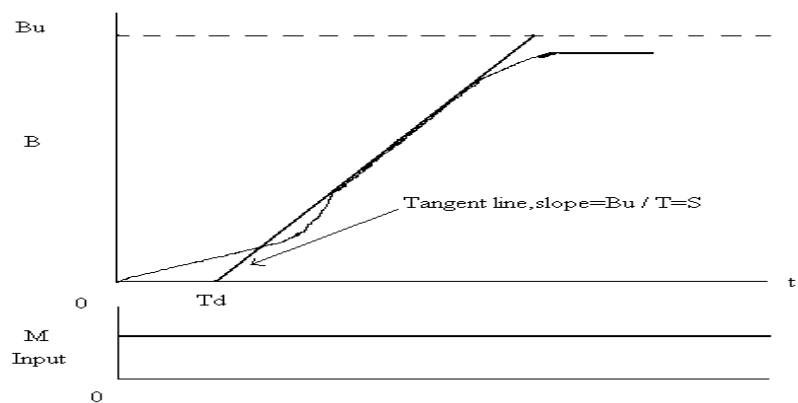


Figure 4: S-shape process reaction curve

Table 2: Cohen and Coon setting for PI controller

Controller	Parameter setting
Proportional-plus-integral (PI)	$K_P = \frac{1}{K_s} \frac{T}{T_d} \left( \frac{9}{10} + \frac{T_d}{12T} \right)$ $\tau_I = T_d \frac{30 + \frac{3T_d}{T}}{9 + \frac{20T_d}{T}}$

## Simulation Results

Here simulation results of three processes such as stirred tank heat exchanger, electric oven temperature control and coupled tank process are shown and discussed in detail. PI controller and Smith Predictor are used to control these processes and how the performance of these controllers is influenced by the variation in dead time is also discussed in this section.

## Stirred Tank Heat Exchanger

The FOPDT model of stirred tank heat exchanger (S.K. Singh, 2009) is

$$G(s) = \frac{e^{-0.0396s}}{0.202s + 1} \quad (4)$$

In the FOPDT of stirred tank heat exchanger the dead time is very small. Figure 5, shows its step response when PI controller is applied on it. Left part and right part of figure 5 shows responses for step changes in the reference signal  $y_{sp}$  and disturbance signal  $d$  respectively.

In this case tuning parameters are  $K_p = 0.01$  and  $T_i = 0.1$  which are chosen using Cohen and Coon tuning rule.

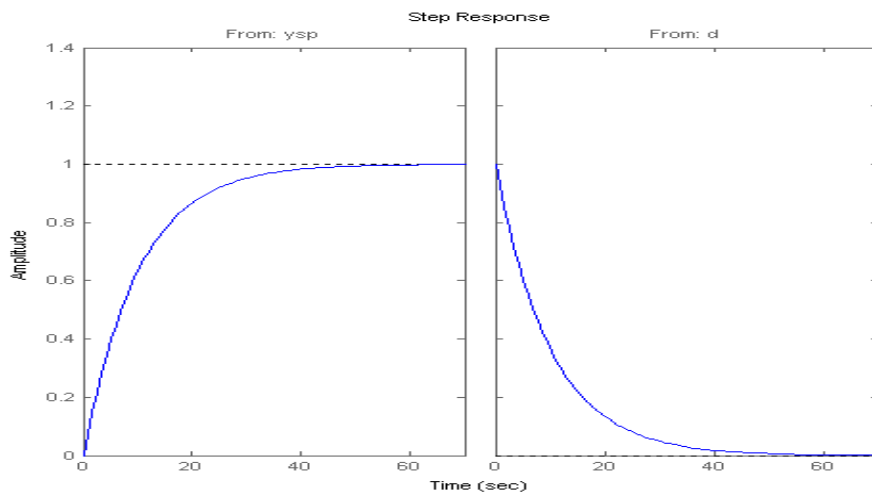


Figure 5: Step response with  $K_p = 0.01$  and  $T_i = 0.1$

For Smith Predictor tuning parameters are same as above for PI controller. Actually the control algorithm in a Smith Predictor is normally a PI controller. Here  $F = \frac{1}{0.202s + 1}$  is used as a filter to remove dead time estimation errors. Actually the filter which is used to remove dead time dead time estimation errors is in the form  $F = \frac{1}{1 + sT_f} = \frac{1}{1 + s\varepsilon L}$  where  $\varepsilon = 0.5$  and  $T_f = \frac{L}{2}$  where

$L$  is the dead time (J.E. Normey-Rico and E.F. Camacho, 2007). From figure 6, it is clear that Smith Predictor provides much faster response as compared to PI controller also Smith Predictor rejects the disturbance earlier as compared to PI controller.

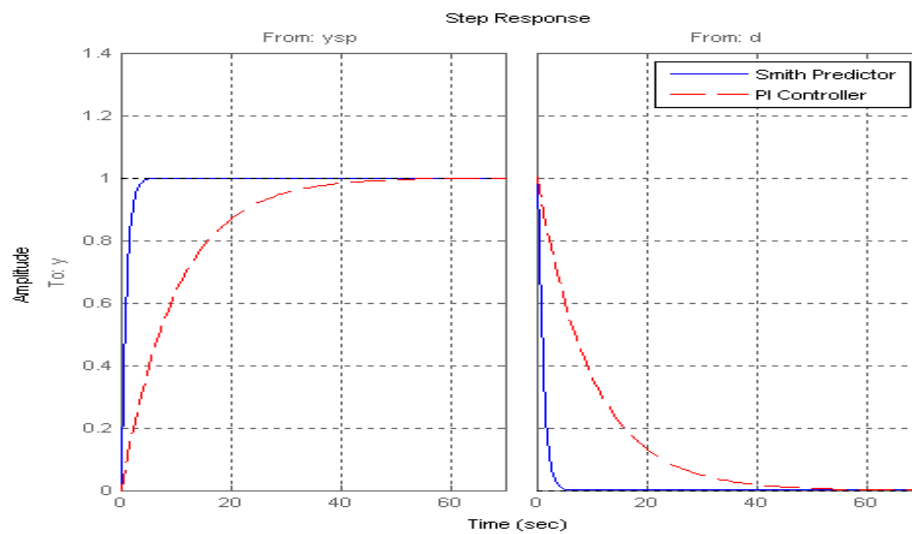


Figure 6: Step response, PI v/s Smith Predictor

In the above analysis, the internal model  $G_n(s)e^{-L_n s}$  matched the process model  $P(s)$  exactly but in practical situations the internal model is only an approximation of the true process dynamics. So it is important to understand how robust the Smith Predictor is to uncertainty on the process dynamics and dead time. Therefore two perturbed models of  $G(s)$  are formed

$$P_1(s) = \frac{0.8e^{-0.0390s}}{0.198s + 1} \quad (5)$$

$$P_2(s) = \frac{1.2e^{-0.0400s}}{0.210s + 1} \quad (6)$$

From figure 7, it is clear that both the designs are sensitive to model mismatch. The Smith Predictor which is used here is acting on the real process and the perturbed process models and is named as Smith Predictor 1. Now to reduce the Smith Predictor's sensitivity to modelling errors stability margins for the inner loop and outer loop are checked and stability margins of the outer loop are improved using a filter that rolls off earlier and more quickly. For this purpose

after some trial and error the filter chosen is  $F = \frac{1 + 0.101s}{1 + 1.01s}$ . Result is shown in figure 8. For improving robustness the Smith Predictor used is named as Smith Predictor 2.

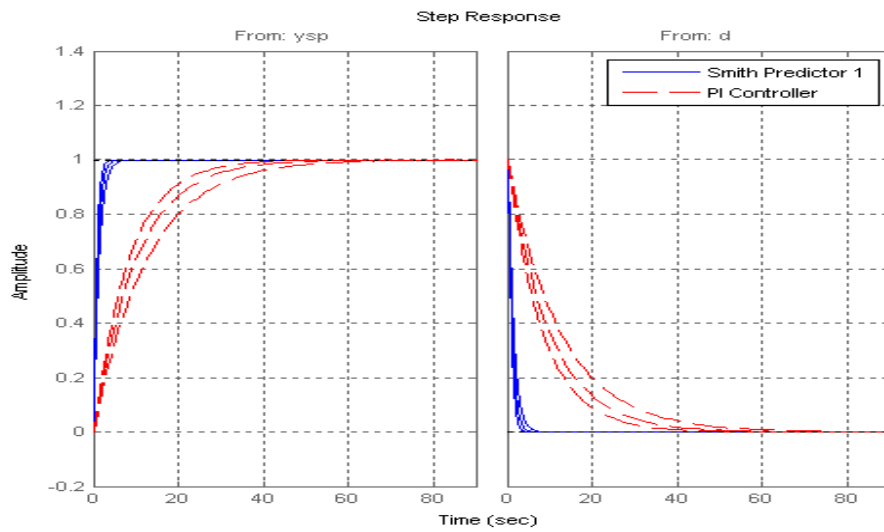


Figure 7: Robustness to model mismatch

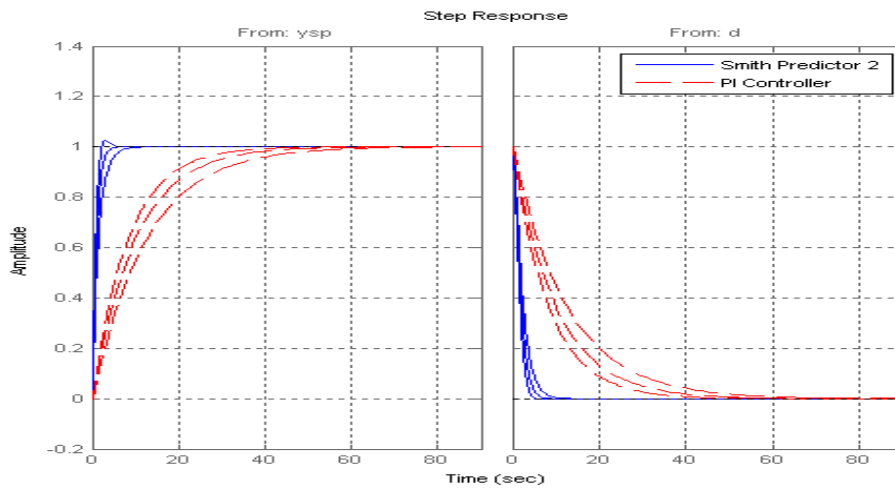


Figure 8: Improving robustness

To improve disturbance rejection a phase lead approximation of  $e^{L_n s}$  is used

$$e^{\tau s} = \frac{1 + K(s)}{1 + K(s)e^{-\tau s}} \quad (7)$$

where  $K$  is a low pass filter with the same time constant as the internal model  $G_n(s)$ . With

$K = \frac{0.005}{0.202s + 1}$ , result is shown in figure 9. For improving disturbance rejection the Smith Predictor used is named as Smith Predictor 3.

In figures 8 and 9, improvement in robustness and disturbance rejection is not quite visible. Especially in figure 9, improvement in disturbance rejection by Smith Predictor 3 is less than Smith Predictor 2 and generally a good trade off between robustness and performance is obtained which is not visible in figures 8 and 9. The reason behind this is the difference in the performance is more evident when dead time is dominant or large. Generally improvement in



the set-point tracking is more noticeable than in disturbance rejection response (J.E. Normey-Rico and E.F. Camacho, 2007).

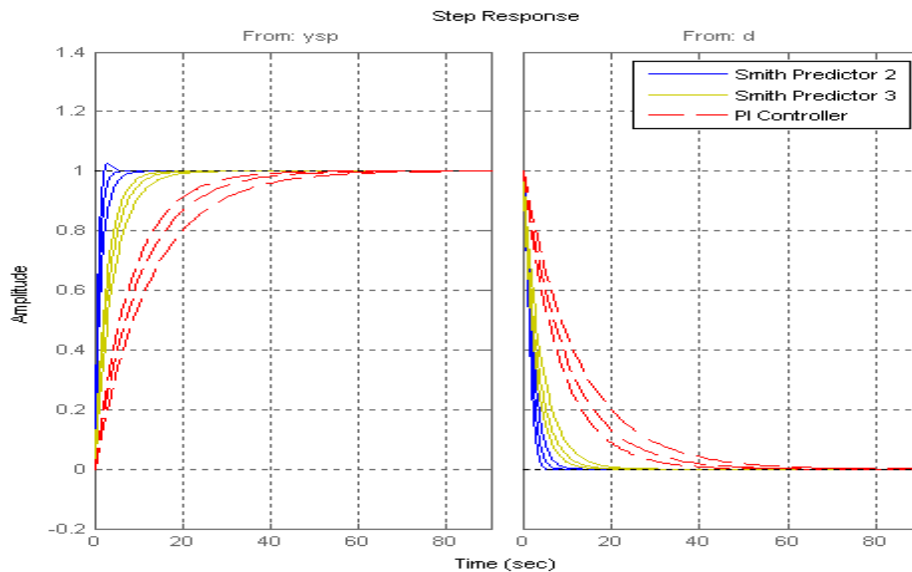


Figure 9: Improving disturbance rejection

To analyze improving robustness and disturbance rejection points clearly example of electric oven temperature control system is taken into account whose FOPDT model has long dead time.

### Electric Oven Temperature Control System

The FOPDT model of electric oven temperature system (S.K. Singh , 2009) is

$$G(s) = \frac{1.63e^{-270s}}{1 + 3480s} \quad (8)$$

This system has a long dead time. Now when a PI controller is applied on this system with  $K_p = 3.5$  and  $T_i = 773.63$  using Cohen and Coon tuning rule the result obtained is shown in figure 10.

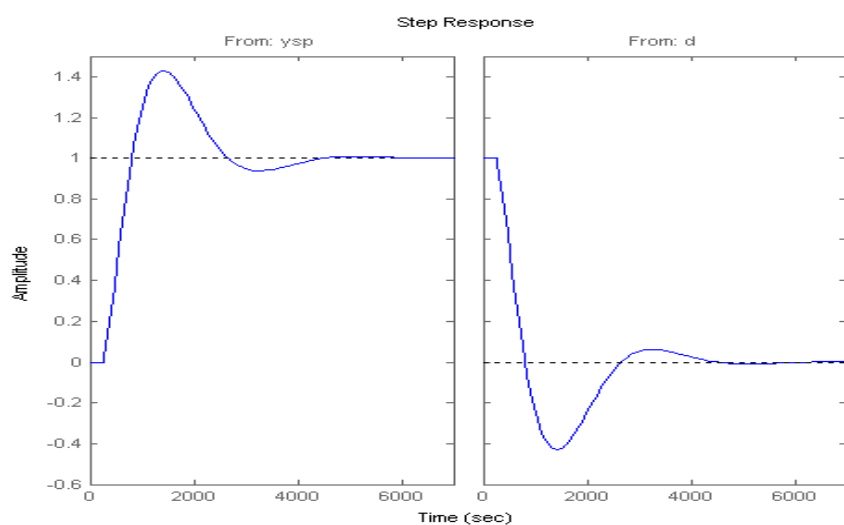


Figure 10: Step response with  $K_p = 3.5$  and  $T_i = 773.63$

Now different values of  $K_p$  i.e. 3.5, 4.5 and 5.5 are used and variation in step response is shown in figure 11. From figure 11, it is clear that increasing the proportional gain  $K_p$  speeds up the response but also significantly increases overshoot and leads to instability. Figure 12 shows step response, PI v/s Smith Predictor. For Smith Predictor tuning parameters are same as above for PI controller. According to selection procedure above described for stirred tank heat exchanger. Here  $F = \frac{1}{20s+1}$  is chosen as a filter to remove dead time estimation errors.

From figure 12, it is clear that Smith Predictor provides much faster response as compared to PI controller also Smith Predictor rejects the disturbance earlier as compared to PI controller.

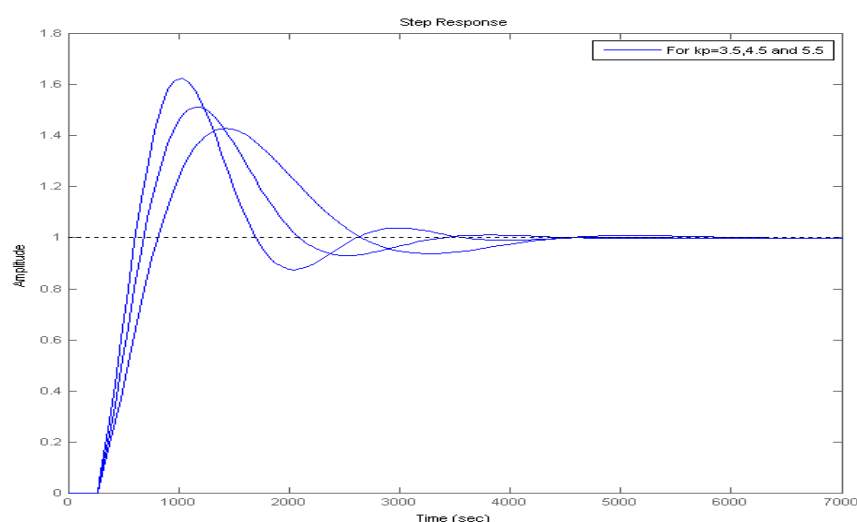


Figure 11: Step response with different values of  $K_p$

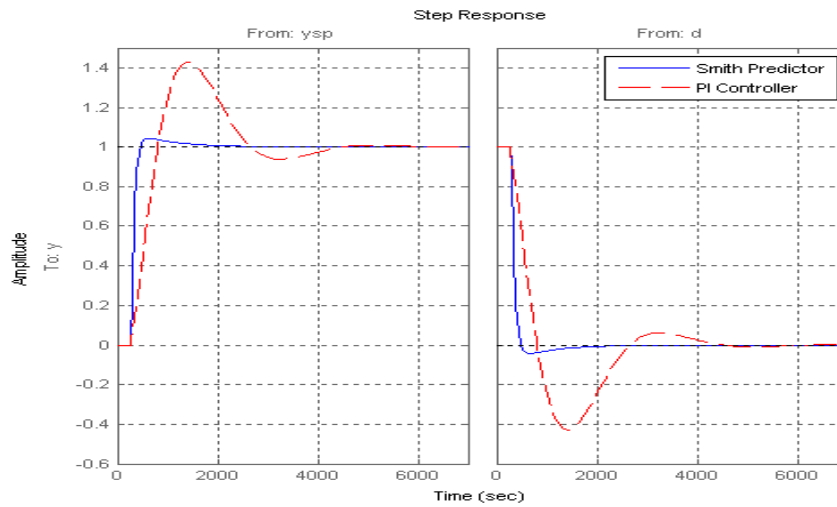


Figure 12: Step response, PI v/s Smith Predictor

In the above analysis, the internal model  $G_n(s)e^{-L_n s}$  matched the process model  $P(s)$  exactly but in practical situations the internal model is only an approximation of the true process dynamics. So it is important to understand how

robust the Smith Predictor is to uncertainty on the process dynamics and dead time. Now consider two perturbed models of  $G(s)$

$$P_1(s) = \frac{e^{-265s}}{1 + 3475s} \quad (9)$$

$$P_2(s) = \frac{1.8e^{-275s}}{1 + 3485s} \quad (10)$$

From figure 13, it is clear that both the designs are sensitive to model mismatch.

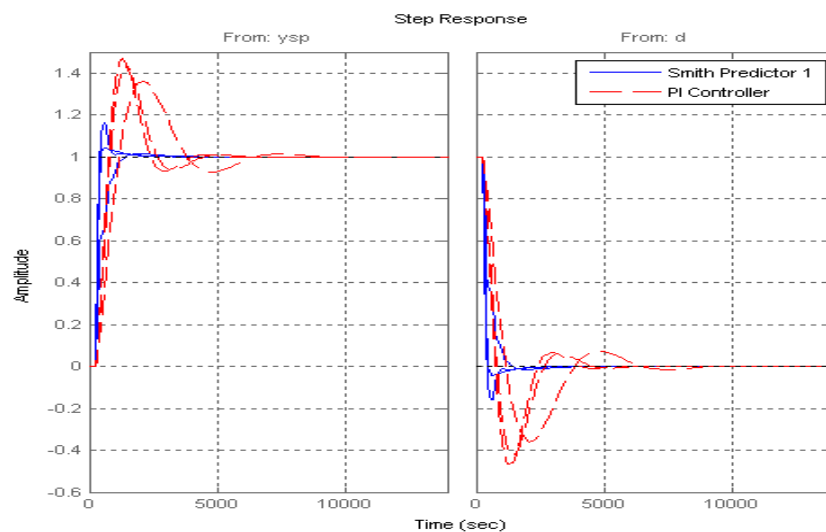


Figure 13: Robustness to model mismatch

Now to reduce the Smith Predictor's sensitivity to modelling errors stability margins for the inner loop and outer loop are checked and stability margins of the outer loop are improved using a filter that rolls off earlier and more quickly. For this purpose after some trial and error

the filter chosen is  $F = \frac{1+50s}{1+500s}$  Result is shown in figure 14. From figure 14, it is clear that the

modified design provides more consistent performance at the expense of a slightly slower nominal response. As described above the Smith Predictor rejects the disturbance rejection earlier than the PI controller and a good trade off between robustness and performance is obtained therefore from figure 14, it is observed that when robustness is improved then disturbance rejection is deteriorated. Here PI controller is rejecting the disturbance rejection earlier as compared to Smith Predictor.

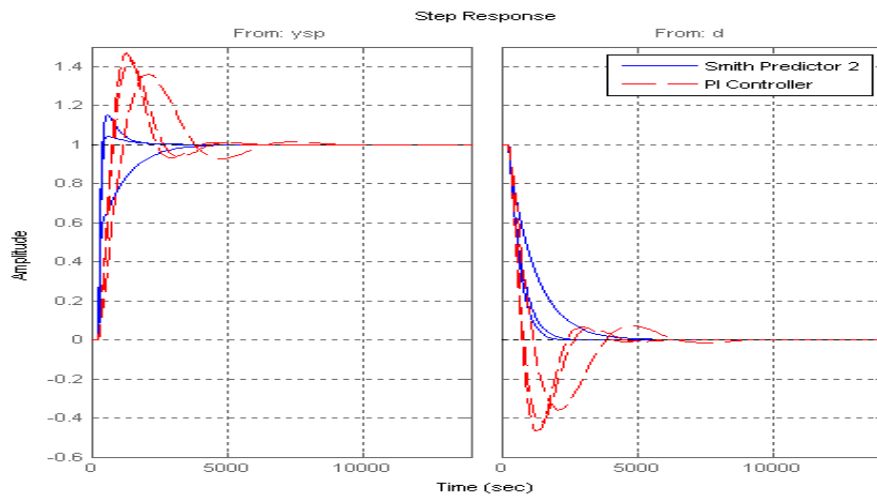


Figure 14: Improving robustness

To improve disturbance rejection a phase lead approximation of  $e^{L_n s}$  is used as shown by equation 7. By using  $K = \frac{0.1}{3480s+1}$ , which is a low pass filter with the same time constant as the internal model  $G_n(s)$ , result obtained is shown in figure 15. Now comparing figure 15 with figures 13 and 14, it is clear that our last design speeds up disturbance rejection at the expense of slower set point tracking.

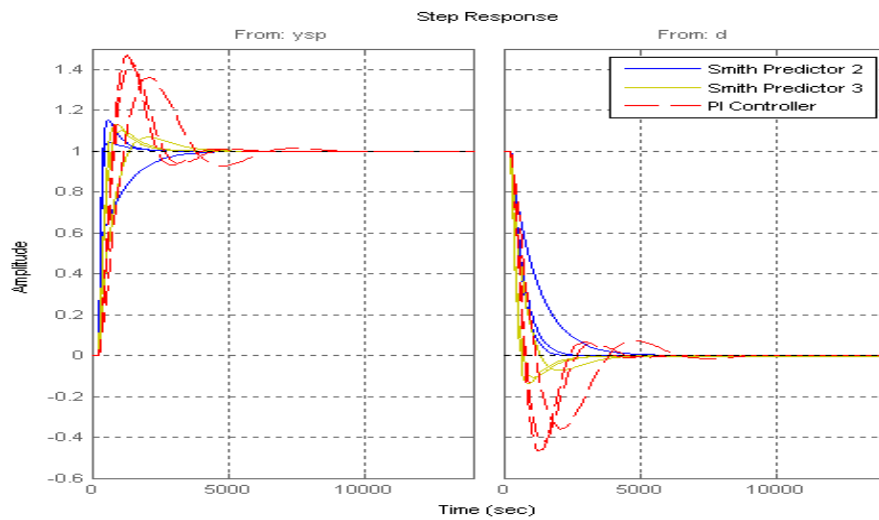


Figure 15: Improving disturbance rejection

### Coupled Tank Process

The results discussed above are for FOPDT models. Here a SOPDT model of a coupled tank process (Mohd Fua'ad Rahmat & Sahazati Md Rozali, 2008) is considered i.e.

$$G(s) = \frac{0.0331e^{-0.4s}}{s^2 + 0.0315s + 0.0248} \quad (11)$$

When a PI controller is applied on above system described by above equation with  $K_p = 0.0315$  and  $T_i = 30.21$  which are chosen using Zeigler-Nicholas tuning method the result obtained is shown in figure 16.

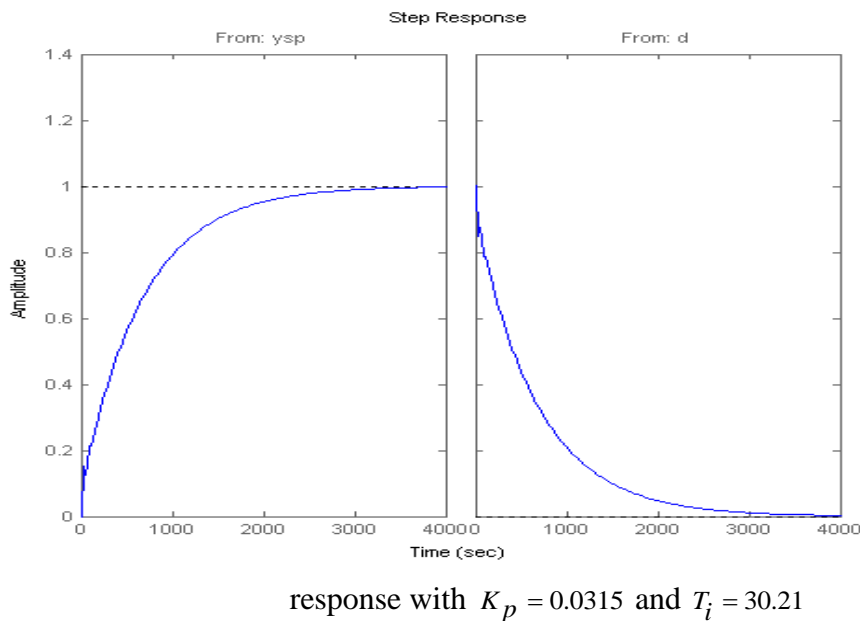


Figure 16: Step

response with  $K_p = 0.0315$  and  $T_i = 30.21$

Figure 17, shows how step response is affected by increase in dead time. Here four different values of long dead times are used. For best results equation 11, is modified with long delay i.e

$$G(s) = \frac{0.0331e^{-600s}}{s^2 + 0.0315s + 0.0248} \quad (12)$$

Equation 12 is used for further analysis.

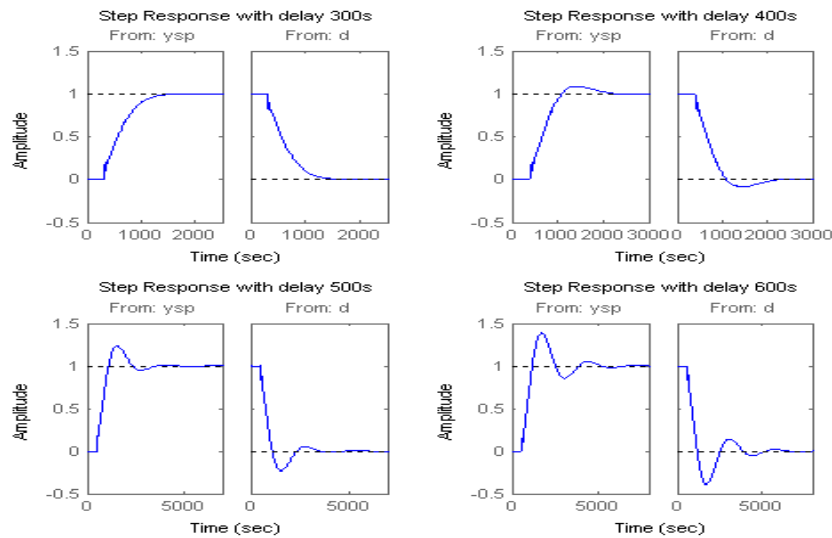


Figure 17: Step response with different delays

Figure 18, shows step response showing comparison between PI controller and Smith Predictor. The tuning for PI controller used in the Smith Predictor is same as used for PI controller above.

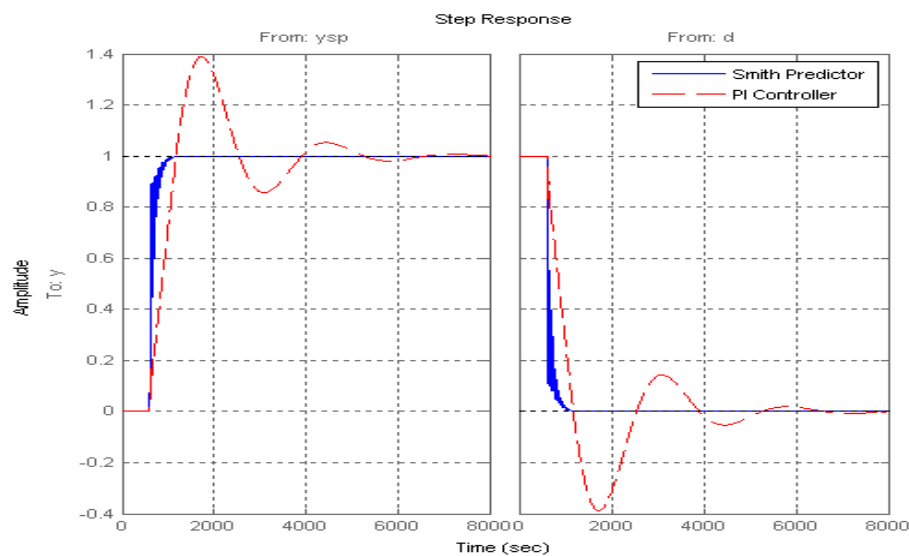


Figure 18: Step response, PI v/s Smith Predictor

Here  $F = \frac{1}{300s+1}$  is used as a filter to remove dead time estimation errors with the same selection procedure as described for stirred tank heat exchanger. From figure 18, it is clear that Smith Predictor provides much faster response as compared to PI controller also Smith Predictor rejects the disturbance earlier as compared to PI controller. In the above analysis, the internal model  $G_n(s)e^{-L_n s}$  matched the process model  $P(s)$  exactly but in practical situations the internal model is only an approximation of the true process dynamics. So it is important to understand how robust the Smith Predictor is to uncertainty on the process dynamics and dead time. Now consider two perturbed models of  $G(s)$

$$P_1(s) = \frac{0.0325e^{-590s}}{s^2 + 0.0310s + 0.0245} \quad (13)$$

$$P_2(s) = \frac{0.0335e^{-610s}}{s^2 + 0.0320s + 0.0250} \quad (14)$$

From figure 19, it is clear that both the designs are sensitive to model mismatch. Now to reduce the Smith Predictor's sensitivity to modelling errors stability margins for the inner loop and outer loop are checked and stability margins of the outer loop are improved using a filter that rolls off earlier and more quickly. For this purpose after some trial and error the filter used is

$$F = \frac{1+50s}{1+500s}. \text{ Result is shown in figure 20.}$$

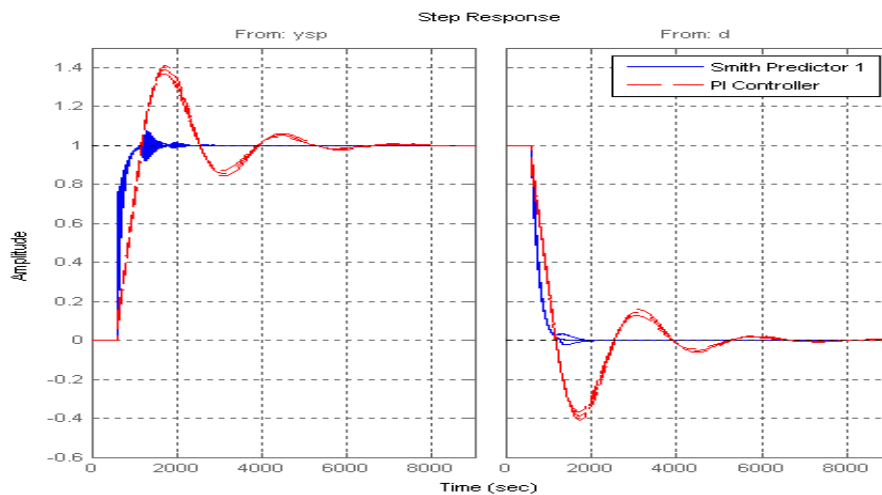


Figure 19: Robustness to model mismatch

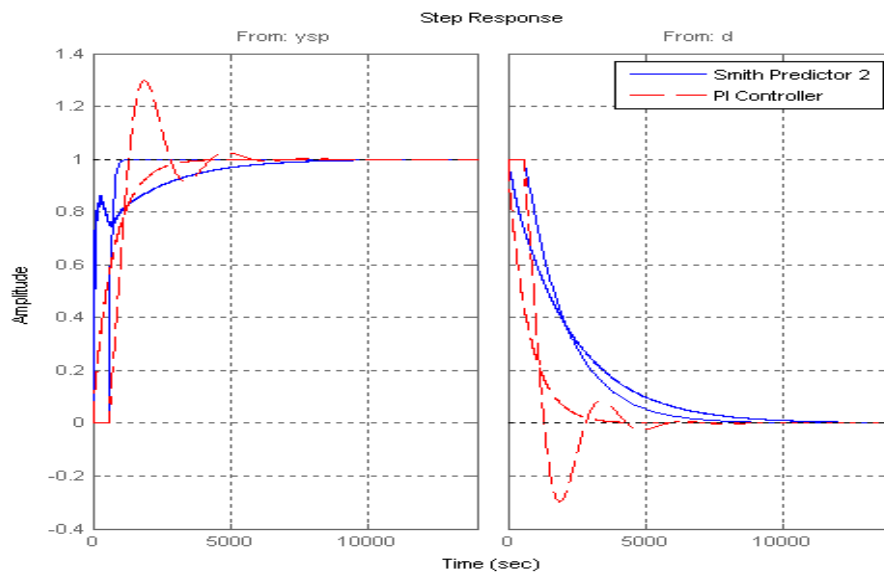


Figure 20: Improving robustness

From figure 20, it is clear that the modified design provides more consistent performance at the expense of a slightly slower nominal response. As described above the Smith Predictor rejects the disturbance rejection earlier than the PI controller and a good trade off between robustness and performance is obtained. From figure 20, it is observed that when robustness is improved then disturbance rejection is deteriorated. Here PI controller is rejecting the disturbance rejection earlier as compared to Smith Predictor. To improve disturbance rejection a phase lead approximation of  $e^{L_n s}$  is used as shown by equation 7. By using  $K = \frac{0.005}{0.0315s + 1}$ , which is a low pass filter with the same time constant as the

internal model  $G_n(s)$ , result obtained is shown in figure 21. Now comparing figure 21 with figures 19 and 20 it is clear that our last design speeds up disturbance rejection at the expense of slower set point tracking.

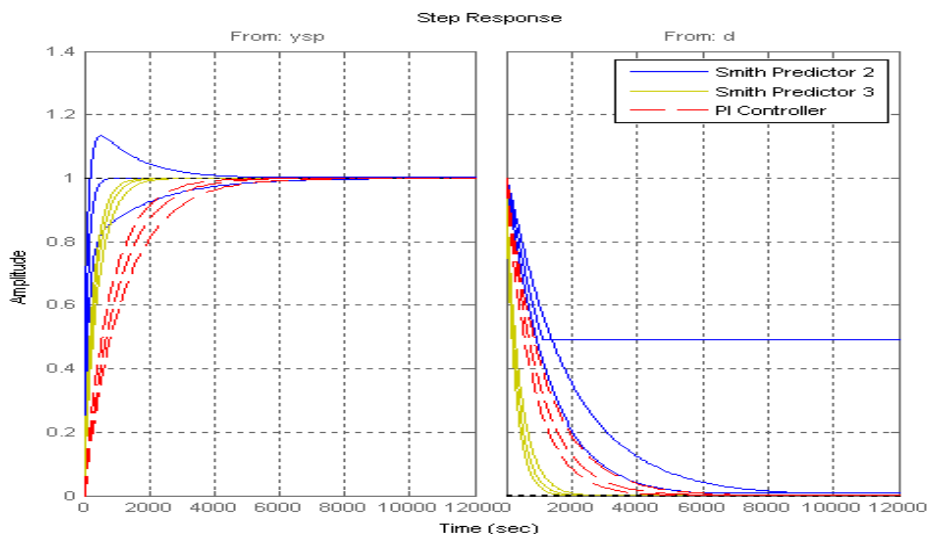


Figure 21: Improving disturbance rejection



## CONCLUSIONS

PI controller and Smith Predictor are good dead time compensators for long dead time processes. The control algorithm in a Smith Predictor is a PI controller and it also uses the idea of prediction. When a comparison is made between the performance of PI controller and Smith Predictor for long dead time processes, best results are obtained with Smith Predictor. Smith Predictor eliminates the effect of the dead time in the set point response. A good trade-off between robustness and performance can be obtained by appropriate tuning of primary controller. Smith Predictor cannot be used with integrative and unstable processes. When the process exhibits integral dynamics the classical Smith Predictor fails to provide a null steady state error in the presence of a constant load disturbance. For Smith Predictor disturbance rejection response cannot be faster than that of the open loop. This can be important when the dead time is non dominant. Advantages of Smith Predictor are more evident when high order models are used.

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