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COMPARATIVE ANALYSIS OF SOME DISTRIBUTIONS ON THE CAPITAL REQUIREMENT DATA FOR THE INSURANCE COMPANY

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ABSTRACT: *In this work, we review the equitable transfer of the risk of a loss in insurance company by preparing a mitigating plan for risks that are chosen to be mitigated and the purpose of the mitigation is to describe how this risk will be handled to hedge against a contingent or uncertain loss. Here we examine their implications on capital requirements in insurance company a risk measurement go hand-in-hand with setting of capital requirements minima by companies as well as regulators but a special attention is given to some risk distributions as well as commendations on the case of a capital requirements in insurance industry using the distributions.*

KEYWORDS: Capital requirement, Tukey-Lambda distribution, Birnbaum –Saunders distribution. Double-exponential distribution, risk management.
MSC: 62H10; 60E07; 62G30.

INTRODUCTION

The subject of the determination of a capital has been on active interest to researchers, to regulators of financial institutions and of direct interest to commercial vendors of financial product and services.

In general terms, the capital requirement risk measure is the amount of capital required to ensure that the enterprise does not become technically insolvent and the promotion of certain risk concepts as value at risk has prompted the study of risk measures by numerous authors (see for example, Wang, 1996, 1997). Specific desirable properties of risk measures were proposed as axioms in connection with risk pricing (Wang, et al, 1997) and more generally in risk measurement (Artnzer et al 1999).

This paper presents an over view of capital requirement measures by describing the structure of the measures of distributions and the considerations that went into their design, after performing a set of rough calculations.

We identify some of their significant benefits that leads to regulatory capital charges that conform more closely to insurance true risk exposure which the information generated by the models of the distribution will allow the regulators and financial participants to compare risk exposures over time and across institutions.

The distributions discussed having been adduced good enough for modelling financial data are compared in this study using Vuong test and ratio of maximized likelihood (RML) test on the collected data to assert their effectiveness and state the best of the distributions.

MATERIAL AND METHOD

In this criterion, data was collected from American International Insurance Company (AIICO) and Niger Insurance Company and fit into four distributions; particularly the aim is to determine which of these distributions models better capital requirements for the risk exposure. The distributions are;

The Birnbaum – Saunders Distribution

The Birnbaum – Saunders distribution, also known as the fatigue life distribution, is a probability distribution used extensively in reliability applications to model failure times. There are several alternative formulations of this distribution in the literature. It is named after Z.W. Birnbaum and S.C. Saunders, (1969). This distribution was developed to model failure due to cracks. The probability that the crack does not exceed a critical length ω

$$p(x > \omega) = \phi\left(\frac{\omega - n\mu}{\sigma\sqrt{n}}\right), \qquad (1)$$

where μ=mean or expectation or median and mode, σ =standard deviation.

The more usual form of Birnbaum – Saunders distribution is;

$$f(x;\alpha,\beta) = \emptyset\left(\frac{1}{\alpha}\left[\left(\frac{x}{\beta}\right)^{0.5} - \left(\frac{\beta}{x}\right)^{0.5}\right]\right) \qquad (2)$$

Here α is the shape parameter and β is the location parameter and $\emptyset(.)$ is the normal distribution.

Since $\emptyset(.)$ is normal, equation (2) becomes

$$e^{-\left(\frac{1}{\propto}\left[\left(\frac{x}{\beta}\right)^{0.5} - \left(\frac{\beta}{x}\right)^{0.5}\right]\right)} \qquad (3)$$

PROPERTIES OF BIRNBAUM- SAUNDERS DISTRIBUTION:

The Birnbaum- Saunders distribution is unimodal with a median of β. The mean (μ), variance (σ^2), Skewness (γ) and Kurtosis (K) are as follows:

Mean (μ) $\quad \beta\ (1 + \frac{\alpha^2}{2})\ 4)$ (4)

Variance (σ^2) $\quad (\alpha\beta)^2\ (1 + \frac{5\alpha^2}{4})$ (5)

Skewness (γ) $\quad \frac{16\alpha^2\ (11\alpha^2 + 6)}{(5\alpha^2 + 4)^3}$ (6)

Kurtosis (k) $\quad 3 + \frac{6\alpha^2\ (93\alpha^2 + 41)}{(5\alpha^2 + 4)^2}$ (7)

Given a data set that is thought to be Birnbaum-Saunders distributed the parameters values are best estimated by maximum Likelihood. If T is Birnbaum-Saunders distributed with parameters α *and* β, then T^{-1} is also Birnbaum-Saunders distributed with parameters α *and* β^{-1}

Transformation:

Let T be Birnbaum-Saunders distributed variate with parameters α *and* β. A useful transformation of T is

$$X = \frac{1}{2}\left[\left(\frac{T}{\beta}\right)^{0.5} - \left(\frac{T}{\beta}\right)^{-0.5}\right]. \qquad (8)$$

Equivalently:

$$\mathrm{T} = \left(1 + 2\mathrm{x}^2 + 2\mathrm{x}\,(1 + \mathrm{x}^2)\right)^{0.5}, \qquad (9)$$

so that, X is then distributed normally with a mean of zero and a variance of $\frac{\alpha^2}{4}$.
The general formula for the probability density function (PDF) is

$$f(x) = \frac{\sqrt{\frac{x-\mu}{\beta}} + \sqrt{\frac{\beta}{x-\mu}}}{2\,\gamma\,(x-\mu)}\ \phi\,\frac{\sqrt{\frac{x-\mu}{\beta}} - \sqrt{\frac{\beta}{x-\mu}}}{\gamma}\ , x > \mu;\ \gamma, \beta > 0, \qquad (10)$$

where γ is the shape parameter, μ is the location parameter, β is the scale parameter, and ϕ is the probability density function of the standard normal distribution.

The case where $\mu = 0$, and $\beta = 1$ is called the standard fatigue life distribution.

The PDF for the standard fatigue life distribution reduces to

$$f(x) = \frac{\sqrt{x} + \sqrt{1/x}}{2\gamma x}\ \phi\left(\frac{\sqrt{x} - \sqrt{1/x}}{\gamma}\right), x > 0;\ \gamma > 0, \qquad (11)$$

And the cumulative distribution function is given as

$$f(x) = \phi\left(\frac{\sqrt{x} - \sqrt{1/x}}{\gamma}\right) x > 0;\ \gamma > 0 \qquad (12)$$

where ϕ is the cumulative distribution function of the standard normal distribution.

Log-normal distribution:
In probability theory, a log-normal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed.
If x is a random variable with a normal distribution, then $y = \exp(x)$ has a log-normal distribution. A random variable which is log-normally distributed takes only positive real values.
Log-normal is also written, log-normal or lognormal. The distribution is occasionally referred to as the Galton distribution or Galton's distribution, after Francis Galton (Aitchson and Broom,1957). The log-normal distribution also has been associated with other names, such as McAlister, Gibrat and Cobb-Douglas (Robert et al ,1994).
Hence the distribution function (f(x)) of lognormal distribution is;

$$F(x) = \frac{1}{x\sqrt{2\pi\sigma^2}}\, e^{-\frac{(in\, x-\mu)^2}{2\sigma^2}} \qquad (13)$$

Properties of Log-normal distribution
The log normal distribution is commonly used to model the lives of units whose failure modes are of a fatigue nature.

$$\text{Mean } (\mu) = e^{\mu + \frac{\sigma 2}{2}} \qquad (14)$$

$$\text{Variance } (\sigma^2) = \left(e^{\sigma^2} - 1\right) e^{2\mu + \sigma^2} \qquad (15)$$

$$\text{Skewness } (\gamma) = \left(e^{\sigma^2} + 2\right)\sqrt{e^{\sigma^2} - 1} \qquad (16)$$

Kurtosis $(k) = e^{4^{\sigma^2}} + 2\, e^{3^{\sigma^2}} + 3\, e^{2^{\sigma^2}} - 6$ (17)

The log-normal distribution is a good companion to the Weibull distribution when attempting to model these types of units.

Double Exponential Distribution:
A double exponential distribution is a constant raised to the power of an exponential function. The general formula is

$$f(x) = a^{b^x} \quad (18)$$

which grows much more quickly than an exponential function (Aho and Sloane, 1973). Hence the general formula for the probability distribution function of the double-exponential distribution is:

$$f(x) = \frac{e^{-\frac{|x-\mu|}{\beta}}}{2\beta} \quad (19)$$

where μ is the location parameter and β is the scale parameter. The case where $\mu = 0$ and $\beta = 1$ is called the standard double exponential distribution. Therefore, the equation for the standard double exponential distribution is:

$$f(x) = \frac{e^{-|x|}}{2} \quad (20)$$

Properties of Double – exponential distribution

The mean (μ), Variance (σ^2), Skewness (γ), and Kurtosis (k) are as follows;

Mean μ
Variance $2\beta^2$ (21)
Skewness 0
Kurtosis 6

Tukey Lambda Distribution:
Tukey Lambda distribution is a continuous probability distribution defined in terms of its quantity function formalized by John Tukey (Vasicek Oldrich, 1976). It is typically used to identify an appropriate distribution and not used in statistical models directly (Shaw and McCabe, 2009).
The Tukey Lambda distribution has a shape parameter λ. As with other probability distribution, the Tukey Lambda distribution can be transformed with a location parameter, μ and a scale parameter, σ.
Hence the distribution function for Tukey Lambda distribution is;

$$f(x) = (e^x + 1)^{-1} \quad (22)$$

Properties of Turkey Lambda Distribution:
Notation Turkey (λ)
Parameters $\lambda \epsilon R$ – shape parameter
Support $x \epsilon \left[\frac{-1}{\lambda}, \frac{1}{\lambda}\right]$ $for\ \lambda > 0$
$x \epsilon R\ for\ \lambda \leq 0,$
PDF $\big(Q\ (p;\lambda)\big), Q'(p;\lambda)^{-1}, 0 \leq p \leq 1$

CDF $(e^{-x} + 1)^{-1},\ \lambda = 0$ (23)

Mean $0, \lambda > -1$

Variance $\frac{2}{\lambda^2}\left(\frac{1}{1+2\lambda} - \frac{\Gamma(\lambda+1)^2}{\Gamma(2\lambda+2)}\right),\ \lambda > {-1}/{2}$ (24)

$\frac{\pi^2}{3}, \lambda = 0$

Skewness $0, \lambda > {-1}/{3}$

Ex-kurtosis $\frac{(2\lambda+1)^2}{2(4\lambda+1)} \frac{g_2^2(3g_2^2 - 4\,g\,g_3 + g_4)}{g_4\left(g_1^2 - g_2\right)^2}$ (25)

Where $g_k = \Gamma(k\lambda + 1) and\ \lambda > {-1}/{4}$

It is necessary to test these four distributions to know which of them models better capital requirement for the risk exposure. For this purpose, Vuong test and ratio of maximized likelihood (RML) test would be useful.

Vuong Test

We apply this test as in Osu and Onwuegbulam (2012).

If

$E_o\{\log h_0\left(Y_t|K_t\right)\} - E_o\left[log\ h_o\ \left(\left(Y_t|K_t\right); r *\right)\right],$ (26)

then $h(.|.)$ is the conditional distribution of y_t given k_t.

E_0 is expectation of the condition distribution of (Y, K) of the correct model.

τ* is the pseudo-true value of τ. The best model is the model closest to the true conditional distribution.

Firstly, the null hypothesis under the young test observed when

H_0: $E_o\left\{log\ \frac{f(Y_{t/K_t}\,;\alpha^*)}{g(Y_t/K_t\,;\mu^*)} = 0\right\}$ (27)

meaning that $F_\propto$ and G_μ are similar.

while H_o : $E_o\left\{log\ \frac{f(Y_{t/K_t}\,;\alpha^*)}{g(Y_t/K_t\,;\mu^*)} > 0\right\}$ (28)

meaning that F_α are better than G_μ

On the other hand

H_0: $E_o\left\{log\ \frac{f(Y_{t/K_t}\,;\alpha^*)}{g(Y_t/K_t\,;\mu^*)} < 0\right\}$ (29)

meaning that G_μ are better than F_α

The distribution function of model F_α given by previous equation for n independent identical distribution (iid) random variables, the Likelihood Ratio (LR) is measured by:

$\mathrm{L}(x|\alpha,\beta,\mu,\sigma^2) = \prod_{i=1}^{n} \sum_{i=1}^{n} f\left(x_{1,} \dots, x_n\middle|\alpha,\beta,\mu,\sigma^2\right)$ (30)

In addition, the distribution of model G_μ calculated using equation (3) above for n iid random variables, the Likelihood Ratio is:

$\mathrm{L}(x|\mu,\sigma^2) = \prod_{i=1}^{n} \sum_{i=1}^{n} g\left(x_{1,} \dots, x_n\middle|\ \mu,\sigma^2\right)$ (31)

Besides that, the Likelihood Ratio (LR) statistics is measured as;

$\mathrm{LR}_{\mathrm{r}}\left\{(\propto,\beta,\mu,\sigma^2),(\mu,\sigma^2)\right. = \left.\sum_{i=1}^{n}\right\} log\ \frac{f(x_1,\dots,x_n|\propto,\beta)}{g(x_1,\dots,x_n|\mu,\sigma^2)}$ (32)

The variance statistics given by:

$$\widehat{W}_n^2 = \frac{1}{n}\sum_{i=1}^{n} log\left[\frac{f(x_1,\ldots,x_n|\propto,\beta)}{g(x_1,\ldots,x_n|\mu,\sigma^2)}\right]^2 - \left[\frac{1}{n}\sum_{i=1}^{n} log\left[f\left(\frac{f(x_1,\ldots,x_n|\propto,\beta)}{g(x_1,\ldots,x_n|\mu,\sigma^2)}\right)\right]\right]^2 \quad (33)$$

The test statistics for Vuong test is calculated using the following formulae:

$$K = n^{-1/2}\, LR_n((\propto,\beta),(\mu,\sigma^2))/\widehat{W}_n \longrightarrow N(0,1) \quad (34)$$

For a chosen critical value V (which is obtained from the Tukey Lambda Distribution), H_0 can be rejected when the Vuong test is higher than absolute value of V.

The discrimination procedure is conducted as follows;

We choose any of the four distributions if the test statistics K≥V otherwise will be rejected.

RATIO OF MAXIMIZED LIKELIHOOD (RML)

We assume here that we have a sample $x_i,\ldots,x_n$ from one of the four distribution functions B_{in} (α,β). The likelihood functions, assuring that the data follow G_A (α,β) or W_E (θ,λ) are:

$$L_{GA}(\alpha,\beta) = \prod_{i=1}^{n} f_{GA}(x_i\,;\,\alpha,\beta) \quad (35)$$

and

$$L_{WE}(\alpha,\beta) = \prod_{i=1}^{n} f_{WE}(x_i\,;\,\theta,\lambda) \quad (36)$$

respectively. The logarithm of RML is defined as:

$$T = I_n\left[\frac{L_{BS}(\alpha,\beta)}{L_{TL}(\theta,\lambda)}\right] \quad (37)$$

and (θ,λ) are Maximum Likelihood Estimators (MLEs) of L (α,β) and L (θ,λ), respectively based on the sample $x_i,\ldots,x_n$. The logarithm of RML can be written as follows:

$$T = n[\hat{\alpha}\, In\, (\hat{\alpha}\ \tilde{x}\ \bar{x}) + \hat{\lambda}\, In\, (\hat{\theta}|\ \bar{x}) - \hat{\alpha} + 1 - In\, \Gamma(\hat{\alpha} - In\hat{\lambda})] \quad (38)$$

Here x̃ and x̄ are arithmetic mean respectively of $x_i,\ldots,x_n$.

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \text{ and } \tilde{x} = \left[\prod X_i\right]^{1/n}, \quad (39)$$

for Birnbaum-Saunders distribution

$$\hat{\beta} = \bar{x}\,|\hat{\alpha} \quad (40)$$

and for Turkey-Lambda θ and λ satisfy the following relation:

$$\hat{\lambda} = \left[\frac{\sum_{i=1}^{n} x_i^{\hat{\lambda}}}{n}\right]^{1/\hat{\theta}} \quad (41)$$

Fearn and Nebenzahl (1991) consider using the ratio maximized Likelihood approach to choose between Gamma and Weibull distribution. The selection relies on log of RML which is T. If $T > 0$ model will be selected as well as Birnbaum- Saunders distribution as it is used extensively in reliability application to model failure times and if $T < 0$ Gamma model is choose.

Moreover, the probability of correct selection (PCS) was determined to identify the probability for both probability models.

The PCS relies on scale parameter which β for Birnbaum –Saunders distribution and λ for Turkey-Lambda distribution.

APPLICATIONS

We take the end-of-year capital requirement data (see table 1 below) of American International Insurance Company (AIICO) and Niger Insurance Company (NIC) (see table 2 below) as sample data. Hence it is desirable to test and check mate these four distributions to know the best in modeling the capital requirement for the risk exposure on the data.

TABLE 1: ANNUAL CAPITAL REQUIREMENT DATA AIICO INSURANCE COMPANY

Initial claims	Amount in billions	Monthly change	Percentage claims
Claims on lands	20 bn	881 A	25 %
Dividends payable	30 bn	362 B	6 %
Cash and equivalent	7 bn	2,647 X	1 %
Liabilities on investment	8 bn	578 Y	-2 %
Cash reserve	3 bn	775 E	4 %
Premium payable	2 bn	1,696 M	5 %
Debt. Securities	5 bn	14,212 K	-15 %
Ordinary loans	15 bn	143,786 C	-16 %
Equity	4 bn	38,845 D	10 %
Mortgage loans	6 bn	14,695 W	2 %

For the above data, variance is approximated to 73. From equations (5) and (6), the value for α and β are 17 and 0.07 when μ and σ^2 are 10 and 73.

TABLE 2: ANNUAL CAPITAL REQUIREMENT DATA FOR NIGER INSURANCE COMPANY

Initial claims	Amount in Billions	Monthly Change	Percentage Claims
Claim on lands	10bn	1.06	13%
Dividends payable	3bn	488.09	48%
Cash and equivalent	1bn	76.51	76%
Liabilities on investment	2bn	13.7	19%
Cash reserves	7bn	1.975	71%
Premium payable	4bn	498.67	6%
Debt securities	6bn	20.225	10%
Ordinary loans	5bn	15.230	29%
Equity	3bn	10.886	5%
Mortgage loans	9bn	6.7283	7%

The method was applied to Birnbaum – Saunders distribution which is as follows: 0.8132, 0.7383, 0.6865, 0.6460, 0.6125, 0.5838, 0.5586, 0.5361, 0.5159 and 0.4975

$$\bar{X} = 10, \sigma^2 = 73, \alpha = 17, \beta = 0.07, \tilde{X} = 5, \sigma^2 = 8, \alpha = 5.6567, \beta = 0.2632$$

$\gamma = 0.0470;\ \ \lambda = 0.090684;\ \ \theta = 39.872544; T = -0.13123 < 0; K = 0.3883; \quad V = 0.8132$.

Here $T < 0$, and $K < V$, then Birnbaum- Saunders distribution was not selected.

The method was applied to log-normal distribution which is as follows; 0.0235, 0.0129, 0.0090, 0.0070, 0.0058, 0.0049, 0.0043, 0.0038, 0.0034 and 0.0031.
$\bar{X}$=10, $\sigma^2 = 73, \beta = 0.07, \tilde{X}$=5, σ^2=8, λ=0.953, θ=22.3675, T=-0.17926< 0,$V = 0.0235$, $K = 0.0146 < V$, Lognormal was not selected.

The method was applied to Double-exponential distribution which is as follows: 0.1839, 0.0677, 0.0249, 0.0092, 0.0034, 0.0012, 0.0005, 0.0002, 0.0001 and 0.0000.
$\gamma = 0.5264$, $\bar{X}$=10, $\beta = 0.07$, $\lambda = 0.6345$, $\tilde{X}$=5,$\theta = 36.2756$, $\beta = 0.2632$, $T = -0.00478 < 0$, $K = 0.0677$, $V = 0.1839$. $K < V$, and was not selected.

The method was applied to Turkey Lambda distribution which is as follows: 0.7311, 0.8808, 0.9526, 0.9820, 0.9933, 0.9975, 0.9991, 0.9997, 0.9999 and 1.0000.
$\bar{X}$=10, σ^2=73, $\alpha = 17$, $\beta = 0.07$, $\tilde{X}$=5, σ^2=8, $\alpha = 5.404$, $\beta = 0.3205$, $\theta = 35.1148$, $\lambda = 0.6778$, $\Gamma = 3.6667$, $T = 1.5940262 > 0$; $K = 1.7511$, and $V = 1.0000$. Therefore, $K > V$. Turkey-Lambda was selected over the others. The figures below are plots of the comparative performances of these four distributions over one another.

Graphs of Birnbaum-Saunders Distributions

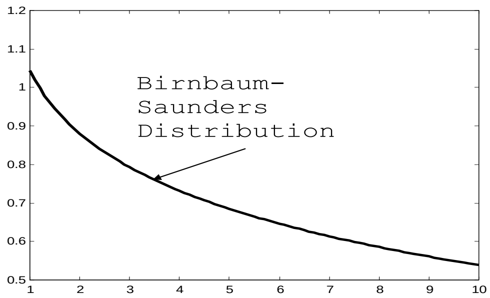

Fig 1: The expected values for small and high values of X for Birnbaum-Saunders distribution (with α = shape parameter, β = location parameter, and Ø ( ) = normal distribution).

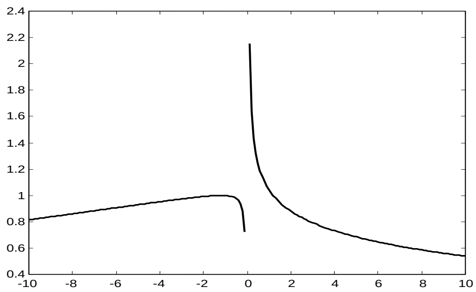

Fig. 2: The expected values for small and high values of x for Birnbaum-Saunders distribution (taking the negative values into consideration).

Figures 1 and 2 of the Birnbaum-Saunders Distribution show that the company's risk cycle leads to an increase in risk as the capital requirement decreases meaning that heavier tail is implied with decrease in capital requirement, hence the probability of that heavy tail or risk does not exceed a critical length. Meaning that as the risk increases, lower capital is involved and as it decreases higher capital is involved. Considering the negative values, the capital requirement for the risk is undefined at zero because at that point there is a big jump. So there should be more than enough capital on reserve for risk to avoid heavy short fall.

Graphs of Log-Normal Distributions

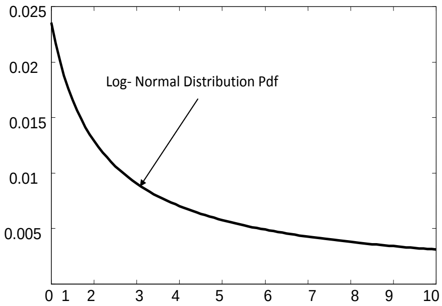

Fig. 3: The expected values for small and high values of x for log-normal distribution (with $\mu = 0$, $\sigma^2 = 73$) where μ = mean and σ^2 = variance.

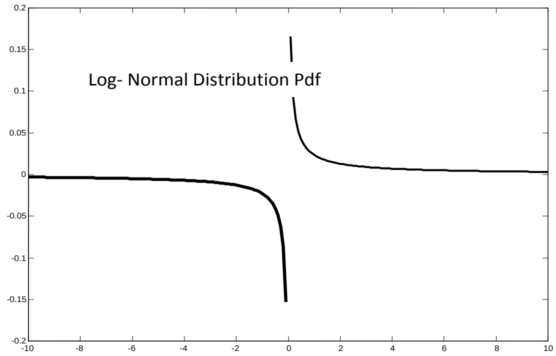

Fig. 4: The expected values for small and high values of x for log-normal distribution (taking negative values into consideration).

The Log-Normal Distributions (figure 3 and 4) is used extensively to model fatigue or bad times in reliability application. So the risk depends on the capital or cash reserve scenarios and do not depend on the unobservable post time risk futures of the scenario. So it is constant with time meaning that low capital on reserve or capital requirement implies high risk and undertaking lower risk implies adding more and more cash to the portfolio or capital on reserve. Considering the negative values, this shows that it is a step function with discontinuity at zero, at that point the capital required for the risk is not differentiable meaning that it is undefined. So to prevent shortfall, the company tends to hold more than enough capital on reserve in case an outburst of risk or worst case occurs at any point in time.

Graphs of Double Exponential Distributions

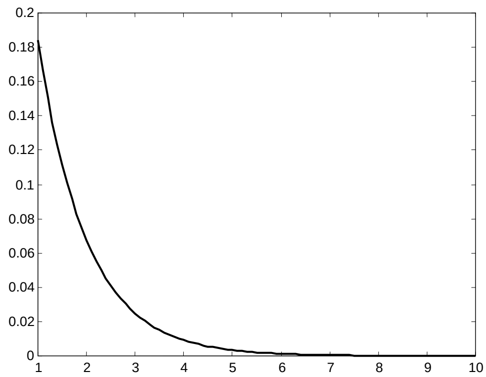

Fig. 5: The expected values for small and high values of x for Standard Double exponential distribution (with $\mu = 0$, $\beta = 1$).

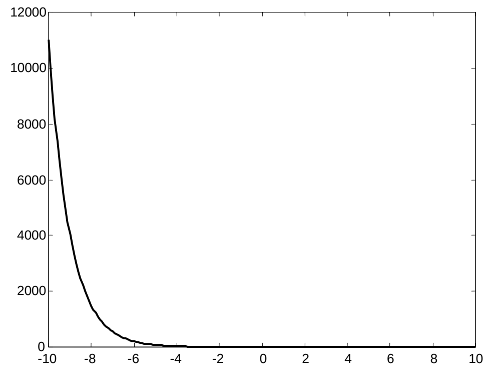

Fig. 6: The expected values for small and high values of x for standard double exponential distribution (taking the negative values into consideration).

In figures 5 and 6 of the Double Exponential Distributions, as the capital requirement is low, heavy tail is implied but as it increases lower tail is implied. So enough capital should as well be kept on reserve

Considering the negative values, higher tail is implied when the capital requirement is negative but as it approaches positive, it reduces and normalized. Also enough capital should also be on reserve when it is negative.

Graphs of Tukey Lambda Distributions

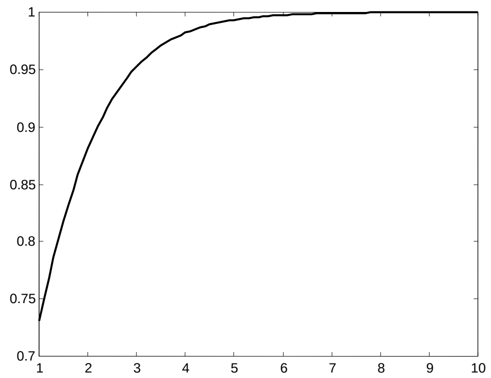

Fig. 7: The expected values for small and high values of x for Tukey Lambda distribution.

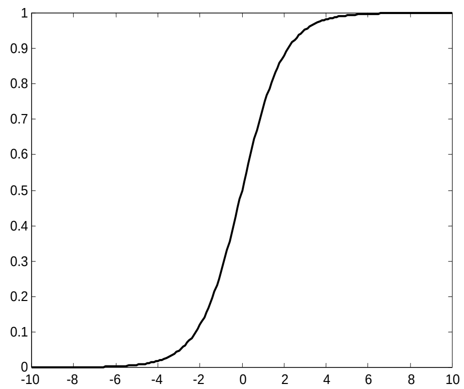

Fig. 8 The expected values for small and high values of x for Tukey Lambda distribution (taking negative values into consideration).

Figures 7 and 8 for the Turkey Lambda distribution, imply that as the risk goes higher, the capital requirement also increases but the increment stabilizes at a point 1 which is the equilibrium (stability). So enough capital on reserve must also be ensured to maintain the system to prevent financial crises. Considering the negative values, as the capital requirement for the risk goes to negative values (bearest minimum value), the worst case will be zero because at zero the risk increases and stabilizes at a point 1. So the system is not stable from the beginning hence a maximum capital on reserve for risk should be ensured using this measure.

CONCLUSION

In this research, a comparison of the log-normal distribution, Birnbaum-Saunders distribution, double exponential distribution and Tukey Lambda distribution is made on the capital requirement data for the insurance company. It is observed that the Tukey Lambda distribution is better to risk based capital taken to estimate the expected loss the insurers would suffer in the face of a catastrophic financial event. And using Tukey Lambda distribution, we see that the size of that expected loss represents the risk based capital required to deal with the loss.

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