CONCEPT MAP AND TARGET TASK APPROACHES AS METHOD COMBINATION THERAPY FOR TEACHING ALGEBRA IN SCHOOLS

Awodeyi, A. F.
Department of Science Education
University of Uyo
Akwa Ibom State.

ABSTRACT: This paper formulated and discussed a Method Combination Therapy involving concept map and target task approaches to teaching factorisation of quadratic expressions to secondary school students (age 15 to 16). The paper considered the Type A and Type B forms of quadratic expressions making clear distinctions between both. The quadratic expressions having been factorised were applied to finding the solutions to quadratic equations. Sufficient activities were presented for both the teacher and the learners for mastery. The Method Combination Therapy is recommended for wider publication and usage in of teaching algebra in secondary schools.

KEYWORDS: Concept Map, Target Task Approaches, Therapy, Algebra, School

INTRODUCTION

Mathematics is generally believed to be the bedrock of Science. It is also true that mathematics is a tool for everyday activities of man. The importance of the subject is acknowledged in the policy of government of the Federal Republic of Nigeria where mathematics is made a core subject in all primary and secondary schools nationwide (FME, 2004). It is also the policy of government that prospective candidates for admission into the tertiary institutions for science or science related courses must have obtained a credit level pass in mathematics prior to their admission. Furthermore, students’ admission into the Universities had been fixed for science and Arts in the ratio of 60 to 40 as a matter of policy (FRN, 2004). Earlier in the country, a general poor performance of pupils at the Federal common entrance examination for the placement of candidates into Federal Government Colleges had triggered public outcry nationwide; because many pupils could not secure admission. Parents had decried the introduction of ‘modern’ mathematics into the curriculum of schools without adequate preparation for its success. The public outcry prompted government to summon mathematicians and mathematics educators for conference in Benin (FME, 1977). This was also the era when modern and traditional mathematics programs were going on simultaneously in schools, leading to a confusion of a sort in the system and mass failure of students at the West African Senior School Certificate Examination (WASSCE).

Major decisions at the Benin Conference include: the production of mathematics curricula for schools with the tags ‘modern’ or ‘traditional’ removed; the teaching of the curricula by competent teachers who are capable of securing the interest of learners in the subject. In
particular, seven objectives were stated for teaching mathematics at the secondary school level on page 4 of the original conference report. These are:

- To generate interest in mathematics and to provide a solid foundation for everyday living;
- To develop computational skills;
- To foster the desire and ability to be accurate to a degree relevant to the problem at hand;
- To develop precise, logical and abstract thinking;
- To develop ability to recognise problems and to solve them with related mathematical knowledge;
- To provide necessary mathematical background for further education;
- To stimulate creativity (FME, 1977:4).

The report was matched with action as appropriate curricula were consequently produced for schools at the levels of primary, junior secondary and senior secondary levels (FME, 1979; 1982; 1985). These curricula had been revised twice between 1985 and now by the Nigerian Education Research and Development Council (NERDC) (FME, 2006a, 2006b, 2007a and 2007b). The Mathematical Association of Nigeria (MAN) and other scholars had prepared textbooks to match various levels of the curricula. It was anticipated therefore that students’ achievement will be enhanced significantly following the curriculum review.

The Problem
The achievement of students in school mathematics as measured by their performance in the West African Senior School Certificate Examination (WASSCE) and the National Examination Council (NECO) is still poor. The output is neither commensurate with inputs of government in aspects of curriculum review, nor reflect the efforts of institutions such as the Mathematical Associations of Nigeria (MAN) and the Science Teachers Association of Nigeria (STAN) that holds conferences and workshops yearly, during which mathematics teachers should become conversant with best practices of mathematics teaching. It would appear that many school teachers were not availing themselves of the opportunity provided by the conferences and workshops to upgrade their skills. The major problem obviously is that of uninspired mathematics teaching.

Purpose of this Paper
The general purpose of this paper is to present a peer reviewed work on best practice in mathematics teaching for wider accessibility to the academic community at large for perusal, application and inputs. Specifically, the paper presents a Method Combination Therapy (MCT) which involves concept map, and target task approaches, for teaching factorisation of algebraic expressions and the solution of quadratic equations to secondary school students. The approach was first presented at the workshop of MAN during her 49th National Annual Conference at Abuja, 2nd - 7th September, 2012 for peer review.
Key Terms Used in this Paper

Concept Map:

A concept map is an informative lesson plan. It is an excellent way to teach a particular concept. It can be used for teaching some mathematical concepts. It makes use of graphical tools that organise and represent knowledge in a sequential order. In a concept map, ideas are organised by enclosing them in boxes or circles. They are connected by lines showing relationship between steps. The relationship between boxes may be represented by writing proper phrases on connecting lines. (Awodeyi, 2000; Eshiet, 2007; James, 2012; Novak, 1990 and 1984).

Target Task Approach (TTA)

Task Based Instruction (TBI) or Task Based Language Teaching (TBLT) was employed long ago to facilitate learning in communicative language (Ellis, 2003). The Task Based Instruction has its root in progressivism - a philosophy based on the idea of progress, which asserts that advancement in science, technology, economic development, and social organisation are vital to improve the human condition. Progressivism holds that active learning is more effective than passive learning (Pica, 2016). Progressivism therefore serves as theoretical framework for the current study.

The TBI in the present study is found suitable in science teaching and it is therefore adapted as Target Task Approach (TTA) to facilitate mathematics learning among students. There are five essential components of TTA as used in the present study. These are:

- A task involves a primary focus in problem solving
- A task has some kind of gap such as reasoning gap
- The gap which must be filled is often represented with a rectangular figure
- The problem solver or student choose the rule of engagement (e.g. a proposition, theorem, or law) needed to complete the task
- A task has a clearly defined outcome.

Target Task Approach requires a pre-task during which the teacher will present what will be expected of the student in the task (i.e. specific objective). In the process of the task, students perform the task typically in small groups with the teacher merely facilitating that is a student centred methodology (Awodeyi and Harbour-Peters, 1993). The teacher may prime the students with the key rule. An example is expected to be discussed with the students. The TTA is a procedure that necessarily requires immediate result at a level of solution before proceeding to the next. It is guidance oriented.

Method Combination Therapy:

The Method Combination Therapy as used in the present paper, is a blend of the Concept Map method and the Target Task Approach, among others to bring about an enhanced learning of algebra in schools.

Quadratic Equations by Method of Factorisation:

The Chief Examiners Reports of the West African Examinations Council (WAEC) are annual publication reports of the marking exercise that comes up immediately after the conclusion of
marking of scripts. In the reports, questions which were not adequately answered by candidates were highlighted and suggestions for improvement given. Students problems that had been reported bother on the inability of candidates to be accurate to a given specification, or making precise logical and abstract thinking. The manipulative skills of candidates were also found to be suspect in some cases (West African Examination Council Chief Examiners Report, 2010). Observed laxities in students’ answer scripts as reported can only be corrected if future students would be motivated intrinsically to learn mathematics. Factorisation and its application to quadratic equations are strategic in the curriculum for mathematics in secondary schools. The topic contains abstract thinking, precise logical thinking, and manipulative skills that would prepare students adequately for mathematics/algebra at their future tertiary education.

**Procedure of Method Combination Therapy:**

Certain materials are required for the preparation of concept map. These are pen and paper; electronic board or the computer if the teacher is computer literate. The structure of the MCT is such that solution to the given problem is presented from the general problem to the specific answers, in descending order. The specifics are the targets which occur at some stages en-route the final solution. The tasks required are to obtain the targets accurately.

**Illustration of Application of Method Combination Therapy to Algebra:**

The quadratic expressions in this work are categorised into two. These are types A and B. It is so organised for convenience. Type A is of the form \( ax^2 + bx + c \), where \( a=1, \ b, \ c \in I \), and Type B is of the form \( ax^2 + bx + c \), where \( a \neq 1, \ b, \ c \in I \). The two types are illustrated in examples below.

Type A: Factorisation of the form \( ax^2 + bx + c \), where \( a = 1, \ b, \ c \in I \) are constants.

**Example1. Factorise** \( x^2 +5x + 6 \)

The required activities in this example are illustrated in Table1. The Table contains the skills, the game, the leisure and the specifications which the teacher would facilitate students to acquire. The first target is to consider the coefficient of the middle term of the quadratic expression and make of an intelligent guess of possible pairs of numbers whose sum will be equal to it. In this example, we consider \( \pm 1 \) and \( \pm 4; \pm 2 \) and \( \pm 3 \). The process of making this intelligent guess is itself a logical reasoning, which is an objective of mathematics teaching.
Table 1: Factorisation Game of $x^2 + 5x + 6$, where $b= +5$ and $c= +6$.

<table>
<thead>
<tr>
<th>s/n</th>
<th>Possible pairs of numbers</th>
<th>Sum of pairs</th>
<th>Product of pairs</th>
<th>Specification required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 4</td>
<td>5</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1, 4</td>
<td>5</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1, -4</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-1, -4</td>
<td>-5</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2, 3</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-2, 3</td>
<td>1</td>
<td>-6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2, -3</td>
<td>-1</td>
<td>-6</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-2, -3</td>
<td>-5</td>
<td>6</td>
<td>*</td>
</tr>
</tbody>
</table>

* **Initial target** i.e. the only pair of numbers whose sum is 5 and product is 6.

The required activity of the teacher in the lesson is to supply values to a few of the boxes while students complete the rest. The teacher would draw attention of students to s/n 5 on Table 1, and guide them to link this up with the second and third boxes on the concept map in figure 1.

**The Concept Map of Factorisation of $x^2 + 5x + 6$:**

In the concept map below, the targets to meet are indicated by rectangular boxes. The boxes are to be filled accurately. Following the identification of 2 and 3 as the specific pair of numbers required, the middle term of the quadratic expression (i.e. 5x) can be split as $2x + 3x$ and put into use in the concept map (Figure 1).

**The Concept Map of solution to: $x^2 + 5x + 6$**

![Concept Map](image)

Figure 1: Factorisation of $x^2 + 5x + 6$ in sequence

**Application of obtained factors to quadratic equation $x^2 + 5x + 6 = 0$**

If given that $x^2 + 5x + 6 = 0$, then we set $(x + 2)(x + 3) = 0$.

Either $(x + 2) = 0$ or $(x + 3) = 0$. This implies that $x + 2 = 0$ or $x + 3 = 0$. 

Print ISSN: ISSN 2054-6297, Online ISSN: ISSN 2054-6300
It follows that \( x = -2 \) or \( -3 \) and the roots of the quadratic equation are \( -2 \) and \( -3 \).

This application is an exhibition of precise logical and abstraction reasoning.

A second example of Type A is given below:

**Example 2: Factorise** \( x^2 - 5x + 6 \)

The difference between examples 1 and 2 must be noted. The middle term of the quadratic expression in the first example is \( 5x \), while that of the second is \(-5x\).

**Teacher and students’ activities in example 2:**

The teacher is expected to take students through the process of identifying a pair of factors of 5 whose sum is \(-5\) and whose product is \(+6\) as shown on Table 2. Students would on the long run master the science of identifying very quickly the required pairs of factors to be adopted on the concept map. Gifted students may achieve this after the first example in Table 1, while it may take others the second example in Table 2 or much longer before mastery.

**Table 2: Factorisation table of** \( x^2 - 5x + 6 \)

<table>
<thead>
<tr>
<th>s/n</th>
<th>Possible pairs of numbers</th>
<th>Sum of pairs of numbers</th>
<th>Product of pairs of numbers</th>
<th>specification required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 4</td>
<td>5</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1, 4</td>
<td>3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1, -4</td>
<td>-3</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-1, -4</td>
<td>-5</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2, 3</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-2, 3</td>
<td>1</td>
<td>-6</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2, -3</td>
<td>-1</td>
<td>-6</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-2, -3</td>
<td>-5</td>
<td>6</td>
<td>*</td>
</tr>
</tbody>
</table>

*Initial target i.e. the only pair of factors of 5 whose sum is \(-5\) and product is \(+6\).*

Following the identification of \(-2\) and \(-3\) as the specific pair of numbers required, the middle term of the quadratic expression (i.e. \(-5x\)) will split as \(-3x\) and \(-2x\). The coefficients of these (i.e. \(-2\) and \(-3\)) are put into use in the concept map (Figure 2).
The Concept Map of solution to: \( x^2 - 5x + 6 \)

![Concept Map](image)

Application of obtained factors to the quadratic equation: \( x^2 - 5x + 6 = 0 \)

When given that \( x^2 - 5x + 6 = 0 \), as a quadratic equation problem and it is required to find the roots of the equation: Set \((x - 3)(x - 2) = 0\). It’s either \((x - 3) = 0\) or \((x - 2) = 0\).

That is, \(x - 3 = 0\) or \(x - 2 = 0\). Thisimplies that \(x = 3\) or \(x = 2\).

The roots of the quadratic equation are 3 and 2.

Class Quiz:
Which of the following quadratic expressions may be factorized using the examples given above? State your reasons.

(i) \( x^2 - 8x + 3 \)
(ii) \( x^2 + 3x - 2 \)
(iii) \( x^2 + 2x - 2 \)
(iv) \( x^2 + 4x + 4 \)
(v) \( x^2 + 3x \)

The essence of the class exercise is to let students know the limitation of method of factorisation. If the coefficient of the middle term of the quadratic expression cannot be split into two as illustrated then the factorisation method breaks down, and another method is sought. Other methods are: completing the square, quadratic formula, and graph. These other methods are not discussed in this paper.

Type B: The form \( ax^2 + bx + c \), where \( a \neq 1, b, c \in I \).

In type B, the rule is that you compute \( ac \) (i.e. the product of \( a \) and \( c \)). Then you search for two pairs of numbers whose sum is \( b \), and whose product is \( ac \).
Example 3: Factorise $2x^2 - x - 1$

In this example, $a = 2$, $b = -1$, $c = -1$ and $ac = -2$. The intelligently guessed out pairs of numbers are $\pm 1$ and $\pm 2$ and are adopted in the preparation of Table 3.

### Table 3: Factorisation Game of $2x^2 - x - 1$, where $b = -1$ and $ac = -2$.

<table>
<thead>
<tr>
<th>s/n</th>
<th>Possible pairs of numbers</th>
<th>Sum of pairs</th>
<th>Product of pairs</th>
<th>specification required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 2</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1, -2</td>
<td>-1</td>
<td>-2</td>
<td>*</td>
</tr>
<tr>
<td>3</td>
<td>-1, -2</td>
<td>-3</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

* Initial target i.e. the only pair of numbers whose sum is $-1$ and product is $-2$.

Following the identification of $+1$ and $-2$ as the required pair of numbers required, the middle term of the quadratic expression (i.e. $-x$) can be split as $-2x + x$ and their coefficients put into use in the concept map (Figure 3).

### The Concept Map of Factorisation of $2x^2 - x - 1$

![Concept Map]

Figure 3: The factors of $2x^2 - x - 1$ are $(2x + 1)$ and $(x - 1)$

**Application of obtained factors to the quadratic equation** $2x^2 - x - 1 = 0$

If $2x^2 - x - 1 = 0$, then $(2x + 1) (x - 1) = 0$. Its either $(2x + 1) = 0$ or $(x - 1) = 0$

This implies that $2x + 1 = 0$ or $x - 1 = 0$. Hence $x = \frac{1}{2}$ or $x = 1$, are the roots.
Example 4: Factorise \(2x^2 - 5x + 3\)

The initial pairs of numbers to be considered are \(\pm 1\) and \(\pm 4; \pm 2\) and \(\pm 3\).

**Table 4: Factorisation table of \(2x^2 - 5x + 3\), where \(b = -5\) and \(ac = 6\).**

<table>
<thead>
<tr>
<th>s/n</th>
<th>Possible pairs</th>
<th>Sum of pairs</th>
<th>Product of pairs</th>
<th>specification required</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 4</td>
<td>5</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1, -4</td>
<td>-5</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-2, -3</td>
<td>-5</td>
<td>6</td>
<td>*</td>
</tr>
<tr>
<td>4</td>
<td>2, 3</td>
<td>5</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

* Initial target i.e. the only pair of numbers whose sum is –5 and product is 6

Following the identification of –2 and –3 as the required pair of numbers required, the middle term of the quadratic expression (i.e. –5x) splits as –2x and –3x, and are put into use as first target in the concept map (Figure 4).

**The Concept Map of solution to quadratic equation \(2x^2 - 5x + 3 = 0\)**

![Image of concept map]

Figure 4: The factors of \(2x^2 - 5x + 3\) are \((2x + 3)\) and \((x - 1)\)

**Application of obtained factors to solution of quadratic equation \(2x^2 - 5x +3= 0\)**

Given that \(2x^2 - 5x + 3 = 0\) and the roots of the quadratic equation is required, then set \((2x + 3) (x - 1) = 0\). It is either \(2x + 3 = 0\) or \(x - 1 = 0\). This gives \(x = -\frac{3}{2}\) or \(x = 1\) as the roots of the quadratic equation.
Class Quiz:
Which of the following quadratic equations may be solved using the method of factorization?
State your reason.
(i) \(2x^2 + 5x + 3 = 0\)
(ii) \(3x^2 - 4x + 1 = 0\)
(iii) \(3x^2 - 5x - 3 = 0\)
(iv) \(6x^2 + 13x + 6 = 0\)
(v) \(5x^2 - 6x - 3 = 0\)

CONCLUSION

The teaching of factorisation to secondary school students using a Method Combination Therapy involving a concept map and the Target Task approaches was discussed in this paper. The application of factorisation to solving quadratic equations was also discussed. It is worthwhile to note that there are other methods of solving quadratic equations which are not discussed in this paper. The Method Combination Therapy sufficiently laid the foundation for students towards solving quadratic equations. Teachers and learners would observe that the teaching of factorisation as carried out in this paper contains much of guided manipulation of numbers and computational skills that should be acquired by students. This is in line with the objectives of mathematics teaching in secondary schools (FME, 1977). Teachers will also note that factorisation of quadratic expressions provide students the opportunity to reason logically, precisely and accurately; especially where students have to find pairs of factors of a number under specified conditions as illustrated in types A and B.

RECOMMENDATIONS

Based on the conclusion drawn above, the following recommendations are made:

- The teaching of factorisation using the Method Combination Therapy (MCT) involving a concept map and the target task approach is recommended for use by secondary school teachers.
- Teachers may find it worthwhile, to compare this approach with the expository with the view of determining existence or otherwise of a significant difference between achievements of students when taught using them.

REFERENCES


