CONCEPTUALIZING TEACHER KNOWLEDGE IN DOMAIN SPECIFIC AND MEASURABLE TERMS: VALIDATION OF THE EXPANDED KAT FRAMEWORK

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ABSTRACT: *This study contributes to discussions on teacher knowledge by providing evidence that teacher knowledge can be conceptualized in domain specific and measurable terms instead of theorizing it in general terms and using proxy measures for it. The groundbreaking attempt at this type of conceptualization was done by the KAT project in the US when they put up a framework that hypothesized three types of knowledge whose intersections were considered blurry. Arguing that the intersections of the three hypothesized knowledge rather produce some form of complex blends of knowledge that cannot be ignored, an expanded form of this original KAT framework has recently been suggested. It is this expanded framework that guided the current study. Using an instrument developed through adaptation of the KAT project's instruments, exploratory factor analysis conducted on data from 252 mathematics teachers in 40 senior high schools in Ghana validated the expanded framework Recommendations for further research in different domains of mathematics and the use of the framework to develop measures of teacher knowledge have been made.*

KEYWORDS: Conceptualization of Teacher Knowledge, Profound Knowledge of School Algebra, Advanced Algebra Teaching Knowledge, School Algebra Teaching Knowledge and Pedagogical Content Knowledge in Algebra

INTRODUCTION

Research is replete with the fact that the teacher is the most important factor that influences students' achievement (see for instance, Begle, 1972; Hanushek, 1972;Eisenberg, 1977; Harbison & Hanushek, 1992; Shulman & Quinlan, 1996; Mullens, Murnane & Willett, 1996; Rowan, Chiang & Miller, 1997; Wilmot, 2009; Yara, 2009). For instance, Yara (2009) argued that what best improves students' academic achievement are teacher characteristics such as his/her teacher's competence, ability, resourcefulness and ingenuity to effectively utilize appropriate language. By this arguement, Yara (2009) alluded to the argument that the teacher is the most important factor in any educational enterprise. However, though researchers are in agreement about the usefulness of teacher knowledge in influencing student performance they are generally not in agreement on how to effectively conceptualise teacher knowledge in order to show which aspect of it bests predicts student performance. For instance, with mathematics, whereas a number of researchers such as Harbison and Hanushek (1992) and Mullens et al (1996), found teachers' subject matter knowledge to be a better predictor of students' achievement than other home based factors, researchers such as (Mullens et al., 1996), Rowan, Correnti & Miller (2002) have asserted that teachers years of teaching experience was a better predictor of students achievement than subject matter competency. Others have also pointed proxy measures of teacher knowledge such as the number of university courses taken, the type of degree the teachers' have (see Darling-

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Hammond, 1999), as well as the possession of advanced degree by teachers as a better predictor of student performance (see Monk. 1994; Rowan et al., 2002).

The attempt to effectively conceptualize teacher knowledge has led to many different conceptualizations (see conceptualizations by Thompson, 1984; Leinhardt & Smith, 1985; Shulman, 1987; Grossman, 1990; Cochran & Jones, 1998; Ma, 1999). For instance, whereas Thompson (1984) argue that the type of knowledge teachers draw upon in teaching could be prejudiced by beliefs, views and predilections about the subject, Leinhardt and Smith (1984), pointed to two types of knowledge namely *Lesson Structure Knowledge* (LSK) which they argued comprise, among other things smooth planning and organization of lessons, and *Subject Matter Knowledge* (SMK) which to them comprise concepts, algorithms, operations, connections among different algorithms and knowledge of the type of errors student make. Shulman and his colleagues (see Shulman, 1987) mentioned seven types of knowledge namely *Content knowledge, General pedagogical knowledge, Curriculum knowledge, Pedagogical content knowledge, Knowledge of learners and their characteristics, Knowledge of educational contexts*, and *Knowledge of educational ends, purposes and values*. Later, Grossman (1990) put forward four types of teacher knowledge namely, *Subject Matter Knowledge, General Pedagogical Knowledge, Pedagogical Content Knowledge* and *Knowledge of Context*. When Ma (1999) came to the research scene she also put forward the idea of *Profound Understanding of Fundamental* Mathematics (PUFM) which according to her is marked by having "multiple perspectives, includes the basic ideas of mathematics and has a longitudinal coherence" (Ma, 1999, p.122).

According to Wilmot (2008), some of these conceptualizations have presented teacher knowledge as a domain neutral construct making it virtually impossible for it to be objectively measured (for example, Shulman, 1987; Grossman & Richert, 1988; & Grossman,1990). Wilmot (2008) has argued that, part of the reasons for the reliance on proxy measures of teacher knowledge has been a result of the fact that several researchers have conceptualized it as a domain neutral construct making it difficult to objectively develop objective measures of it. A good example of such a domain neutral is the *pedagogical content knowledge* (pck) conceptualization by Shulman and his colleagues (see Shulman, 1986b). It is important to state that though Shulman and his colleagues mentioned seven types we focus on their pck conceptualization, because of its prominence so far as the knowledge base for teaching is concerned. According to them pck is the type of knowledge that distinguishes the teacher as a professional from those who claim to be content experts and those who simply comprehend kids. We argue that though common sense tells us that even in mathematics, the pck needed to teach algebra has to be different from that needed to teach calculus, Shulman's conceptualization does help to objectively answer to question as to, for instance exactly how the two are different from each other and how they can be objectively measured differently.

Our point of view is that, instead of relying on proxy measures, there is the need for re-conceptualization of teacher knowledge in ways that is not only domain specific but also allows its components to be measured. At the Senior High School level, researchers in the Knowledge of Algebra for Teaching (KAT) project at Michigan State University in the 2000s proposed one of such ground breaking framework of teacher knowledge (see Ferrini-Mundy, Burrill, Floden, & Sandow, 2003; Ferrini-Mundy, McCrory, Senk, & Marcus, 2005; Ferrini-Mundy, Senk, & McCrory, 2005). From their framework, the KAT project members developed and administered instruments for measuring knowledge for teaching algebra.

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We acknowledge the fact that around the time Ferrini-Mundy and her team on the KAT project were working on their conceptualization, Deborah Ball and her colleagues were also on similar domain specific conceptualization of teacher knowledge (see Ball & Bass, 2000; Hill, Ball & Shilling, 2004; Hill, Rowan & Ball, 2005). The difference is that whereas Ferrini-Mundy and her colleagues where working on the knowledge for teaching Algebra at the high school level, Ball and her colleagues were researching into the knowledge for teaching elementary school mathematics. Deborah Ball and her colleagues also introduced a conceptualization called *Mathematical Knowledge for Teaching* (MKT) and *Specialized Knowledge of Content* (SKC), which according to them is "a specialized knowledge of content made up of several items: representing numbers, and operations, analyzing unusual procedures or algorithms and providing explanations of rules" (Hill, Ball & Schilling, 2004, pp. 27-28). Like the KAT researchers, Ball and her colleagues also developed instruments for measuring knowledge for teaching mathematics at the elementary level.

The point also need to be made that prior to the work by Ferrini-Mundy and her colleagues and Ball and her colleagues, various tests had been developed and used within the US and other countries for teacher certification, in ways that suggest that passing those tests was guarantee that the test taker had a good knowledge for teaching. For instance, in Singapore pre-service mathematics teachers have for some time now been required to pass a mathematics qualifying examination before their graduation by the National Institute of Education. Similarly, in the US over 30 states have used the PRAXIS as a teacher licensing examination. However, in spite of such teacher certification the quality of achievement of K-12 students continues to be of concern to Americans. The RAND Study Panel (2003), that was set up to address such shortfalls recommended that for teacher knowledge in mathematics to be improved there was the need for further clarification of the knowledge base required for teaching mathematics well, the development of instruments for measuring the Mathematical Knowledge for Teaching objectively and isolated Algebra as a vital area of focus in these efforts, perhaps because of the foundational nature of algebra to other domains of mathematics and the fact that Americans have always seen algebra as a right for all its citizens (see Moses, 1995).

It can be argued that the work by Ferrini-Mundy and her colleagues on the KAT project was influenced by such recommendations by the RAND Study Panel (2003). By focusing on the mathematics domain of Algebra, the KAT project members put forward a domain specific conceptualization of teacher knowledge (i.e., knowledge for teaching algebra) by hypothesizing three types of knowledge and, as already said, developed and piloted instruments for measuring them. However, attempts to validate the KAT framework using data from Ghana proved not very successful (see Wilmot, 2008, 2016) as the entire framework could not be corroborated. For instance, though the existence of two of the KAT project's hypothesized knowledge, teaching knowledge and advanced knowledge could be confirmed, there were not enough unique factor loadings to confirm the third. As Wilmot (2016) puts it,

"The fact that Factors 2 and 4 had [three each of] *Teaching Knowledge and Advanced Knowledge* items [uniquely] loading respectively on them point to the fact that these two Factors could be named *Teaching Knowledge and Advanced Knowledge* respectively. In addition, it is worth noting that on Factor 6 the only two items that loaded were *School Knowledge* items. Factor 6 was however not labelled as *School Knowledge* because of the suggestion by Costello and Osborne (2005) that factors with fewer than three items are considered to be unstable and should not be labeled" (p. 19).

This notwithstanding, since the construct of school knowledge, its nature and boundaries are well outlined in various countries' or states' high school mathematics curricula, a case could be made about the reasonableness of the three hypothesized knowledge types in the original framework except that there were insufficient number of items to fully validate them. The other aspect of the KAT framework that was not corroborated by Wilmot (2008) was the claim by the KAT project team members that the intersections of their three hypothesized knowledge was blurry (see Ferrini-Mundy, Burrill, Floden, & Sandow, 2003; Ferrini-Mundy, McCrory, Senk, & Marcus, 2005; Ferrini-Mundy, Senk, & McCrory, 2005.). This claim suggests that the intersections are not significant or not necessary to be defined. On the contrary, the factor analysis performed by Wilmot (2016) revealed two item loadings of combinations of items of two of the KAT project's hypothesized knowledge types. Though such two item loadings did not permit Wilmot (2016) to name those factors, he argued that "the findings of [this] study point to the possibility that the boundaries … may not be blurry as initially conjectured in the KAT framework" Wilmot, 2016, p. 20).

From the foregoing, one question that needs to be answered is, "to what extent can research validate the Expanded KAT framework proposed by Wilmot (2016)? This is the question that the present study was designed to investigate.

Conceptual framework

In an attempt at conceptualizing teacher knowledge in domain specific terms, researchers of the Knowledge of Algebra for Teaching (KAT) project used Algebra as the mathematics domain of focus and hypothesized three type of knowledge, Advanced Knowledge, School Knowledge, and Teaching Knowledge as the three key types of knowledge necessary for teaching algebra (see Ferrini-Mundy, McCrory, Senk, & Marcus, 2005; Wilmot, 2008). The strength of the KAT conceptualization lies in the fact that their hypothesized knowledge are measurable and provides a good basis for measuring teacher knowledge in domain specific terms instead of relying on a proxy measure of it. However, instead of relying on the original KAT framework, the current study is designed to validate the expanded conceptualization posited by Wilmot (2016). Consequently, expanded conceptualization by Wilmot (2016) is the conceptual framework that guided the study.

The main standpoint is that unlike the original KAT framework, which concluded that the intersection of their three hypothesized knowledge types was burry and by implication not of any evidential value or necessary to be studied (Ferrini-Mundy, McCrory, Senk, & Marcus, 2005), the expanded framework of Wilmot (2016) suggests that such intersections produce some form of complex combination knowledge (see also Putnam 1987) that cannot be ignored. In this specific direction, the most distinguishing characteristics of the enhanced KAT framework by Wilmot (2016) are the four new types of knowledge arising from the intersection of the three measurable types of knowledge originally hypothesized by Ferrini-Mundy and her colleagues on the KAT project. The argument in this paper is that these four new additions together with the three hypothesized knowledge in the original KAT framework need to be re-examined in some further detail and consequently form the basis for this current empirical study. Consequently, in this current study this expanded KAT framework formed the conceptual framework. Figure 1 presents a schematic diagram of the expanded framework, which guided this study.

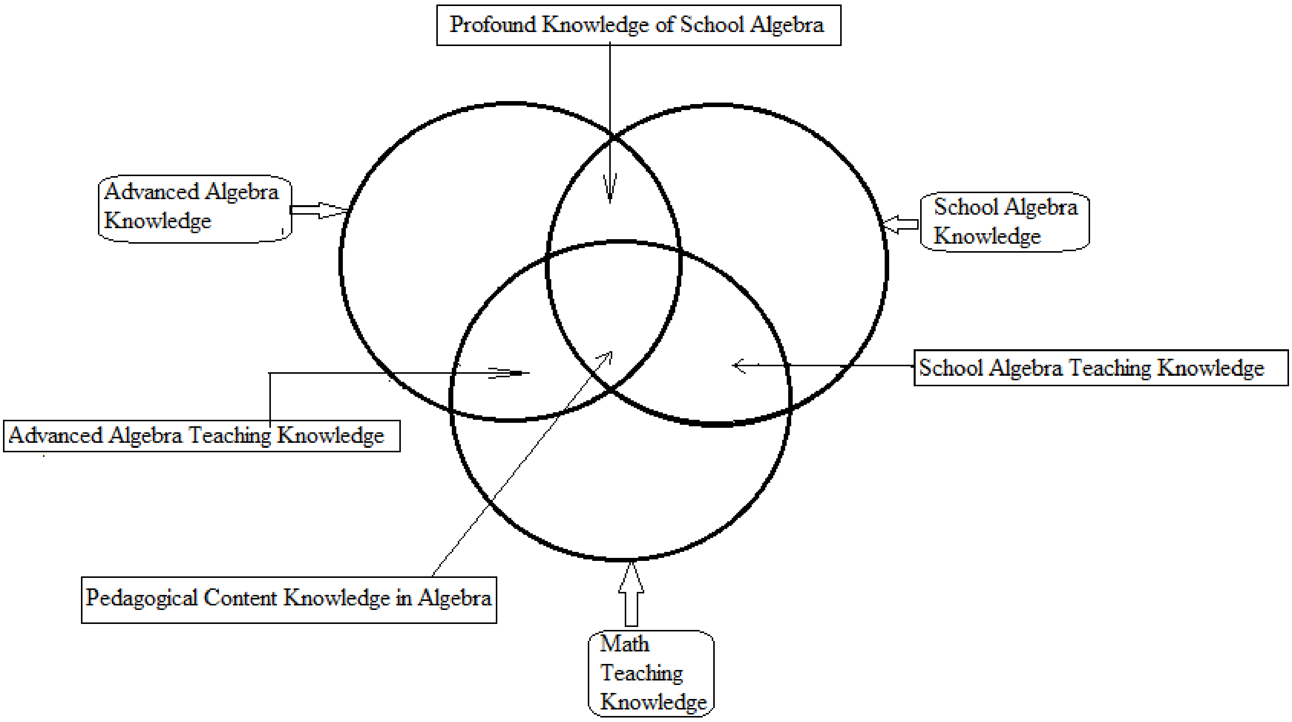

Figure 1: Expanded framework for reconceptualization of domain specific teacher knowledge

As shown in Figure 1, in the enhanced framework particular attention is still paid to the three hypothesized knowledge type that were originally considered by the KAT project (see Ferrini-Mundy, McCrory, Senk & Marcus, 2005) as potentially influencing knowledge of algebra needed for teaching (See Figure 1). Two differences can be pointed out between the original framework hypothesized by Ferriny-Mundy and her KAT researcher and the expanded framework suggested by Wilmot (2016). First, in the expanded framework, the original KAT hypothesized knowledge types have been qualified with "algebra" to emphasize the domain of mathematics being focused on. So for instance instead of *School Knowledge* as in the original framework we now have *School Algebra Knowledge*. The definitions ascribed to the original three types of knowledge continue to be the same in the expanded framework. Second, the outstanding features of the enhanced framework has to do with the intersections among the three key areas, namely, *Profound Knowledge of School Algebra*, *Advanced Algebra Teaching Knowledge*, *School Algebra Teaching Knowledge* and *Pedagogical Content Knowledge in Algebra* (Wilmot, 2016, p. 18).

Now, we return to a description of the nature of the enhanced features that were interrogated in this present study. First, *Profound Knowledge of School Algebra*, the type of knowledge formed as a result of the intersection of the School and Advanced Algebra Knowledge types. It is the type of teacher knowledge that makes a teacher demonstrate a deep understanding of algebra. It shows, for instance, in a teacher's ability to provide "alternate definitions, extensions and generalizations of familiar theorems, and a wide variety of applications of high school algebra" (Wilmot 2016, p. 18).

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Second, is *School Algebra Teaching Knowledge,* the type of knowledge formed as a result of the intersection of Mathematics Teaching Knowledge and the School Algebra Knowledge (i.e., Teaching Knowledge and School Knowledge respectively in the original KAT framework). It provides an indication that beyond the broad spectrum of school knowledge the teacher who possesses this type of knowledge has the ability to fall on the general knowledge of how to teach other domains of school mathematics to make instructional diversity possible when teaching school algebra. Specifically, the possession of this type of fused knowledge is what makes the teacher able to make connections with a range of mathematics and allied topics to school algebra, simplify concepts in algebra while maintaining standards of algebra teaching and finally, able to make learners comprehend algebra in the midst of any complexities that might exist.

Third, is the construct of *Advanced Algebra Teaching Knowledge* (formed by the intersection of Advanced Algebra Knowledge and Mathematics Teaching Knowledge). According to Wilmot (2016), a teacher's ability to bridge, trim and decompose algebraic concepts even at a stage more advanced that school algebra is the evidence of the possession of this type of knowledge. Thus, teachers who possess this type of knowledge have a good understanding of advanced algebra and are able to teach it when it becomes necessary.

Fourth, *Pedagogical Content Knowledge in Algebra* is an amalgamated intersection of the three measurable knowledge types that share the fine features of the enhanced KAT framework and rests at the very centre of the framework (See Figure 1). It is this type of knowledge that results from a complex interaction of the content (School Algebra Knowledge and the Advanced Algebra Knowledge) and pedagogical type of knowledge (i.e., the Mathematics Teaching Knowledge) of updated KAT framework.

Instrumentation

Wilmot (2008) had earlier adapted the original instrument developed and piloted by the Knowledge of Algebra for Teaching (KAT) project. His adaptation involved modifying the US contexts used in the original KAT instrument into Ghanaian contexts to make it meaningful for the Ghanaian participants of his study. However, Factor Analysis conducted by Wilmot (2008 & 2016) revealed a number of cross loadings that made it impossible for him to describe all the extracted factors and corroborate the three hypothesized knowledge in the KAT framework to the fullest. He therefore recommended, among other things, that more items be developed for a replication of his study. On the basis of this recommendation, a further adaptation of the KAT instrument was made before being used for this present study. This second adaptation involved merging the two instruments into one, developing multiple choice items out of the open ended ones and developing extra multiple ended items based on the KAT framework to increase the number of items. After this adaptation, the resultant instrument developed for this study comprised 80 items in all. In addition, since the original KAT items were found to be consistent with the content of algebra in the senior high school syllabus (both core and elective mathematics) in Ghana, at the time of the study, care was taken to ensure that the extra items that were added were also based on the content of algebra in the senior high school syllabus. Increasing the number of items on the instrument this way ensured that it covered a wider range of content high school algebra, as well as a wider range of advanced and teacher knowledge issues. It must be stated, however, that one major challenge with this instrument is the time frame required for participants to respond to the entire items on the instrument. Participants in the Wilmot (2008) study had used 60 minutes

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in completing their forms but in this study, participants had to be allowed to spend 3 hours because of the increased number of items.

According to Wilmot (2008) the reliability of the original KAT instruments was 0.837 for Form 1 and 0.842 for Form 2 when the instruments were piloted in the US. However, after the first adaptation and pilot in Ghana by Wilmot (2008) the reliabilities were reduced to 0.521 and 0.643 respectively. The resultant instrument developed for this study (see Yarkwah, 2018) was also piloted in one Region of Ghana. The pilot was done on 50 teachers in 10 different schools of similar characteristics as those used in the main study and yielded a reliability coefficient of 0.786 using the KR-20 formula. After factor analysis of the pilot data six items were eliminated due to their multiple loading and so a 74-item instrument was what eventually got used for the study.

Content validity of the instruments was established by subjecting the instrument for further review by two other mathematics education Professors in the Department of Mathematics and I.C.T. Education, University of Cape Coast and three doctoral students who had not less than 10 years experience each teaching Core and Elective Mathematics at the high school level in Ghana.

Selection of participants

Wilmot (2008) reported using only high performing schools in Ghana's educational system and recommended the need for future replication of the study to include schools in all the categorizations by the Ghana Education Service and also to ensure the schools selected are from more than one region. At the time of the study, senior high schools in Ghana had been into four categories: A, B, C and D (Ghana Education Service, 2009). The categorizations were based on the resources available to the schools. Following the recommendation by Wilmot (2008), the current study was conducted in senior high schools in three regions (Ashanti, Central, and the Western Regions) of the country (see Yarkwah, 2018).

A multi-stage sampling technique was resorted to in order to obtain the schools that participated in the study. In the various regions, a municipality or a district was chosen using the simple random sampling technique. Thereafter the stratified random sampling technique was used to put the various schools into strata using the GES categorization. From each of the selected categories, schools that were focused on were randomly selected. Altogether, 252 mathematics teachers from forty schools, from all the three regions and schools from all the four categories, (categories A, B, C, and D) in accordance with the Ghana Education Service (GES) classification participated in the study. These comprised thirty co-educational schools, five single-sex female and five single-sex male schools.

Data collection Procedure

The primary purpose of this research was to determine the extent to which the knowledge types hypothesized by Wilmot (2016) could be corroborated. To ensure confidentiality, names of teachers who participated in the study were not recorded on the instruments.

An initial visit was paid to the forty schools, which were finally involved in the research. During the visit, audience was sought from heads of the schools and the teachers who were going to be involved in the study. At the meeting, the purpose of the study, its duration, and potential benefits were explained to the heads and teachers for their consent to participate in

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the study and also allow the study to take place in their schools. Also, at these meetings, decisions about dates and times for the administration of the instrument were taken.

Altogether, administration of these instruments lasted for twenty weeks as two schools were covered in one week. In each school, the participating teachers were brought together in the staff common room after the close of classes so as not to disrupt normal class hours. Each session lasted for about three hours.

Data analysis and Discussion

As already explained, data for this study was obtained from teachers who teach Core Mathematics or Elective Mathematics or both in 40 schools within three Regions of Ghana using an adapted and expanded instrument from the KAT project. Exploratory factor analysis was performed on the data obtained for this validation.

Factor analysis was used in this study for three main reasons in agreement with what Bryman & Cramer (2001) have summarized. These are: 1) to try to make sense of the bewildering complexity of teachers' knowledge for teaching algebra by reducing it to a more limited number of factors or variables, 2) to find out the extent to which items measuring the same concept could load together on specific factors and, and 3) the degree to which the number of factors can be reduced to a more limited number in order to make decision, in this case, about the dominant factors as far as senior high school teachers' knowledge for teaching algebra is concerned. In other words, in this study, factor analysis was used to derive the variables, called factors, which gave better understanding about the data collected.

To begin the process of identifying the reasonable number of extracted factors from the data, Table 1 was used. This table shows the variance explained when the various numbers of factors (components) are retained and the corresponding eigenvalues. In other words, it reveals the number of possible factors that could be extracted from the data to explain the variation among the scores and their corresponding eigenvalues. The "% of Variance" column explains the variation in scores of the items explained by that component or factor when it is the only one retained in the analysis, while the cumulative percentage column reveals the total/cumulative variation in scores when the corresponding number of items is retained in the analysis. As would be expected in such factor analyses, extracting all the components or factors, should point to the explanation of all the variation in scores of the items. Consequently, factor 74 corresponds to a cumulative percentage variance of 100% since there were 74 items on the instrument. The eigenvalues give an indication of the strength level of each of the extracted number of factors.

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Table 1: Total Variance Explained

Component	Initial Eigenvalues		
	Total	% of Variance	Cumulative %
1	8.653	11.693	11.693
2	3.320	4.486	16.179
3	2.956	3.994	20.174
4	2.702	3.651	23.824
5	2.442	3.301	27.125
6	2.315	3.129	30.253
7	2.174	2.937	33.191
8	2.109	2.851	36.041
9	1.903	2.571	38.612
10	1.844	2.492	41.104
11	1.788	2.417	43.521
12	1.666	2.252	45.772
13	1.635	2.210	47.982
14	1.576	2.130	50.112
15	1.526	2.062	52.174
16	1.455	1.967	54.141
17	1.376	1.859	56.000
18	1.352	1.827	57.827
19	1.318	1.781	59.608
20	1.267	1.712	61.320
21	1.211	1.636	62.956
22	1.156	1.562	64.519
23	1.101	1.488	66.006
24	1.087	1.469	67.476
25	1.033	1.397	68.872
26	1.026	1.387	70.259
27	.986	1.332	71.592
28	.961	1.299	72.891
29	.928	1.254	74.144
30	.899	1.215	75.360
31	.849	1.148	76.507
32	.827	1.118	77.626
33	.791	1.069	78.695
34	.747	1.009	79.704
35	.738	.998	80.702
36	.720	.973	81.675
37	.694	.938	82.614
38	.663	.896	83.509
39	.635	.858	84.367
40	.612	.827	85.194
41	.607	.820	86.014
42	.569	.768	86.783
43	.562	.760	87.542
44	.531	.717	88.259
45	.486	.657	88.917
46	.475	.641	89.558
47	.456	.617	90.175
48	.445	.601	90.776
49	.426	.575	91.351

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Component	Initial Eigenvalues		
	Total	% of Variance	Cumulative %
50	.413	.558	91.909
51	.402	.544	92.452
52	.392	.530	92.982
53	.375	.506	93.488
54	.350	.474	93.962
55	.345	.466	94.428
56	.319	.431	94.859
57	.308	.416	95.275
58	.296	.400	95.675
59	.279	.377	96.052
60	.272	.368	96.420
61	.258	.349	96.769
62	.244	.330	97.099
63	.231	.312	97.411
64	.219	.296	97.706
65	.216	.291	97.998
66	.209	.282	98.280
67	.202	.274	98.553
68	.194	.262	98.815
69	.172	.232	99.047
70	.166	.224	99.272
71	.146	.197	99.468
72	.137	.186	99.654
73	.133	.180	99.834
74	.123	.166	100.000

In terms of the eigenvalues, since in general, a low eigenvalue for a given component implies that factor's contribution to the explanation of variances in the variables is small and may be ignored, the first decision about the number of extracted factors was based on the Kaiser-criterion (also referred to as the K-1 rule) of retaining only the factors with eigenvalues greater than 1.0 (see the second column of Table 1).

A cursory look at Table 1 shows that applying the K-1 rule would have resulted in settling on 26 as the number of retained factors and these together would have explained approximately 70.3% of the variance. However, since the theoretical framework guiding this study hypothesizes seven knowledge types, (3 main and 4 form the regions of overlap) it was concluded that applying the Kaiser criterion would lead to an exaggeration. Consequently, the scree-test plot was used for further check. Figure 2 shows this scree plot.

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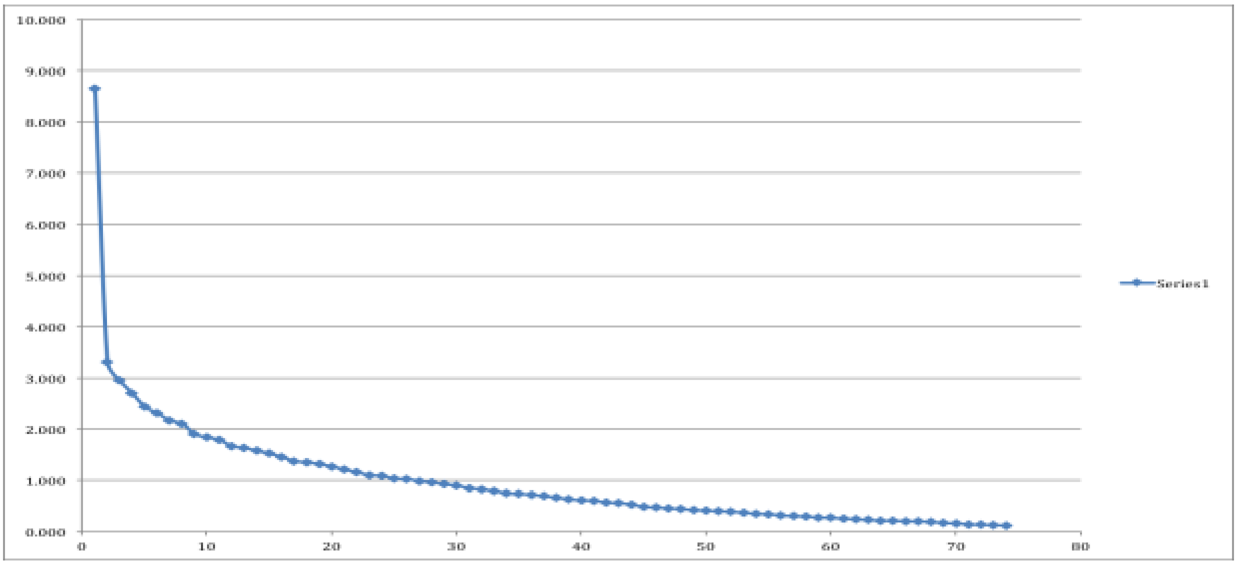

Figure 2: Scree plot of the factor loadings

From Figure 2, it will be observed that the elbow of the graph, or the sharp break point as Cattel (1993) calls it, can be seen to exist at either factor number 7, 8 or 9. Since, such interpretations from scree plots are based on visual observation of the elbow of the graph, it is mostly subjective (Hayton, Allen & Scarpello, 2004), making it possible for factor 7, 8 or 9 to be settled on depending on the person doing the analysis. To remove this subjectivity, we resulted to applying the suggestion by Nelson (2005) of superimposing the regression line on the scree plot. The resultant graph is presented in Figure 3.

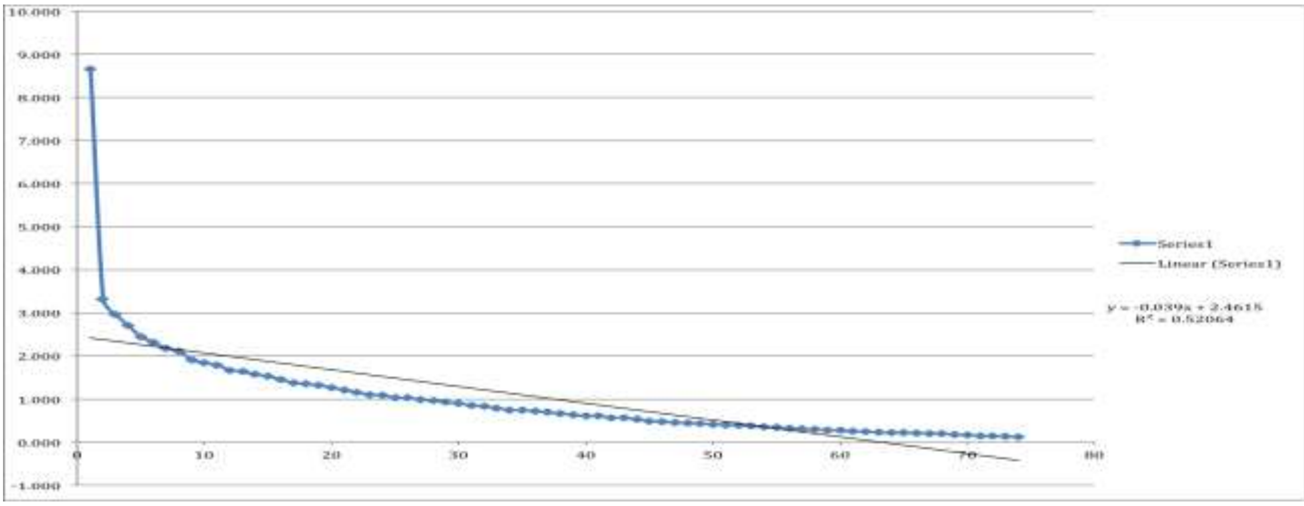

Figure 3: Graph of the regression line superimposed on the scree plot

From Figure 3, it is clear that moving from left to right the regression line intersects the scree plot on the left hand side at factor 7. For further analysis, it was therefore concluded that 7

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factors were retained. Looking back at Table 1, it can be seen that, together, the seven factors contributed to explaining 33.191% of the variance of the scores. The implication of this is that other factors, not focused on in this study, also have the potential of causing and therefore accounting for part of the variation in the scores.

Next, in an attempt at examining the nature of the seven factors factor loadings from the analysis were focused on. To do this, it was considered that loadings of absolute value above 0.30 were strong enough to be indicative of the nature of the factor (Guadagnoli & Velicer, 1988)). In addition, items with cross loadings were dropped from the analysis since it was impossible to uniquely assign any item that loaded strongly on more than one factor to any of the factors on which it loaded. Table 2 shows how the items loaded on the seven factors. The resultant item loadings after these considerations are presented in Table 2.

Table 2: Item loadings on the seven retained factors

	Component						
	1	2	3	4	5	6	7
Q18	.656						
Q20	.597						
Q39	.554						
Q32	.546						
Q30	.525						
Q35	.512						
Q57	.509						
Q29	.509						
Q63	.469						
Q31	.464						
Q53	.464						
Q3	.448						
Q23	.443						
Q25	.420						
Q46	.398						
Q56	.334						
Q27		.660					
Q12		.625					
Q24		.558					
Q26		.501					
Q59		.459					
Q1		.448					
Q45		.420					
Q11		.361					
Q52		.313					
Q62			.670				
Q71			.547				
Q68			-.392				
Q22			.349				
Q9			.346				
Q38			-.332				
Q49				.644			
Q51				.573			
Q65				.462			
Q70				.354			

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	Component						
	1	2	3	4	5	6	7
Q64				.348			
Q43					.505		
Q66					.436		
Q36					.428		
Q7					.361		
Q60					.354		
Q61						.550	
Q50						.515	
Q37						.363	
Q4							.460
Q72							.437
Q16							-.360

extraction method: principal component analysis.
rotation method: varimax with kaiser normalization.
a. rotation converged in 13 iterations.

A cursory look at Table 2 reveals that, 46 items uniquely loaded on the 7 retained factors (after removing items that loaded strongly on more than one factor). In addition, each of the 7 retained factors had not less than three items uniquely loading on them. It therefore became necessary to take a closer look at the nature of the items uniquely loading on each factor (using the item categorizations) to see the extent to which they lend support to the naming of the factors. The items that uniquely loaded on each of the factors presented in Table 3.

Table 3: Summary of item loadings with item categorizations

No. OF LOADED ITEMS	**Factors (with item categorization in parenthesis)**						
	1	**2**	**3**	**4**	**5**	**6**	**7**
1	18(SK)	27(AK)	71(TK)	49(TK)	43(TK)	61(AK)	4(AK)
2	20(SK)	12(AK)	68(TK)	51(TK)	66(TK)	50(AK)	72(SK)
3	39(SK)	24(AK)	22(SK)	65(AK)	36(TK)	37(AK)	16 (TK)
4	32(SK)	26(SK)	9(SK)	70(AK)	7(SK)*		
5	30(SK)	59(SK)	38(SK)	64(TK)	60(TK)		
6	35(SK)	1(SK)					
7	57(SK)	45(AK)					
8	29(SK)	11(SK)					
9	63(SK)	52(AK)					
10	31(SK)						
11	53(SK)						
12	3(SK)						
13	23(SK)						
14	25(SK)						
15	46(SK)						
16	56(SK)						

*outlier

To discuss Table 3, we first concentrate on the naming of those factors that correspond to the three main hypothesized knowledge types from the original KAT framework, which formed

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the basis of item formulation for this study. After this, a discussion of the item loadings that confirm that the intersections of the three types of knowledge are not blurry, and which together with the three hypothesized knowledge validates the conceptual framework that guided this study is presented.

A cursory look at Table 3 reveals that all the 16 items that uniquely loaded on Factor 1 were, prior to the fieldwork, categorized as School Knowledge items. Similarly, all the three items that loaded on Factor 6 were Advanced Knowledge items. Using the conceptual framework that guided the study, Factors 1 and 6 were therefore named as the **School Knowledge** and **Advanced Knowledge** factors respectively. On Factor 5, however, there was an outlier. Of the five items that uniquely loaded on this factor, four were previously categorised as Teaching Knowledge items and one (Item number 7, the outlier) had been categorised as a School Knowledge item. This Item 7 was formulated as:

> *"Given a set D whose elements are odd integers, positive and negative, (zero is not an odd integer). Which of the following operations when applied to any pair of elements will yield only elements of D?"*
>
> i. *Addition*
>
> ii. *Multiplication*
>
> iii. *Division*
>
> iv. *Finding the arithmetic mean*
>
> *The correct answer is*
>
> A. *i and ii only*
>
> B. *ii and iv only*
>
> C. *ii, iii, and iv only*
>
> D. *ii and iii only*
>
> E. *ii only*

This item was categorized as a School Knowledge item because it was felt that the component concept of odd integers was a concept in the curriculum of school mathematics. In addition, the idea of "closure" which was being tested was one of the things taught in school mathematics under the topics, *Operations*, at the Senior High School level in Ghana. However, a discussion of how this question was answered by 50 of the participants randomly selected revealed that most of them who had it right put themselves in a teaching situation. The following vignette exemplifies how a typical teacher (one of the most eloquent participants) explained her approach:

Researcher: Did you answer this question? (pointing to Item 7)

Participant: Oh yes

Researcher: Please explain to me how you got your answer.

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Participant: It is possible to add two odd integers and get zero (for example –2 and 2). Division can also result in fractions. (for example 1 dived by 3). Finally, the arithmetic mean of two odd integers can be an even integer (for example, the arithmetic mean of 3 and 5 is 4). Hence, Addition, Division and Arithmetic Mean are all out. So the answer is E (the product of any two odd integers is always an odd integer). As teachers we ought to teach students to know this.

Responding to this question in this manner pointed to the possibility that the cognitive demands of this question is perhaps those of the type presented in the framework under the Teaching Knowledge category. It was therefore not surprising that Item 7 loaded with four of the Teaching Knowledge items. In fact, the manner in which it was answered makes it possible for Item 7 to even be re-categorised as a Teaching Knowledge item. From the foregoing, Factor 5 was named as the **Teaching Knowledge** factor.

Also it is clear from Table 3 that the other factors, Factors 2, 3, 4 and 7 are the factors that confirm that the intersections of the three types of knowledge are not blurry. For instance Factor 2 had five Advanced Knowledge and four School Knowledge items uniquely loading on it, Factor 3 had three School Knowledge and two Teaching Knowledge items while Factor 4 had three Teaching Knowledge and two Advanced Knowledge items loading on it. These three factors were thus respectively named as in the light of the conceptual framework that guided the study. Finally, of the three items that loaded on Factor 7, one each was School Knowledge, Advanced Knowledge, and Teaching Knowledge. Thus, Factor 7 was considered the type of knowledge representing the intersection of the three main types of knowledge. It was thus named the **pedagogical content knowledge in algebra knowledge** (refer to Figure 1).

CONCLUSIONS AND RECOMMENDATIONS

Analyses of data from this study led to a number of conclusions. First, it is clear that the Expanded KAT framework proposed by Wilmot (2016) has been validated. This is significant to the discussion of issues of conceptualization of teacher knowledge because, undertaking this study with algebra, a specific domain of mathematics, means that rather than discussing teacher knowledge in a generalized and mostly qualitative manner (as in for example Shulman, 1986a, 1987; Wilson, Shulman & Richert, 1987 Ball, 1988; Grossman, 1990) the findings of this study points to the need to shift such discussions into more domain specific terms. A replication of this study is therefore recommended using the other domains of mathematics as focus and even in the other school subjects.

A second conclusion, which is implicit in the aforementioned, is that results of this study have also fully validated the three types of knowledge originally hypothesized by the KAT researchers. The implication of this for teacher educators is that the need to provide opportunities for each type of knowledge to be developed by prospective teachers, as well as practicing teachers is recommended. In addition, validation of the three main knowledge types paves the way for research into which aspects of teacher knowledge could best predict student performance; a finding that would have massive effect on curricula emphasis of teacher education programmes and other professional development programmes for teachers.

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A third conclusion from this study is that, contrary to the claim by the KAT researchers that the intersection of the three main knowledge types is blurry, the validation of the Expanded KAT framework points to the fact the intersections are not blurry. This finding of the non-blurry nature of the intersections is in line with what Putnam (1987) refers to as "curriculum scripts". The idea is that in the course of their work, experience teachers do not rely solely on discrete knowledge packages but on amalgams of the knowledge, such as the intersection of the three knowledge types in the KAT framework, to support students' learning. As Wilmot (2008) argues, "it is these curriculum scripts, which shape teachers' agenda for teaching and not … (intuitive knowledge of students). The curriculum scripts that experience teachers possess enable them to adopt flexible and interactive approaches to teaching and enhance their efficiency" (p. 39-40).

Fourth, it must be emphasized that the present study relied on an instrument comprising items developed based on the three types of knowledge in the framework originally hypothesized by the KAT researchers. Thus, the KAT framework therefore provided the basis for developing the items in the instrument used in this study. Using an instrument comprising such specific content items in this study has added to the discussion on how to measure knowledge in the field by providing data to support the fact rather than rely on proxy measures, teacher knowledge in algebra (and by extension any domain of mathematics or any school subject) can be more objectively measured. Adaptation of the ideas used to develop the instrument used in this study (as is apparent in the original KAT framework) is thus, recommended for future studies and educators interested in developing measures for teacher certification.

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