COSMOLOGICAL PLASMA FILAMENTS INSTABILITY AND UNIVERSE CLIMATE SYSTEMS

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ABSTRACT: *The language of differential geometry used to study instability of inhomogeneous plasma filaments by those techniques where the curve's motion is fully described by Frenet torsion and curvature. The case studied is time depending frame changes according to free motions of charged particles. Longitudinal modes describing low-frequency waves in plasma in the medium are obtained by applying chaotic flows which describe cosmological perturbations to Magneto-Hydrodynamic (MHD) equations of inhomogeneous plasma.*

KEYWORDS: Inhomogeneous plasma, Instability, Frenet frame, Alfven waves.

INTRODUCTION

In plasma science, It's known that the studying of astrophysics and solar is severe related to Geometrical hydrodynamic and Magneto-hydrodynamics (MHD) instability studies [1, 3]. So chaotic flows and twisted filamentary magnetic structures has been developed and applied in solar physics [4, 5].

Since the essential concept behind MHD is those magnetic fields can influence currents in a moving conductive media, some proses like this produce forces can impact on the media and also changes the magnetic field itself.

Topology and Riemannian geometry roles was investigated to MHD dynamos so it important for use in astrophysics and solar systems [6, 7]. In this paper the generalization of filamentary structures considered to investigate instability.

MHD and Alfven waves

Since about 99% of baryonic matter contented in universe consists of plasma then MHD quite well applies to astrophysics and cosmology so we can start from the MHD field equations which take the form

$$\left.\begin{aligned}&\frac{d\rho}{dt}+\rho\nabla.V=0\\&\rho\frac{dV}{dt}+\nabla\rho-j\times B=0\\&E+V\times B=0\\&\frac{d}{dt}\left(\frac{p}{\rho^{\Gamma}}\right)=0\end{aligned}\right\}\qquad(1)$$

Here Γ represent the specific heats ratio, and ρ the density of plasma mass.

Since that the plasma is highly conducting, and if the motion is sufficiently fast then both viscosity and heat conduction can be plausibly neglected, so the MHD equations valid when

$$\delta^{-1}v_t \geq V \geq \delta v_t$$

Here δ is the parameter of magnetization, V is the velocity associated with the plasma dynamics under investigation, and v_t is the thermal velocity,

Then we can see that the MHD equations describe relatively violent, large-scale motions of highly magnetized plasmas in general. Strictly, the MHD equations are only valid in collisional plasmas (Path – free).

Now by combining set (1) we can form a closed set of equations:

$$\left.\begin{aligned}&\frac{d\rho}{dt}+\rho\nabla.V=0\\&\rho\frac{dV}{dt}+\nabla p-\frac{(\nabla\times\mathrm{B})\times\mathrm{B}}{\mu_0}=0\\&-\frac{\partial B}{\partial t}+\nabla\times(\mathrm{V}\times\mathrm{B})=0\\&\frac{d}{dt}\left(\frac{p}{\rho^{\Gamma}}\right)=0\end{aligned}\right\}\qquad(2)$$

Linearize above equations (2) (equilibrium quantities, velocity and plasma current, to be zero) gives us:

$$\left.\begin{aligned} &\frac{\partial \rho}{\partial t} + \rho_0 \ \nabla . V \ = 0 \\ &\rho_0 \frac{\partial V}{\partial t} + \nabla p - \frac{(\nabla \times \mathrm{B}) \times \mathrm{B}_0}{\mu_0} = 0 \\ &-\frac{\partial B}{\partial t} + \nabla \times (\mathrm{V} \times \mathrm{B}) = 0 \\ &\frac{\partial}{dt} \partial \left(\frac{p}{p_0} - \frac{\Gamma_\rho}{\rho_0} \right) = 0 \end{aligned}\right\} \qquad (3)$$

Note that; ρ_0 , p_0 , and B_0 constants in a spatially uniform plasma.

Now by looking for wave like solutions of equations (3) in which perturbed quantities vary like

$$\exp[\mathrm{i}(\mathrm{k.r} - \omega \mathrm{t})]$$

It follows that

$$-\omega\rho + \rho_0 k . V \ = 0 \qquad (4)$$

$$-\omega\rho_0 V \ + kp - \frac{(k \times B) \times B_0}{\mu_0} = 0 \qquad (5)$$

$$\omega B + k \times (V \times B_0) = 0 \qquad (6)$$

$$-\omega \left(\frac{p}{p_0} - \frac{\Gamma_\rho}{\rho_0} \right) = 0 \qquad (7)$$

If $\omega \neq 0$, above equations (4) yield

$$\rho = \rho_0 \frac{k.V}{\omega}$$

$$p = \Gamma p_0 \frac{k.V}{\omega}$$

$$B = \frac{(k.V)B_0 - (k.B_0)V}{\omega}$$

Substitution into the linearized equation of motion, Eq. (5), gives us

$$\left[\omega^2 - \frac{(k.B_0)^2}{\mu_0 \rho_0} \right] V = \left\{ \left[\frac{\Gamma p_0}{\rho_0} + \frac{B_0^{\ 2}}{\mu_0 \rho_0} \right] k - \frac{(k.B_0)}{\mu_0 \rho_0} B_0 \right\} (k.V) - \frac{(k.B_0)(V.B_0)}{\mu_0 \rho_0} k \qquad (8)$$

Now by assuming that θ be angle between B_0 and k . Equation (8) can reduced to the eigenvalue equation

$$\begin{pmatrix} \omega^2 - k^2 V_A^2 - k^2 V_s^2 \sin^2\theta & 0 & -k^2 V_s^2 \sin\theta\cos\theta \\ 0 & \omega^2 - k^2 V_A^2 \cos^2\theta & 0 \\ -k^2 V_s^2 \sin\theta\cos\theta & 0 & \omega^2 - k^2 V_s^2 \cos^2\theta \end{pmatrix} \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = 0 \qquad (9)$$

Here,

$$V_A = \sqrt{\frac{B_0^2}{\mu_0 \rho_0}}$$

denote the Alfven velocity, and

$$V_s = \sqrt{\frac{\Gamma p_0}{p_0}}$$

is the sound speed.

The solubility condition for Eq. (9) that $\Delta = 0$. This yields the dispersion relation

$$(\omega^2 - k^2 V_A^2 \cos^2\theta)\left[\omega^4 - \omega^2 k^2 (V_A^2 + V_S^2) + k^4 V_A^2 V_S^2 \cos^2\theta\right] = 0 \qquad (10)$$

this have three independent roots according to the three different types of wave that can propagate through MHD plasma. The

$$\omega = kV_A \cos\theta, \qquad \textit{with} \text{ associated eigenvector } (0, V_y, 0) \qquad (11)$$

$$\omega = kV_+, \qquad \textit{with} \text{ associated eigenvector } (V_x, 0, V_z) \qquad (12)$$

$$\omega = kV_-, \qquad \textit{with} \text{ associated eigenvector } (V_x, 0, V_z) \qquad (13)$$

Where

$$V_\pm = \left\{ \frac{1}{2}\left[V_A^2 + V_s^2 \pm \sqrt{(V_A^2 + V_s^2)^2 - 4 V_A^2 V_s^2 \cos^2\theta} \right] \right\}^{\frac{1}{2}}$$

The second root (12) is generally termed the *fast* magnetosonic wave; whereas the third root (13) is usually represent the *slow* magnetosonic wave.

So, these waves where plasma motion parallel or perpendicular to the magnetic field are describe non-zero perturbations in density and pressure of the plasma. The observation suggests that the dispersion relations (12) and (13) are likely to endure significant modification in collision less plasmas.

Perturbations in structures of MHD filaments

Recall that the equilibrium quantities of set (1) are:

$$\left.\begin{aligned} &B = B_0 t \\ &J_0 = 0 \\ &v_0 = 0 \\ &p = p_0 + p_1 \\ &\rho = \rho_0 + \rho_1 \end{aligned}\right\} \qquad (14)$$

by $B = B_{s,n,t'}$ we define magnetic field B along the filament , and components over the arc length s of the filament depending at time by $B_{s,t}$.

Consider that B_0 doesn't depended on time and also doesn't depend on normal coordinates n so $B_0(s)$ .The vectors t and n along with binormal vector b together form Frenet frame which satisfy the Frenet-Serret theorem

$$\left.\begin{aligned} &t' = \kappa n \\ &n' = -\kappa t + \tau b \\ &b' = -\tau n \end{aligned}\right\} \qquad (15)$$

Now by assumption that the Frenet frame depend on the degrees of freedom then the gradient operator becomes

$$\nabla = t\frac{\partial}{\partial s} + n\frac{\partial}{\partial n} + b\frac{\partial}{\partial b} \qquad (16)$$

Now recall that the other Frenet frame expressions are

$$\frac{\partial}{\partial n} n = -\theta_{n,s} t - (\mathrm{div} b)b$$

$$\frac{\partial}{\partial n} t = \theta_{n,s} n + (\Omega_b + \tau)b$$

$$\frac{\partial}{\partial n} b = -(\Omega_b + \tau)t - (div\ b)n$$

$$\frac{\partial}{\partial b} t = \theta_{bs} b - (\Omega_n + \tau)n$$

$$\frac{\partial}{\partial b} n = (\Omega_n + \tau)t - \kappa + (\mathrm{div}\, n)b$$

$$\frac{\partial}{\partial b} b = -\theta_{bs} t - (\kappa + \mathrm{div}\, n)n$$

So Frenet frame time evolution has form:

$$\left.\begin{aligned} t' &= -\tau\kappa n + \kappa' b \\ n' &= -\tau\kappa t \\ b' &= -\kappa' t \end{aligned}\right\} \qquad (17)$$

Solution's instability of MHD plasma filaments

Appling above equations into the LHS of magnetic equation give

$$\nabla \times B_0 = \mu_0 . J_0 = 0 \qquad (18)$$

dilation this on the Frenet frame yields

$$B_0 \left[n \times \partial_n t + b \times \partial_b t \right] = 0 \qquad (19)$$

where we have used that B_0 is constant. Substitution of the above relations for the Frenet frame yields geometrical restriction

$$\Omega_n = \; — \tau = \; 0 \qquad (20)$$

For this planar filaments stipulation implies that torsion $\tau = 0$. The remaining constraint is

$$\kappa = \Omega_b$$

Note that filament bundles are geodesic if $\Omega_s = 0$, also we consider that the curvature and torsion are uneasy but we assume that the torsion remains zero after instability so the motion is forced or obliged to be perturbed in the plane. Since the current density is written as $J = \rho v$ we can obtain

$$\nabla \times B_1 = \mu_0 J_1 \qquad (21)$$

which yields

$$\partial B_1 + (\theta_{sn} + \theta_{nb}) B_1 = 0 \qquad (22)$$

this can reduce to:

$$(iK_\perp - \kappa_0) B_0 = \mu_0 J_1 \qquad (23)$$

Now call that $\theta = (\omega t - K_{\square} s + K_\perp n)$, where $\omega = \mathrm{Re}\,\omega + i\ \mathrm{Im}\,\omega$, into equation (23) we have (using Euler's formula):

$$iK_\perp B_1^0 (\cos\theta - i\ \sin\theta) = (\kappa_0 B_1^0 + \mu_0 J_1^0)(\cos\theta - i\ \sin\theta) \qquad (24)$$

real equations of above has solution

$$K_{\perp}B_1^0 = (K_0 B_1^0 + \mu_0 J_1^0)^2 \qquad (25)$$

Maxwell equations $\nabla.B = 0$ become:

$$\partial_s B_0 + (\theta_{sb} + \text{div } b)B_0 = 0 \qquad (26)$$

with solution

$$B_0 = -c_0 e^{\int(\theta_{bs}+\theta_{ns})ds} \qquad (27)$$

Now the perturbed equation is

$$\nabla \cdot B_1 = 0 \qquad (28)$$

which can reduce to the expression

$$\partial B_1 + (\theta_{bs} + div\ b)B_1 = 0 \qquad (29)$$

This can output complex expression

$$iK_{\parallel}B_1^0(\cos\theta - i\ \sin\theta) = -B_1^0(\cos\theta - i\ \sin\theta) \qquad (30)$$

which analogous to equation (24) and by same proses solved to yield

$$K_{\parallel} = \pm\left[\theta_{\beta s} + \theta_{vs}\right] \qquad (31)$$

Now solving the conservation of mass solution as

$$i\,\omega\rho_1 = \upsilon_1\rho_0\, div\ b \qquad (32)$$

By expanding this we have

$$-i\,(\text{Re}\,\omega + \text{iIm}\,\omega)\rho_1 = \upsilon_1\,\rho_0\, div\ b \qquad (33)$$

thus

$$(\text{Re}\,\rho_1^0)^2 = (\text{Im}\,\omega\rho_1^0 + \upsilon^0\rho_0)^2 \qquad (34)$$

solution of this equation where $\text{Re}\,\omega = 0$ allows investigation of the plasma filaments instability, which is

$$\text{Im}\,\omega = \frac{\upsilon_1^0\rho_0}{\rho_1^0} \qquad (35)$$

if $\text{Im}\omega > 0$ instability is possible this implies

$$\frac{\upsilon_1^0\rho_0}{\rho_1^0} > 0 \qquad (36)$$

By assumption that the mass densities ρ of the fluid are always positive then the constraints on the velocity $\upsilon_1^0 > 0$ are imposed by instabilities.

Now by expansion of the magnetic field we go on to investigate the remaining Maxwell MHD equations.

$$\omega_0 := \operatorname{Im}\omega = \pm\frac{\kappa_{\square} L\, B_0 (k_0 + div\ \beta)}{B_1^0} \qquad (37)$$

Here $L = \int ds$ the length of the filament. In the case of $L = \pi R$ (solar loops) one can write the Alfven waves frequency as $\omega_0^2 = (\kappa_{\square}\,\mathrm{V}_a)^2$ and comparing it with (37) we have the Alfven velocity V_A as

$$V_A = (\frac{LB_0(\kappa_0 + \mathrm{div}\, n)}{B_1^0})^2 \qquad (38)$$

This can simplify to

$$V_A = \pm\left[\frac{LB_0\kappa_0}{B_1^0}\right] \qquad (39)$$

by assumption that $div\ n = 0$.

So this show propagation of Alfven waves over the filament. The last equation gives an expression between pressure's components as

$$\frac{\rho_1^0}{\rho_0} p_0 = p_1^0 \qquad (40)$$

The solution given here is well suitable for plasma filaments where the curvature perturbed or not.

CONCLUSIONS

Electromagnetic fields generalization is possible in the case of filaments instability. So Instability of plasma MHD is investigated in the inhomogeneous Frenet frame. Alfven waves, where the velocity is represented in terms of the Frenet curvature of planar filaments are found. Thus it plays an important role in the studies of universe climate systems.

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