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COMPUTATION OF NUCLEAR BINDING ENERGY AND INCOMPRESSIBILITY WITH A NEW M3Y-TYPE EFFECTIVE INTERACTION

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ABSTRACT: *This paper presents the computation of nuclear binding energy per nucleon and incompressibility of infinite nuclear matter with a new effective interaction obtained on the basis of the lowest order constrained variational approach. Using the interaction in its density-dependent form, a binding energy per nucleon* ε *= -16MeV was reproduced at a saturation density,* $\rho_0 = 0.17fm^{-3}$*. It has also been used in a zero-range pseudo-potential approximation to obtain a range of values of incompressibility (*$K\infty$ *= 301 - 307 MeV) based on a choice of a narrow range of acceptable values of saturation density. The results of the computation, when compared with previous work, have impressively proven to be in good agreement, suggesting that the new interaction is viable and might do well in folding calculations with an appropriate inclusion of density and energy dependence like its M3Y-Reid counterpart.*

KEYWORDS: Computation, Nuclear Binding Energy, M3y-Type Effective Interaction

INTRODUCTION

In recent years, successful efforts have been made within the domain of Nuclear Physics to develop a number of effective interactions based on approaches ranging from empirical fit of experimental data to microscopic derivation from the bare nucleon-nucleon (NN) interaction. Some of these approaches are the Gmatrix approach [1, 4], lowest order constrained variational (LOCV) principle [9], relativistic mean-field theory (RMF) [27], chiral quark models based on quantum chromodynamics (QCD) [32] and effective field theory (EFT)[25].

Irrespective of the manner of derivation, these effective interactions have the infinite nuclear matter as an important testing ground and a source of invention of new tools with which to treat the quantitative relationship between the two-body forces and nuclear properties [5,18]. The main applications of nuclear matter have so far concerned the nuclear binding energy, equilibrium density and neutron star [16, 20, 26, 33]. The reproduction of these values together with the incompressibility remains one of the aims of infinite nuclear matter calculations [10]. This has ever been a fundamental and meaningful test for all effective interactions and techniques for many-body problem to pass to be used for a successful prediction of nuclear properties of finite nuclei. This is essentially the reason for applying our new effective interaction to the infinite nuclear matter herein.

It is common knowledge that the saturation of density and energy is a basic property of nuclei [19]. Since the nuclear matter is characterized by a density ρ_0= 0.17fm^{-3} and a binding energy per nucleon, ε =-16MeV [6], efforts are geared towards exploring these properties of symmetric nuclear matter in this work. Therefore, in applying the new effective interaction to nuclear matter, the first and foremost concern of this paper is to use it to reproduce the binding energy

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per nucleon of symmetric nuclear matter at saturation; and the incompressibility is determined afterwards. Realizing that the famous M3Y-Reid interaction has been used for similar calculations previously [2, 17], the performance of our new effective interaction is compared afterwards with previous work.

In a large number of researches on nulear matter [3, 11, 13, 16, 21, 26, 31] over the last four decades, there have been two major approaches invoving M3Y-type effective interactions, the pseudo-potential and full-potential approaches, which have all along been used. In the pseudo-potential approach, Hartree-Fock (HF) calculations of nuclear matter are performed using the M3Y-type effective interaction supplemented by zero-range density dependent exchange term [3]; whereas the direct and exchange components are explicitly used for nuclear matter (NM) calculation in the full-potential approach [17, 26]. We have chosen for use in this work the pseudo-potential approach for the computation of nuclear incompressibility as a first approximation. In this approximation, the single-nucleon exchange is included by adding the zero-range pseudo-potential to the effective NN interaction. Such a potential involving the effective interaction (M3Y-Reid) based on the G-matrix constructed from the Reid nucleon-nucleon potential is known to have been successfully used in folding models for nucleon-nucleus and nucleusnucleus [15, 23, 29] calculations at low and medium energies. This success is an additional motivating reason to use our new effective interaction in its density dependent form, supplemented by the zero-range pseudo-potential, to compute nuclear incompressibility in expectation that the results will be such that they will indicate the likelihood of future success in folding calculations.

The nuclear incompressibility is of special interest in Nuclear Physics because it characterizes the nuclear equation of state (EOS) in a definite manner; and its computation is very important for the study of properties of nuclei (radii, masses, giant resonances etc.) supernova collapse, neutron stars, emission of neutrionus in supernoava explosions and heavy-ion collisions [10, 26]. Many different methods of experimental determination based on giant monopole resonances and production of hard protons in heavy-ion collisions [3] have reported various ranges of values of nuclear incompressibility, K which have been corroborated in some cases by some theoretical calculations. Amongst other things, this theoretical computation is meant to determine the position of our new effective interaction amongst other effective interactions used for similar theoretical calculations and also to determine the extent of its agreement with experimental values that have been widely accepted as standards.

This paper is organized such that Section 2 discusses in summary the derivation of our new effective interaction while Section 3 explains the procedure for the computation of binding energy per nucleon and incompressibility; and Section 4, presenting and discussing the results of computation, is followed by Section 5 which gives concluding remarks.

Effective Interaction

The matrix elements of our effective interaction were calculated in a harmonic oscillator basis using the lowest-order constrained variational (LOCV) method. The details of the calculation were reported in [8, 9] where the matrix elements have been shown to be of the form:

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$$E_2' = \left\langle \Phi \left| \sum_{i>j} f(ij) V_{ij} f(ij) \right| \Phi \right\rangle \quad (1)$$

where $\langle\Phi|$ represents a two-body (harmonic oscillator) wave function and $f(ij)$ are the correlation operators which are meant to take care of the effect of the strong repulsion of the nucleon-nucleon interaction, making the matrix elements finite at short inter-nucleon distances and V_{ij} is the Reid soft-core potential. The effective nucleon-nucleon interaction suitable for calculations involving nuclear matter and finite nuclei has been defined in [4, 9] to have a central (V_C), a spinorbit (V_{LS}) and a tensor (V_T) component expressed as:

$$\begin{aligned} V_C &= \sum_k V_k Y\left(\frac{r_{ij}}{R_k}\right) \\ V_{LS} &= \sum_k V_k Y\left(\frac{r_{ij}}{R_k}\right) \\ V_T &= \sum_k V_k r_{ij}^2 Y\left(\frac{r_{ij}}{R_k}\right) \boldsymbol{S}_{ij} \end{aligned} \quad (2)$$

where $Y\left(\frac{r_{ij}}{R_k}\right)$ is a Yukawa potential function of the form [4, 9]:

$$Y\left(\frac{r_{ij}}{R_k}\right) = \frac{exp\left(-\frac{r_{ij}}{R_k}\right)}{\left(\frac{r_{ij}}{R_k}\right)} \quad (3)$$

V_k in equation (2) are the strengths of the interaction to be determined by fitting the two-body matrix elements of equation (1) to those of the sum of Yukawa functions with different ranges; R_k are the ranges which are chosen to be 0.25, 0.40, 0.70 and 1.414 fm [1, 2]; and r_{ij} is the separation between the i and j nucleons; the tensor operator S_{ij} is [14]:

$$Sij = 3(\sigma i.rij)(\sigma j.rij) - \sigma i\sigma j \quad (4)$$

with σ_i and σ_j representing Pauli spin matrices; and the spin-orbit operator L.S has an expectation value proportional to [5, 22]:

$$2\langle L.S\rangle = j(j+1) - l(l+1) - s(s+1), \quad (5)$$

where $L = \sqrt{l(l+1)}$, $S = \sqrt{s(s+1)}$ and $J = \sqrt{j(j+1)}$.

The strengths of the effective interaction, V_k, determined by fitting the Yukawabased matrix elements of equation (2) to the two-body effective potential of equation (1), were separated into various angular momenta channels; namely, the singlet even (SE), singlet odd (SO), triplet even (TE), triplet odd (TO) along with spin-orbit and tensor channels. Our effective interaction

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arising from these strengths is M3Y-type and it has a radial form expressed in terms of three Yukawa functions as [4, 28]:

$$v^{D(EX)}(r) = \sum_{i=1}^{3} Y^{D(EX)}(i) \frac{exp(-\mu_i r)}{\mu_i r}, \tag{6}$$

where the functions $Y^{D(EX)}$ are represented in terms of SE, TE, SO, TO channels as [4, 28]:

$$\begin{aligned} Y^D &= \frac{1}{16}\left[3t^{(SE)} + 3t^{(TE)} + 1t^{(SO)} + 9t^{(TO)}\right] \\ Y^{EX} &= \frac{1}{16}\left[3t^{(SE)} + 3t^{(TE)} - 1t^{(SO)} - 9t^{(TO)}\right] \end{aligned} \tag{7}$$

The direct (v_D) and exchange (v_{EX}) components of the central part of the M3Y NN effective interaction, in terms of spin σ,σ' and isospin τ,τ' of the nucleons, are expressed as [17]:

$$\begin{aligned} v^{D(EX)}(r) = v_{00}^{D(EX)}(r) + v_{10}^{D(EX)}(r)(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}') + v_{01}^{D(EX)}(r)(\boldsymbol{\tau}\cdot\boldsymbol{\tau}') \\ + v_{11}^{D(EX)}(r)(\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}')(\boldsymbol{\tau}\cdot\boldsymbol{\tau}') \end{aligned} \tag{8}$$

where r is the inter-nucleon distance and ρ is the nuclear density around the interacting nucleon pair, σ,σ' are the spins and τ,τ' are the isospins of two nucleons participating in the interaction. For the cold symmetric nuclear matter that is spin-saturated only the first term dominates in the effective interaction. Thus, the radial strengths (in MeV) of the direct and exchange components of our new effective interaction are given in terms of three Yukawas respectively as [9]:

$$\begin{aligned} v^{D}(r) &= \frac{10472.13e^{-4r}}{4r} - \frac{2203.11e^{-2.5r}}{2.5r} \\ v^{EX}(r) &= \frac{499.63e^{-4r}}{4r} - \frac{1347.77e^{-2.5r}}{2.5r} - \frac{7.8474e^{-0.7072r}}{0.7072r} \end{aligned} \tag{9}$$

The interaction strengths used for constructing the new effective interaction are taken from Table V of Reference [9]

Since it is intended in this work to compare the results of our calculation with previous work [2] done with the famous M3Y-Reid interaction, its explicit radial form expressed as [4, 16]:

$$\begin{aligned} v^{D}(r) &= \frac{7999.00e^{-4r}}{4r} - \frac{2134.25e^{-2.5r}}{2.5r} \\ v^{EX}(r) &= \frac{4631.375e^{-4r}}{4r} - \frac{1787.125e^{-2.5r}}{2.5r} - \frac{7.8474e^{-0.7072r}}{0.7072r} \end{aligned} \tag{10}$$

Computational Procedure

The first test of functional viability of semi-microscopic interactions such as the new M3Y-type effective interaction is to reproduce the saturation properties of the cold symmetric nuclear matter. But it is a well-known fact that the nuclear matter generated with the Yukawa forces in a non-relativistic Hartree-Fock (HF) calculation is unstable as the saturation condition cannot be achieved owing to the attractive character of the M3Y interaction. For an improved description of the saturation condition to be obtained, it has been shown [17] that the introduction of a density dependence into the original M3Y interaction is the necessary and sufficient condition. In this case, the density-dependent M3Y interaction is the original M3Y interaction multiplied by the density-dependent factor, F(ρ)in the form:

$$vD(EX)(\rho,r) = F(\rho)vD(EX)(r) \quad (11)$$

where for the computation of the binding energy per nucleon,

$$F(\rho) = C(1 + Ae^{-\beta\rho}) \quad (12)$$

The parameters C, A and β of the density dependence are such that the saturation condition of the cold infinite nuclear matter is achieved at ρ_0=0.17fm^{-3}. With the inclusion of the density dependence, the binding energy per nucleon of the infinite nuclear matter is [16]:

$$\frac{E}{A}(\rho) = \frac{3\hbar^2 k_F^2}{10m} + F(\rho)\frac{\rho}{2}\left(J_{\mathrm{D}} + \int [\hat{j}_1(k_F r)]^2 v^{\mathrm{EX}}(r) d^3 r\right) \quad (13)$$

where m is the bare nucleon mass, J_D is the volume integral of the direct part of the interaction and $\hat{j}_1(x) = 3j_1(x)/x$, with $j_n(x)$ as the nth-order spherical Bessel function.

This particular computation involves the use of the direct and exchange components of the M3Y-type interaction in equation (9).

The incompressibility, K_∞, of the cold infinite nuclear matter is expressed as:

$$K_\infty(\rho) = 9\rho^2 \frac{\delta^2}{\delta\rho^2}(\frac{E}{A}(\rho))|_{\rho=\rho_0} \quad (14)$$

where ρ is the nucleonic density and k_F is the fermi momentum which has the mathematical expression:

$$k_F{}^3 = 1.5\pi^2\rho \quad (15)$$

In order to compute the incompressibility, K_∞, of the infinite nuclear matter, the zero-range pseudo-potential instead of the full-exchange potential approach. In this approach, the chosen M3Y-type effective interaction is assumed to be energy and density dependent so that

$$v^D(r,\rho,\varepsilon) = v^D(r,\varepsilon)F(\rho,\varepsilon), \quad (16)$$

where

$$v^D(r,\varepsilon) = \frac{10472.13e^{-4r}}{4r} - \frac{2203.11e^{-2.5r}}{2.5r} + J_{00}(\varepsilon)\delta(r) \quad (17)$$

Here, the zero-range pseudo-potential term which represents the single-nucleon exchange term is given by [7, 23]:

$$J_{00}(\varepsilon) = -276(1 - 0.005\varepsilon)MeV.fm^{-3} \quad (18)$$

and the density dependent part, having the form [2]:

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$$F(\rho,\varepsilon) = C(1-\beta(\varepsilon)\rho^{\frac{2}{3}})$$

8 , (19)

represents higher-order exchange and Pauli blocking effects while C and $\beta(\varepsilon)$ are constants which are adjusted to attain saturation. Based on this approach, ε is defined as [2]:

ε = 3______ħ2*kF*2 + $F(\rho,\varepsilon)\rho J$____D (20)

10*m* 2

where m = 931.4943 MeV/C^2 is the nucleonic mass and J_D is the volume integral of the M3Y-type interaction, supplemented by the zero-range pseudo-potential,

expressed as:

$$J_D(\varepsilon) = 4\pi \int v^D(r) r^2 dr + J_{00}(\varepsilon) \quad (21)$$

The equilibrium density is obtained from the saturation condition:

$$\frac{\delta\varepsilon}{\delta\rho} = 0 \quad (22)$$

which is the same as:

$$\frac{\delta\varepsilon}{\delta\rho} = \frac{\hbar^2 k_F^2}{5m\rho} + \frac{J_D C}{2}\left(1 - \frac{5}{3}\beta(\varepsilon)\rho^{\frac{2}{3}}\right) = 0 \quad (23)$$

When the density dependence of equation (19) is substituted in equation (20), ε becomes

$$\varepsilon = \frac{3\hbar^2 k_F^2}{10m} + \frac{\rho J_D}{2} C(1-\beta(\varepsilon)\rho^{\frac{2}{3}}) \quad (24)$$

Solving equations (23) and (24) simultaneously at saturation, the density dependent parameters are [2]:

$$C = \frac{-2\hbar^2 k_F^2}{5mJ_D\rho\left(1 - \frac{5}{3}\beta(\varepsilon)\rho^{\frac{2}{3}}\right)} \quad (25)$$

$$\beta(\varepsilon) = \left[\frac{(3-3p)}{(9-5p)}\right] / \rho^{\frac{2}{3}} \quad (26)$$

$$p = \frac{\left[10m\varepsilon\right]}{\left[\hbar^2(1.5\pi^2\rho)^{\frac{2}{3}}\right]}$$

9 (27)

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With the parameters of density dependence determined, the nuclear incompressibility is computed from equation (14) as:

$$K_{\infty} = -\left[\frac{3\hbar^2 k_F^2}{5m} + 5J_D C\beta(\varepsilon)\rho^{\frac{5}{3}}\right]_{\rho=\rho_0} \quad (28)$$

RESULTS

The results of the present calculations are shown in Table 1 and Figure 1 respectively. Figure 1, representing a plot of binding energy per nucleon, shows that the new effective interaction has acceptably reproduced the nuclear matter saturation point represented by the solid circle on the curve.

For the computation of incompressibility, a narrow range of acceptable values (ρ_0 = 0.17 - 0.15fm^{-3}) of saturation density has been used in consideration of the fact that its values used by different groups differ. The corresponding range of incompressibilities obtained is 307-301 MeV. This is shown in Table 1 in which the results obtained in Reference [2] are presented in brackets underneath those of the present calculation; and the agreement between the two, resulting from the marginal performance gap or difference, is reasonably impressive. An excellent agreement has also been found between the results of the present computation and an experimental estimate based on the production of hard photons from heavy-ion collision establishing that $K\infty$ = 290 ± 50 MeV and a theoretical estimate by infinite nuclear matter model (INM)[24] predicting a well-defined and stable value of incompressibility to be $K\infty$ = 288 ± 20 MeV. Since the results in Ref. [2] were obtained with the M3Y-Reid effective interaction which is known to have done well in nucleon-nucleus and nucleus-nucleus [17] calculations,this agreement is indicative of a future successful use of the new effective interaction in similar folding calculations.

Table 1: Parameters of Density Dependence and Nuclear Incompressibilities obtained with the new M3Y-Type Effective Interaction at various values of Saturation Density. The results obtained in Ref.[2] are in brackets

$\rho(fm^{-3})$	$\beta(\varepsilon)fm^2$	C	K [MeV] ∞
0.170	1.551	2.02 (1.98)	307.3 (309.6)
0.165	1.586	2.06 (2.02)	305.3 (308.2)
0.160	1.624	2.10 (2.07)	304.5 (306.9)
0.155	1.664	2.15 (2.11)	303.6 (305.5)
0.150	1.705	2.20 (2.16)	301.2 (304.0)

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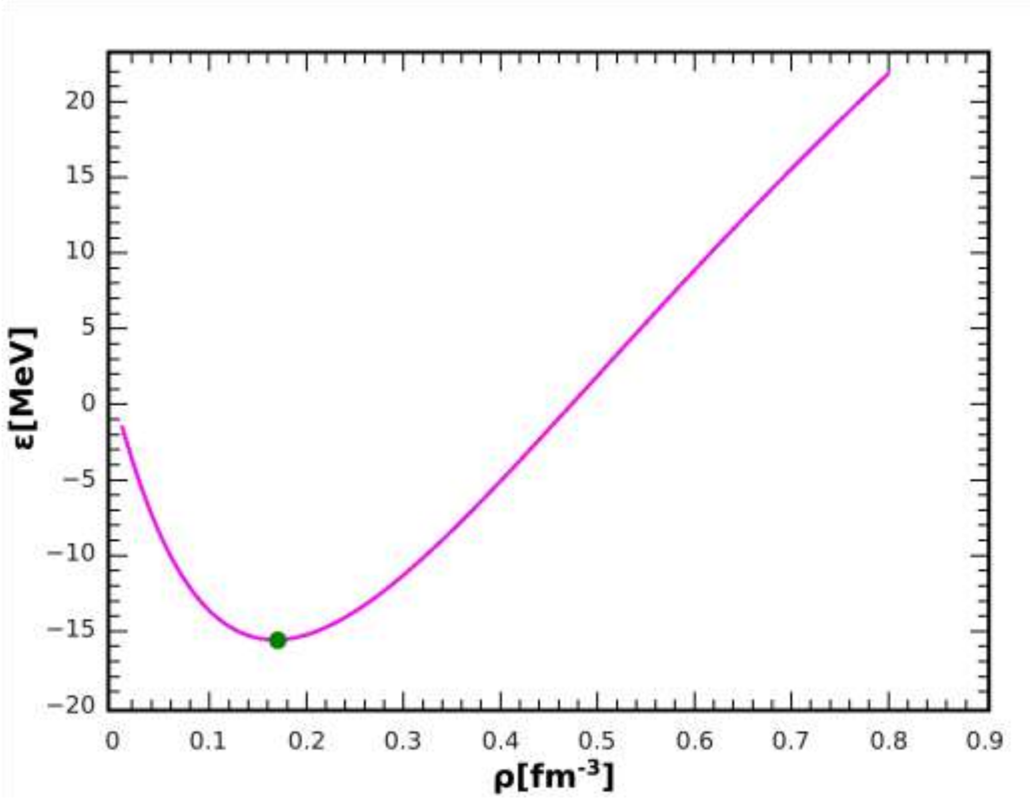

Figure 1: Graph of Nuclear Binding Energy against Nuclear Density with the Saturation Point Represented by the Solid Circle.

CONCLUSION

In this work, a new density dependent effective interaction obtained by a variational method has been used to reproduce the binding energy per nucleon, ε= -16 MeV at the saturation density,ρ_0 = 0.17 fm^{-3}. Supplemented by the zero-range pseudo-potential, the interaction has also been used to compute the incompressibility of the cold symmetric nuclear matter, resulting in $K\infty$ = 301 - 307 MeV which is in impressive agreement with the theoretical calculation in [2] and [24] as well as the experimental estimate that $K\infty$ = 290 ± 50 MeV found in [2]. The impressive agreement recorded herein suggests that the new effective interaction is viable and reliable for correct nuclear matter calculation; and it also expresses the likelihood of a future success in folding calculations. It is hoped that the fullexchange potential instead of the pseudo-potential approximation will be used in the next paper for a broader and better assessment of the character and viability of the new effective interaction.

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