BOOTSTRAP CONFIDENCE INTERVALS FOR VARIANCE COMPONENTS IN THE UNBALANCED RANDOM ONE –WAY MODEL

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ABSTRACT: *The problem of constructing confidence intervals for the ratio of variance components in unbalanced random one-way model is to investigate. In this respect, various different exact methods, which are introduced, by Wald (1947), Bross (1950), Tukey (1951), Anderson, and Bancroft (1952) are used. However, these methods are rather difficult to compute since it involves the solution of non-linear equations. So several approximate methods, which are easier to compute, are discussed. These methods were introduced by Satterthwait (1946), Morigiti (1954), Bulmer (1957), Thomas and Hultiquist (1978), El-Bassiouni (1978), and El-Ganzouri (1986). In this search a Monte Carlo simulation study is conducted to examine the performance of the above mentioned methods. Recently, new alternative methods namely Bootstrap are also, given and included in comparison. Simulation depends on the sizes of the samples and that the basis of comparison are constructed based on the coverage values and average length. After damaged the results of the coverage values and average length are discussed. In design (1) the best method is (SAM) method, because it has appropriate coverage value with a small value of average length for all values of ρ, this method is from bootstrap methods. In design (2) and design (5) the best method is (Wald) method, because it has high coverage with a small value average length with all values of ρ, this method from exact methods. In design (3) and design (4) the best method is (Wald) for small values of ρ, ρ≤0.1and it is an exact method. However, for large values of ρ, ρ>0.1 the best method is (SAM) method and it is a bootstrap method. In applied case sugar -cane experiment the results of limits, the A.ADJ has the biggest upper limit and, (BB) method has the smallest upper limit. While the (A) method has the highest lower limit, where the bootstrap methods (BB), (Bt), and (SAM) have the smallest lower limits. However, for average length, the best method that has the smallest average length, and the best method is (BB) MAETHOD. In addition to. The another applied case wheat experiment the results of limits, the (Bt) has the biggest upper limit and Morigiti and Bulmer have the smallest upper limit .While ,Bross has the highest lower limit ,Where the And Ban has the smallest lower limit. However, for average length, the best method that has the smallest average length, and the best method is Bross method.*

KEYWORDS: Unbalanced One-Way Random Effect Model; Interval Estimation; Variance Components; Bootstrap Confidence Interval; Quadratic Estimators; Monte Carlo Simulation.

INTRODUCTION

The Thesis Problem

This study, discuss the problem of constructing confidence intervals for the ratio of variance components in the unbalanced random one-way model is investigated, Wald (1947) out lined

an exact method for constructing such interval. It is rather difficult to computer since it involves the solution of two non-linear equations.

In recent years, several attempts had been made to use analysis of variance (ANOVA) and the resulting variance components estimates to estimate the reliability of a measurement procedure. Thus, interval estimation of creation function of variance components is of interest to researches in all fields of applications in which the variance components model is used. Several authors have proposed confidence intervals for linear functions and ratios of variance components.

For the most part, these intervals are approximate with unknown exact probabilities associated with their coverage. In this search, a technique is given for the construction of simultaneous confidence intervals for the values of all continuous functions of the variance components in a balanced, general random linear model. These confidence intervals are conservative that is, the actual confidence level cannot be less than any preset value. The proposed technique is easy to apply, as it only requires the optimization of a given continuous function of the variance components over a bounded region. To evaluate the variance among the laboratories and the variance of testing error, a confidence interval on this sum of variances will provide more information about the process than will a simple point estimate.

The problem of constructing bootstrap confidence intervals for percentiles of the model is considered seven bootstrap approximate methods. They are used to construct these confidence intervals. The bootstrap algorithm Efron (1979, 1987, and 1992) is used to construct confidence limits for all methods. The comparison between confidence intervals lengths will be done using small, moderate and large sample sizes.

REVIEW OF PREVIOUS STUDIES

Several methods have been proposed for the construction of confidence intervals for variance components in some particular linear models. These include procedures by Satterthwait (1941), Bross (1950), Tukey (1951), Morigiti (1954), Bulmer (1957), and Williams (1962). These procedures are approximate and do not state the exact probabilities associated with their coverage. Boardman (1974) to investigate the actual probability of coverage for several of these procedures conducted a comparative Monte Carlo study. Satterthwaite (1946) and Wald (1947) continue to be popular among practitioners, and Welch (1956) discussed problems involving linear combinations of variances and offered approximate method for these problems. In a more recent paper, Burdick and Sielken (1978) described a procedure to construct unbiased estimator for a linear combination of variance components in an unbalanced nested classification. They showed that this estimator has an exact chi-square distribution and can be used to form an exact confidence interval on the linear combination of variance components.

An improvement of their procedure with respect to degrees of freedom (d.f) and the particular linear combinations considered was given by Seely (1972). Graybill and Wang (1980) proposed a method for obtaining confidence intervals on non-negative linear combinations, of variance in a balanced random model. In this method, large sample confidence limits, developed on the basis of the asymptotic normality of the uniformly minimum variance

unbiased estimator of the ratio are modified so they might be more exact for small or moderate sample sizes .

Interest in interval estimation of function of variance components, other than linear, has developed during the last two decades. Breomling (1969) using a result by Kimball (1951), obtained conservative confidence interval for the variance ratios of balanced random models. The exact confidence coefficients associated with Breomling`s confidence regions were obtained by Sahi and Anderson (1973) in terms of the upper tail of the probability integrals of the inverted Dirichlet distribution. Graybill and Wang (1979) presented approximate lower and upper confidence intervals for three special functions of the variance components in a balanced random two-fold nested classification model.

Recently, approximate methods have been proposed by Boardman (1974) compared using simulation technique between several exact method for the variance components in the random one-way model. Several approximate methods are also available and easier to compute.

Thomas and Hultiquist (1978) introduced the harmonic mean method while El-Bassiouni (1978) introduced the arithmetic mean method. The terms harmonic and arithmetic arise from the fact that the harmonic and arithmetic mean of eigenvalues of a certain covariance matrix were used to replace the individual eigenvalues. Thus simplifies the computation. El-Bassiouni (1978) outlined also a conservative method along the lines of Breomling (1969), which guarantees at least the nominal coverage.

Graybill and Wang (1980) have proposed other methods, and Khuri (1981) provides a brief review of the research in this area. Previous methods have been developed for designs in which sum of squares are independent and have chi-square distribution. In unbalanced designs, however the sum of squares in general does not pass these properties, and use of these methods may be in appropriate.

El-Ganzouri (1986) conducted a Monte Carlo simulation study, which resulted from modification of arithmetic method as compared with the exact and new approximate method also given and included in the comparison. About that there are a lot of new studies such as, Ming Wang (1990) talked about the lower abound of confidence co efficient for a confidence interval on variance components, Weerahandi (1993) talked about generalizing confidence internalizing confidence intervals, Aitkin (1999) talked about a general Maximum likelihood analysis of variance components in generalized linear models.

More recently, Zhang and Woodroof (2002) talked about credible and confidence sets for the ratio of variance components in the balanced one-way model. Bottai and Orsini (2004) discussed confidence intervals for the variance components of random effects linear models. The latest one was Sonogo (2008) that discussed tolerance intervals in random effects model.

Objective of the Study Research

In this thesis, a Monte Carlo simulation study is conducted to examine and compare the performance of the above-mentioned exact and approximate methods and bootstrap confidence interval methods are given. In chapter (2) covers the basic background .The one-way random effects model for unbalanced data is defined; a mathematical expression for the model is presented and probability distributions for its components are discussed. It also

discuss the exact and approximate confidence interval formulas for the ratio of the variance components (ρ). Chapter (3) covers Bootstrap methods of confidence interval for variance components, and then we will explain the comparison between these different methods, in chapter (4) using Mote Carlo comparison And Numerical results of the simulation between exact approximate methods and bootstrap methods and agriculture application .Finally, in chapter (5) discussion of applied cases.

The Confidence Intervals

Section (2.1) is devoted to the confidence interval for the variance component. Section(2.2) introduces the one-way model, discussed confidence interval formulas for the ratio of variance components in exact methods. While, in section (2.3), the same confidence interval for the approximate methods are introduced.

The One-Way Model.

In a sample of (t) treatments, and that have (n) experimental units, at our disposal to be used to compare their effects. Suppose that the (n) units are divided into (t) groups each having (n_i) units, (i = 1,2,……,t), where, the units in group (i) receive treatment (j), let (y_{ij}) denote the response of the jth units in the ith group. We consider the model:-

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \qquad\qquad , i=1,\ldots\ldots.t \ .$$

, j=1,………n_i .

, n=$\sum_{i=1}^{t} n_i$.

Where,

$Y_{ij}\rightarrow$ is the response of the jth observe in the ith treatment,

$\mu \rightarrow$ is the over-all mean,

$\alpha_i \rightarrow$ is the effect of treatment (i), $\alpha_i \sim NID(0,\sigma_a^2)$.

$\varepsilon_{ij}\rightarrow$is the independently of the random error, $\varepsilon_{ij}\sim NID(0,\sigma_e^2)$,

In the matrix form, we have:-

$$Y = 1_n\mu + A\alpha + \varepsilon$$

Where (1_n) is an (n×1) vector of ones,

$$A=\begin{bmatrix} 1_{n_1} & 0 & \cdots & 0 \\ 0 & 1_{n_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1_{n_t} \end{bmatrix}=\text{Diag}(1_{n_i}) \quad , \qquad\qquad , i = 1,\ldots\ldots.t .$$

$\alpha\rightarrow$is (t×1) vector of the treatment effects.

$\varepsilon\rightarrow$is (n×1) vector of the random error.

We can note that

$$E(Y) = 1_n\mu, \quad \text{and} \quad cov(Y) = \sigma_e^2 I_n + \sigma_a^2 AA`.$$

Hence $Y \sim N(1_n\mu, \sigma_e^2 I_n + \sigma_a^2 AA`)$.

Let (H) be an [n $\times$ (n-1)] matrix with ith column.

$$H_i = (1_i`, -i, 0)`/\sqrt{i(i+1)}$$

The columns of (H) form an orthonormal basis for the subspace of vectors orthogonal to [sp (1_n)].

$$H_i \times sp(1_n) = 0. \qquad H`H = I_{n-1} \qquad , HH` = I - \frac{1}{n}1_n 1_n`.$$

See LaMotte (1976). Seely (1972) showed that $(Z = H`Y)$ is maximal location invariant statistic.

$$If (Y`AY) is \ \mu\_invariant.$$

$$Y`AY = Y`HH`CHH`Y \qquad = Z`H`CHZ \qquad = tr(H`CHZZ`).$$

Where (C) is an (n$\times$n) symmetric matrix .In every $\mu\_invariant$ quadratic in(y)is linear function of $(ZZ`)$,and(y)is normally distributed with mean $(1_n\mu)$ and covariance $(\sigma_e^2 I_n + \sigma_a^2 AA`)$.

Where,

$$E(Z) = 0 \qquad , COV(Z) = \sigma_e^2 I_{n-1} + \sigma_a^2 W \quad \text{And} \quad W = H`AA`H.$$

Hence $\quad Z \sim N_{n-1}(0, \sigma_e^2 I_{n-1} + \sigma_a^2 W)$.

Let the spectral decomposition of (W) be given by $W = \sum_{i=0}^k \lambda_i E_i$, and following Olsen, et.all (1976), let $0 = \lambda_1 < \lambda_2 < \cdots < \lambda_k$ be the (k+1) distinct eigenvalues of $(H`kk`H)$ with multiplicities $r_1, \ldots, r_k$, respectively, and for i =1,......, k $(\lambda_0 = 0)$ because (n > t), where (E_i) denotes the orthogonal projection operator on subspace on the eigen-vectors corresponding to λ_i, Hence the density function of:-

$$F(Z) = ke^{(\frac{-1}{2}Z`(\sigma_e^2 I_{n-1} + \sigma_a^2 w)^{-1} Z)}.$$

$$= ke^{[\frac{-1}{2}[Z`(\sum_{i=0}^k (\sigma_e^2 + \lambda_i \sigma_a^2) E_i)^{-1} Z]} \qquad = ke^{[\frac{-1}{2}\sum_{i=0}^k Z` E_i Z/(\sigma_e^2 + \lambda_i \sigma_a^2)]}.$$

$$\text{Let,} Q_i = Z` E_i Z = Y`HE_i H`Y. \qquad i = 0, \ldots, k \qquad\qquad (2.1)$$

Hence $Q_0, \ldots, Q_k$ constitute a minimal sufficient statistic for the family of distributions induced by (Z). Moreover the sufficient statistic $(Q_0, \ldots, Q_K)$ is complete when (k=1) in case $(n_1 = n_2 = \ldots = n_k)$ (balanced design) or in case (t=2) otherwise (k=1), we see that there

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doesn't exist a complete sufficient statistic in case of unbalanced (un equal sample sizes) designs, unless we have only two groups .from now on we assume that the (n_i) are not all equal and that(t>2).

See LaMotte (1976) and El-Bassiouni (1978) for $i = 0, \ldots \ldots, k$ the following results are established by El-Bassiouni (1978).

$$Q_i/(\sigma_e^2 + \lambda_i \sigma_a^2) \sim \chi^2_{(r_i)}$$

(2.2)

Where, $\quad r_i = rank(E_i) = tr(E_i)$

And, $\quad Q_0, \ldots \ldots, Q_k \ are \ independent$

The quadratic forms $\quad Q_0, \ldots \ldots, Q_k$ are related to (ANOVA) sums of squares .In fact, Olsen, et, all (1976) showed that.

$$Q_0 = \sum_{i=1}^{t} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{i.})^2 = SSW$$

(2.3)

Which the sum of squares is within groups, where,

$\bar{Y}_{i.} = (\frac{1}{n_j}) \sum_{j=1}^{n_i} Y_{ij}$. $\qquad$ Further, since.

$Y^{'} HH^{'} Y = \sum_{i=1}^{t} \sum_{j=1}^{n_i} (Y_{ij} - \bar{Y}_{..})^2$.

Where, $\bar{Y}_{..} = \frac{1}{n} \sum_{i=1}^{t} \sum_{j=1}^{n_i} Y_{ij}$, we have that,

$Q_+ = \sum_{i=1}^{t} Q_i = Z^{'} (\sum_{i=1}^{k} E_i) Z$.

$\qquad = Z^{'} Z - Z^{'} E_0 Z \quad = Y^{'} HH^{'} Y - Q_0$.

$\qquad = \sum_{i=1}^{t} n_i (\bar{Y}_{i.} - \bar{Y}_{..})^2 = SSB$ $\hfill$ (2.4)

Which is the sum of squares between groups, Moreover, the degrees of freedom are given by:-

$$r_0 = tr(E_0) = n - t$$

(2.5)

And

$$r_+ = \sum_{i=1}^{t} r_i = t - 1$$

(2.6)

See El-Bassiouni (1978). From the model:

$$Y_{ij} = \hat{\mu} + \hat{\alpha}_i + \hat{\varepsilon}_{ij}.$$

$$(Y_{ij}-\bar{Y}_{..}) = (\bar{Y}_{i.}-\bar{Y}_{..}) + (Y_{ij}-\bar{Y}_{i.})$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

Total   treatment   error

$$\sum\sum(Y_{ij} - \bar{Y}_{..})^2 = \sum n\,(\bar{Y}_{i.} - \bar{Y}_{..})^2 + \sum\sum(Y_{ij} - \bar{Y}_{i.})^2 \,.\quad \text{SSTOT} \qquad = \qquad \text{SSB} \qquad +$$

SSW.

ANOVA

Source	d.f	SS	MS	EMS	F
Between(α_i)	$t-1=r_+$	SSB	MSB=S_1^2	$\sigma_1^2.$	$\frac{S_1^2}{S_2^2}.$
Within (ε_{ij})	$n-t=r_0$	SSW	MSW=S_2^2	$\sigma_2^2=\sigma_e^2$	
Total	$n-1$	SSTOT	$\hat{\sigma}_a^2 = (S_1^2 - S_2^2)/n_0.$		

From ANOVA table:

$$E(MSB) = \sigma_e^2 + n_i\sigma_a^2 = \sigma_1^2. \qquad E(MSE) = \sigma_e^2 = \sigma_2^2.$$

And,

$$MSB = \frac{SSB}{t-1} = \frac{SSB}{r_+} = S_1^2, \quad \hat{\sigma}_e^2 = MSE = S_2^2 = \frac{SSW}{n-t} = \frac{SSW}{r_0}.$$

$$\hat{\sigma}_a^2 = \frac{MSB - MSW}{n_0} = \frac{S_1^2 - S_2^2}{n_0}. \quad n_0 = \frac{1}{t-1}\left(\sum_{i=1}^t n_i - \frac{\sum_i n_i^2}{\sum_i n_i}\right).$$

And from the model.

$$Y_{ij} = \hat{\mu} + \hat{\alpha}_i + \hat{\varepsilon}_{ij}. \quad \bar{Y}_{..} = \mu + \bar{\alpha}_. + \bar{\varepsilon}_{..}.$$

$$var(\bar{Y}_.) = var(\bar{\alpha}_.) + var(\bar{\varepsilon}_.).$$

$$= \frac{\sigma_a^2}{t} + \frac{\sigma_e^2}{n_i t} \qquad = \frac{n_i\sigma_a^2 + \sigma_e^2}{n_i t}.$$

So

$$E(MSB) = n_i\sigma_a^2 + \sigma_e^2, \text{ and } var\left(\hat{\bar{Y}}_{..}\right) = \frac{S_2^2}{n_i t} = \frac{\hat{\sigma}_e^2}{n_i t}.$$

The Exact Confidence Interval Formulas for the Ratio of the Variance Components (ρ)

This section, introduce exact formulas for constructing confidence interval for the ratio (ρ), which is the between variance components to the within variance components.

$$\rho = \frac{\sigma_a^2}{\sigma_e^2} = \frac{between\ variance\ components}{within\ variance\ components}.$$

There are different methods of obtaining confidence intervals for the among groups component of variance in the design have been presented. This methods include procedures proposed by Wald (1947), Bross (1950), Tukey (1951) , Andrson and Buncroft (1952) , and Williams (1963).

The exact confidence interval for (ρ) given in Wald (1947) to construct a confidence interval for (ρ) is as follows:-

$$var(\bar{Y}_{i.}) = \sigma_a^2 + \frac{\sigma_e^2}{n_i} = \frac{\sigma_e^2}{W_i}. \quad \text{With} \quad W_i = \frac{n_i}{1+\rho n_i} i = 1, \dots \dots \dots, t.$$

The confidence intervals for the ratio (ρ) of the variance components can be constructing to this end. Let $F_{(t-1,n-t,p)}$ be such that $\Pr\{F_{(t-1,n-t)} < F_{(t-1,n-t,p)}\} = p$.

Wald (1947) considered the problem of constructing confidence intervals for (ρ).

$$F(\rho) = \frac{r_0}{r_+}\sum_{i=1}^{k}\frac{Q_i/Q_0}{1+\lambda_i\rho}.$$

He proposed an exact interval whose upper end point is the root in (ρ) of the equation.

$$F(\rho) = F_{(r_+,r_0)} = F_{(r_+,r_0,1-\frac{\propto}{2})} = F_{(t-1,n-t,1-\frac{\propto}{2})}.$$

Where $F_{(r_+,r_0\frac{\propto}{2})}$ be F-distribution with degrees of freedom (r_+) and (r_0) and level of significant $(\frac{\propto}{2})$ where $(r_+ = \sum_{i=1}^{k}r_i)$ on the other hand, the lower end point is the root in (ρ) of the equation.

$$F^{'}(\rho) = F^{'}_{(r_+,r_0)} = F_{(r_+,r_0,\frac{\propto}{2})} = F_{(t-1,n-t,\frac{\propto}{2})}.$$

$$\Pr(\rho) = \Pr\{\frac{r_0}{r_+}\sum_{i=1}^{k}\frac{Q_i/Q_0}{1+\lambda_i\rho}\}.$$

$$F_{(t-1,n-t,1-\frac{\propto}{2})} > \Pr\left\{\frac{n-t}{t-1}\sum_{i=1}^{k}\frac{Q_i/Q_0}{1+\lambda_i\rho}\right\} > F_{(t-1,n-t,\frac{\propto}{2})} .$$

Bross Method

According to Bross(1950) derived the confidence intervals for the ratio (ρ) considered here ,but these limits fail to satisfy certain boundary properties .And his upper limit would be negative if:-

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$$\left[\frac{\acute{F}_{(r_+,r_0)}}{\acute{F}_{(r_+,\infty)}} < F < 1\right] \rightarrow \left[\frac{\acute{F}_{(r_+,r_0)}}{\acute{F}_{(r_+,\infty)}} < \frac{S_1^2}{S_2^2} < 1\right].$$

Where $S_1^2 = MSB = \frac{SSB}{t-1} = \frac{\sum n_i(\bar{Y}_{i.}-\bar{Y}_{..})^2}{t-1} = \frac{\sum n_i(\bar{Y}_{i.}-\bar{Y}_{..})^2}{r_+}$.

And $S_2^2 = MSW = \frac{SSW}{n-t} = \frac{\sum\sum(Y_{ij}-\bar{Y}_{i.})^2}{n-t} = \frac{\sum\sum(Y_{ij}-\bar{Y}_{i.})^2}{r_0}$.

$$\hat{\sigma}_a^2 = \frac{S_1^2-S_2^2}{n_0} \quad ., \quad F = \frac{S_1^2}{S_2^2}.$$

Let $F_{(r_+,\infty)}$ be the upper $(1-\frac{\propto}{2})$ percentage point of the (F) distribution with (r_+) and (∞) degrees of freedom (d.f). And let $\acute{F}_{(r_+,\infty)}$ be the lower $(\frac{\propto}{2})$ percentage point of the (F) distribution.

The confidence interval limit for σ_a^2 is .

Upper   limit$=\hat{\sigma}_a^2\left[\frac{F/\acute{F}_{(r_+,r_0-1)}}{\acute{F}_{(r_+,\infty)}F/\acute{F}_{(r_+,r_0)}-1}\right].$ Lower   limit

$=\hat{\sigma}_a^2\left[\frac{F/F_{(r_+,r_0-1)}}{F_{(r_+,\infty)}F/\acute{F}_{(r_+,r_0)}-1}\right].$

Confidence interval limits for ρ.

Upper limit $= \hat{\rho}\left[\frac{F/\acute{F}_{(r_+,r_0-1)}}{\acute{F}_{(r_+,\infty)}F/\acute{F}_{(r_+,r_0)}-1}\right].$ Lower limit $=\hat{\rho}\left[\frac{F/F_{(r_+,r_0-1)}}{F_{(r_+,\infty)}F/\acute{F}_{(r_+,r_0)}-1}\right].$

Tukey Method

In general servey , paper on "Component in Regression" proposed an alternative to Bross's (1950) fiducially bounds because Bross`s bound did not satisfy the "usual" boundary conditions .Tuke y (1951)proposed a modified version of the upper bound but did not elaborate on it .

-Let $F_{(\infty,r_+)}$ be the upper$(1-\frac{\propto}{2})$ percentage point of the F-distribution with ∞ and r_+ degrees of freedom .And let $\acute{F}_{(\infty,r_+)}$ be the lower $(\frac{\propto}{2})$ percentage point of the F-distribution, and it`s like $F_{(r_+,r_0)}$ and $\acute{F}_{(r_+,r_0)}$ be the upper and lower percentage point of F-distribution with r_+ and r_0 degrees of freedom.

The confidence interval limit for σ_a^2 is

Upper limit $= \dfrac{S_2^2}{n_0}\left(F - F`_{(r_+,r_0)}\right)F_{(\infty,r_+)}$ .Lower limit

$= \dfrac{S_2^2}{n_0}\left(F - F_{(r_+,r_0)}\right)F`_{(\infty,r_+)}$.

Confidence interval limits for ρ is .

Upper limit $= \dfrac{\left((F-F`_{(r_+,r_0)})F_{(\infty,r_+)}\right)}{n_0}$.      Lower limit $= \dfrac{\left((F-F_{(r_+,r_0)})F`_{(\infty,r_+)}\right)}{n_0}$.

Where, $F = \dfrac{S_1^2}{S_2^2}$ ، $S_1^2 = \dfrac{SST}{r_+}$ ، and $S_2^2 = \dfrac{SSE}{r_0}$.

Anderson and Bancroft Method [ANDBAN].

While discussing some of the available procedures for confidence interval for σ_a^2 ,Anderson and Bancroft (1952) proposed modified version of Bross`s procedure which satisfies the boundary conditions unless $(F < F_{(r_+,r_0)})$ .In the unlikely situation , the upper bound will be negative.

Let $F_{(r_+,r_0^{-1})}$ be the upper $(1-\dfrac{\infty}{2})$ percentage point of the F-distribution with r_+ and r_0^{-1} degrees of freedom. And let $F`_{(r_+,r_0^{-1})}$ be the lower $\left(\dfrac{\infty}{2}\right)$ percentage point of the F-distribution.

The confidence interval limit for σ_a^2 is.

Upper limit $= \dfrac{S_2^2}{n_0}\left(\dfrac{F-F`_{(r_+,r_0)}}{F`_{(r_+,r_0)}}\right)$.   Lower limit $= \dfrac{S_2^2}{n_0}\left(\dfrac{F-F_{(r_+,r_0)}}{F_{(r_+,r_0)}}\right)$.

Confidence interval limits for ρ is.

Upper limit $= \dfrac{1}{n_0}\left(\dfrac{F-F`_{(r_+,r_0)}}{F`_{(r_+,r_0)}}\right)$.   Lower limit $= \dfrac{1}{n_0}\left(\dfrac{F-F_{(r_+,r_0)}}{F_{(r_+,r_0)}}\right)$.

Williams Method [WIL]

William (1962) independently obtained a (1-2∝) lower bound on the confidence interval coefficient by studying a graph of $(\sigma_1^2 = \sigma_e^2 + n_i\sigma_a^2)$.Versus (σ_a^2/σ_e^2) with a plot of four lines which bound the two interval sets. After examining a projection of this intersection set on the (σ_a^2) axis, he established lower confidence co-efficient which we have found to be too conservative.

Define $\chi_U^2(r_+)$ and $\chi_L^2(r_+)$ as the upper $(1-\frac{\alpha}{2})$ and lower $(\frac{\alpha}{2})$ percentage points of the Chi-square distribution with (r_+) degrees of freedom respectively.

The confidence interval limit for σ_a^2 is .

Upper   Limit $=\dfrac{r_+}{n_0\chi_L^2(r_+)}\left(S_1^2 - S_2^2 F^{`}{}_{(r_+,r_0)}\right)$.Lower   Limit$=$

$\dfrac{r_+}{n_0\,\chi_U^2(r_+)}\left(S_1^2 - S_2^2 F_{(r_+,r_0)}\right)$.

Confidence interval limits for ρ is.

Upper Limit $=\dfrac{r_+}{n_0\sigma_e^2\,\chi_L^2(r_+)}\left(S_1^2 - S_2^2 F^{`}{}_{(r_+,r_0)}\right)$.

Lower Limit $=\dfrac{r_+}{n_0\sigma_e^2\,\chi_U^2(r_+)}\left(S_1^2 - S_2^2 F_{(r_+,r_0)}\right)$.

Williams (1962) used a result of Tukey (1951) where Williams limits are shown and so carefully drive by Williams and they were suggested by Tukey a number of years earlier (Williams-Tukey method{WILLTUK}).

-Let $F_{(\infty,r_+)}$ be the upper $(1-\frac{\alpha}{2})$ percentage point of the F-distribution with ∞ and r_+ degrees of freedom .And let $F^{`}{}_{(\infty,r_+)}$ be the lower $(\frac{\alpha}{2})$ percentage point of the F-distribution, and it`s like $F(r_+,r_0)$ and $F^{`}(r_+,r_0)$ be the upper and lower percentage point of F-distribution with r_+ and r_0 degrees of freedom.

Confidence interval limits for (F) is:-

Upper limit $=\dfrac{S_2^2}{n_0}\left(\dfrac{F-F^{`}{}_{(r_+,r_0)}}{F^{`}{}_{(r_+,r_0)}}\right)$.   Lower limit $=\dfrac{S_2^2}{n_0}\left(\dfrac{F-F_{(r_+,r_0)}}{F_{(r_+,r_0)}}\right)$.

Confidence interval limits for ρ is.

Upper limit $=\dfrac{\sigma_a^2}{n_0}\left(\dfrac{F-F^{`}{}_{(r_+,r_0)}}{F^{`}{}_{(r_+,r_0)}}\right)$.   Lower limit $=\dfrac{\sigma_a^2}{n_0}\left(\dfrac{F-F_{(r_+,r_0)}}{F_{(r_+,r_0)}}\right)$.

There are many people who discussed the exact tests, Seely and El Bassiouni (1985) considered extensions of Wald`s variance components tests. They discussed confidence intervals for a variance ratio in unbalanced mixed linear models… etc.

In the next section we will reviewed the approximate methods.

Approximate Methods

In this section, we will discuss the confidence interval formulas for the ratio of variance components in approximate methods, and these include procedures proposed by Satterthwait (1946), El-Bassiouni (1978), and EL-Ganzouri (1986), then let $\bar{y}$ denote the unweight mean of group means.

$$\bar{y} = \frac{1}{t}\sum_{i=1}^{t} y_i$$

$$Q_0 = \sum_{i=1}^{t}\sum_{j=1}^{ni}(y_{ij} - \bar{y}_{i.})^2 = SSW \qquad , \bar{y}_{i.} = \frac{1}{n_i}\sum_{i=1}^{t}\sum_{i=1}^{ni} y_{ij}$$

$$S_{\bar{y}}^2 = \sum_{i=1}^{t}(\bar{y}_{i.} - \bar{y})^2/(t-1)$$

(2.7)

$$\tilde{n} = \frac{t}{\sum_{i=1}^{t}\frac{1}{ni}}$$

Where, $(\tilde{n})$ is the harmonic mean of the sample size, and $(S_{\bar{y}}^2)$ is the sample variance of treatment means.

Satterthwait Method [A]

According to Satterthwaite (1946) showed that $(F/(1 + \lambda_A\rho))$ is distributed approximately as F (t-1, n-t) $= F_{(r_+, r_0)}$.

Where

$$F = \frac{S_1^2}{S_2^2} \text{‚} \quad S_1^2 = \frac{SSB}{t-1} \text{‚} \quad \text{and } S_2^2 = \frac{SSW}{n-t}.$$

S_1^2 Is the variance between mean of squares, see (2.3). S_2^2 Is the variance within mean of squares, see (2.4).

Hence $F = \frac{(n-t)SSB}{(t-1)SSW}$. And $\lambda_A = (n - \sum_{i=1}^{t}\frac{n_i^2}{n})/(t-1)$.

The confidence interval for ρ is thus given by:-

$$\left[\left(\frac{1}{\lambda_A}\right)\left(\frac{F}{F_{(r_+, r_0, 1-\frac{\alpha}{2})}} - 1\right) < \rho < \left(\frac{1}{\lambda_A}\right)\left(\frac{F}{F_{(r_+, r_0, \frac{\alpha}{2})}} - 1\right)\right] .$$

And Satterthwaite showed that the distribution of $\sum_{i=1}^{K}(1 + \lambda_i\rho)\chi_{(r_i)}^2$ is approximately the distribution $g(\rho)\chi_{m(\rho)}^2$, where the quantities $g(\rho)$ and $m(\rho)$ are

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chosen. Such that , the first two moments of approximate distribution are equal to those of the exact one.

Moriguti Method

Morigiti (1954)suggested the form of Tukey (1951)as an alternative to Boss's (1950) his procedure with others developed up to that time.

The confidence interval for σ_a^2 where $\sigma_a^2 = \frac{(S_1^2 - S_2^2)}{n_i}$ is given by:-

$$\text{Upper limit} = \frac{S_1^2}{n_0}\left[F_{(\infty, r_+)} - \frac{1}{F} + \frac{F_{(r_0, r_+)} - F_{(\infty, r_+)}}{F^2 F_{(r_0, r_+)}^2}\right] \quad .$$

$$\text{Lower limit} = \frac{S_1^2}{n_0}\left[\frac{1}{F_{(r_+, \infty)}} - \frac{1}{F} - \frac{F_{(r_+, r_0)}}{F^2}\left(\frac{F_{(r_+, r_0)} - F_{(r_+, \infty)}}{F_{(r_+, \infty)}}\right)\right] \quad .$$

Confidence interval limits for ρ is.

$$\text{Upper limit} = \frac{F}{n_0}\left[F_{(\infty, r_+)} - \frac{1}{F} + \frac{F_{(r_0, r_+)} - F_{(\infty, r_+)}}{F^2 F_{(r_0, r_+)}^2}\right]$$

$$\text{Lower limit} = \frac{F}{n_0}\left[\frac{1}{F_{(r_+, \infty)}} - \frac{1}{F} - \frac{F_{(r_+, r_0)}}{F^2}\left(\frac{F_{(r_+, r_0)} - F_{(r_+, \infty)}}{F_{(r_+, \infty)}}\right)\right]$$

Bulmer Method

According to Bulmer (1957)this approximate confidence interval is found for the expected value of the difference between two quantities which are independently distributed proportional to (χ^2) varieties.

The confidence intervals limits forσ_a^2 is:-

$$\text{Upper limit} = \frac{S_2^2}{n_0}\left[\frac{F}{F`_{(r_+, \infty)}} - 1 + \frac{F`_{(r_+, r_0)}}{F}\left(1 - \frac{F`_{(r_+, r_0)}}{F`_{(r_+, \infty)}}\right)\right] \quad .$$

$$\text{Lower limit} = \frac{S_2^2}{n_0}\left[\frac{F}{F_{(r_+, \infty)}} - 1 + \frac{F_{(r_+, r_0)}}{F}\left(1 - \frac{F_{(r_+, r_0)}}{F_{(r_+, \infty)}}\right)\right] \quad .$$

Confidence interval limits for ρ is.

$$\text{Upper limit} = \frac{\sigma_a^2}{n_0}\left[\frac{F}{F`_{(r_+, \infty)}} - 1 + \frac{F`_{(r_+, r_0)}}{F}\left(1 - \frac{F`_{(r_+, r_0)}}{F`_{(r_+, \infty)}}\right)\right]$$

$$\text{Lower limit} = \frac{\sigma_a^2}{n_0}\left[\frac{F}{F_{(r_+, \infty)}} - 1 + \frac{F_{(r_+, r_0)}}{F}\left(1 - \frac{F_{(r_+, r_0)}}{F_{(r_+, \infty)}}\right)\right]$$

Thomas and Hultiquist Method [H]

Thomas and Hultiquist(1978) showed that $[\tilde{n}S_{\bar{y}}^2(S_2^2(1+\tilde{n}\rho)]$ is distributed approximately as $F_{(r_+, r_0)}$. in this case, the confidence interval for ρ is given by :

$$\text{Upper limit} = \frac{1}{\tilde{n}}\left(\frac{\tilde{F}}{F_{(r_+, r_0)}} - 1\right). \qquad \text{Lower limit} = \frac{1}{\tilde{n}}\left(\frac{\tilde{F}}{F`_{(r_+, r_0)}} - 1\right).$$

Where, $\qquad \tilde{F} = \frac{\tilde{n}S_{\bar{y}}^2}{S_2^2} \qquad , \qquad S_2^2 = \frac{SSW}{r_0} \qquad .$

We call this the harmonic method (H).

El-Bassiouni Method

See El-Bassiouni (1978), leads to the following relationship,

$$g(\rho) = 1 + (a_1\rho + a_2\rho^2)/(a_1\rho + r_+),$$

and, $\qquad m(\rho) = \frac{a_1^2\rho^2 + r_+(2a_1\rho + r_+)}{a_2\rho^2 + (2a_1\rho + r_+)} \quad ,$

where ,

$$a_1 = \sum_{i=1}^{k}\lambda_i r_i$$

$$= \text{tr}(H`AA`H) = \text{tr}(AA`HH`)$$

$$= n - \frac{1}{n}\sum_{i=1}^{t} n_i^2$$

And,

$$a_2 = \sum_{i=1}^{k}\lambda_i^2 r_i \quad = \text{tr}(H`AA`H)^2 \quad = \text{tr}(AA`HH`AA`HH`)$$

$$= \sum_{i=1}^{t} n_i^2 - \frac{2}{n}\sum_{i=1}^{t} n_i^3 + \frac{1}{n^2}(\sum_{i=1}^{t} n_i^2)^2.$$

The Satterthwait approximation thought to be fairly accurate for nearly balanced designs, nearly equal $\lambda_i's$ .For such models we note that

$$m^* = a_1^2/a_2 \simeq r_+ = t-1 \quad .$$

Hence $\quad m(\rho) \simeq r_+ = t-1.$ $\qquad$ futher , $g(\rho) \simeq 1 + (a_1/r_+)\rho.$

$$\text{let } \lambda_A = a_1/r_+ = \sum_{i=1}^{k} \lambda_i r_i / \sum_{i=1}^{k} r_i.$$

$$= (n - \frac{\sum_{i=1}^{t} n_i^2}{n})/(t-1).$$

It is interesting to note that λ_A is the arithmetic mean of the λ_i `s, it is also the co-efficient of σ_a^2 in the expression of the expected value of S_1^2. We are going to call this method (based on Satterthwait approximation), the arithmetic mean method A.

EL–Bassiouni (1978) gave a conservative confidence interval as a conservative upper limit for ρ, which is given by:-

$$= \min_{<i<k} \left[\left(\frac{1}{\lambda_i}\right) \left(\frac{Q_i/r_i}{Q_0/r_0} F_{(r_0, r_i, 1-\alpha_i)} - 1\right) \right]$$

Where the $(\alpha_i \text{ 'S})$ are such that $\left[1 - \alpha = \prod_{i=1}^{k}(1 - \alpha_i)\right]$ .On the other hand a conservative lower limit for **ρ** is given by :-

$$= \max_{l<i<k} \left[\left(\frac{1}{\lambda_i}\right) \left(\frac{Q_i/r_i}{Q_0/r_0} F_{(r_0, r_i, \alpha_i)} - 1\right) \right]$$

A numerical study of Sahi and Anderson (1973) reveals that the conservative confidence coefficient are very good approximations to the exact ones.

El-Ganzouri Method [A.ADJ]

According to El-Ganzouri (1986) and from the approximate degrees of freedom $(m^* = \frac{a_1^2}{a_2^2})$ and instead of $(r_{+=t-1})$.El-Ganzouri considered the following confidence interval for (ρ) is as:-

$$\text{Upper limit} = \left(\frac{1}{\lambda_A}\right) \left(\frac{F}{F_{(m^*, r_0, \frac{\alpha}{2})}} - 1\right). \qquad \text{Lower limit}$$

$$= \left(\frac{1}{\lambda_A}\right) \left(\frac{F}{F_{(m^*, r_0, 1-\frac{\alpha}{2})}} - 1\right).$$

This is the arithmetic adjusted mean method (A.A DJ).

BOOTSTRAP CONFIDENCE INTERVAL

Introduction

In (1979) Efron introduced the Bootstrap approach as an alternative resampling methodology to extend the well-known in estimating the bias and standard errors of statistics in order to construct reliable confidence intervals. The mechanism of this approach provides as what is namely the bootstrap distribution as well as the approximate sampling distributions, which is shown to be at least as good as the Gaussion approximation, The Bootstrap distribution is better than the approximate normal distribution, see Efron (1992). In the next chapter we will compare the numerical results between different all d the method of estimation for the confidence interval of ρ .

The Bootstrap estimate of confidence interval

The bootstrap was introduced in (1979) by Efron as a computer_ based method for estimating the interval of ($\hat{\rho} = \frac{\hat{\sigma}_a}{\hat{\sigma}_e}$), it the best to the advantage of being completely automatic.

The Bootstrap methods depend on the notion of a bootstrap sample. Let $\left(\hat{F}\right)$ be the empirical distribution. Putting probability $\left(\frac{1}{n}\right)$ on each of the observed values (y_i) , $i = 1, 2, \ldots \ldots, n$ .A bootstrap sample is defined to be a random sample of size (n) drawn from $\left(\hat{F}\right)$ say $[Y^* = (y_1^*, y_2^*, \ldots \ldots, y_n^*)]$,

$$\hat{F} \rightarrow (y_1^*, y_2^*, \ldots \ldots, y_n^*).$$

(B) Bootstrap samples are generated from the original data set. Each bootstrap sample has (n_i) elements generated by sampling with replacement (n_i) times from the original data set .Bootstrap replicates $[\rho(y_1^*), \ldots \ldots, \rho(y_B^*)]$ are obtained by calculating the value of the statistic $[\rho(y)]$ on each bootstrap sample .Finally, the variance components of the values $[\rho(y_1^*), \rho(y_2^*), \ldots \ldots, \rho(y_B^*)]$ are our estimates of the variance components of $[\rho(y)]$.

The bootstrap algorithm begins by generating a large number of independent bootstrap samples $(y_1^*, y_2^*, \ldots \ldots, y_B^*)$,each of size (n_i) typical value for (B) , corresponding each bootstrap sample is a bootstrap replication of (ρ) .namely $\rho(y_b^*)$,the value of the statistic(s) evaluated for (y_b^*) ,If $\rho(y)$ is the sample median, for instance, then $\rho(y^*)$ is the median of the bootstrap sample . The bootstrap estimate of standard error $\widehat{se}_{boot}$ is the standard deviation of the bootstrap replication as,

$$\widehat{se}_{boot} = \{\textstyle\sum_{b=1}^{B} [\rho(y_b^*) - \rho(.)]^2 / (B-1)\}^{\frac{1}{2}}, \quad \rho(.) = \textstyle\sum_{b=1}^{B} \rho(y_b^*)/B$$

$$prob_F\left\{z^\alpha \leq \frac{\hat{\rho} - \rho}{\widehat{se}} \leq z^{1-\alpha}\right\} = 1 - 2\alpha$$

(3.1)

$$prob_F\left\{\rho \in \left[\hat{\rho} - z^{(1-\alpha)}.\widehat{se} \leq \rho \leq \hat{\rho} - z^\alpha.\widehat{se}\right]\right\} = 1 - 2\alpha$$

(3.2)

The probability calculation (3.1) is done with the true mean equaling ρ, so $\hat{\rho} \sim N(\rho, se^2)$. we will denote confidence intervals by $(\hat{\rho}_{lo}, \hat{\rho}_{up})$, so $[\hat{\rho}_{lo} = \hat{\rho} - z^{(1-\alpha)}.se]$ and $[\hat{\rho}_{up} = \hat{\rho} - z^{\alpha}.se]$ for the interval in(3.2) .In this case, we can see that the interval $[\hat{\rho} - z^{1-\alpha}.se , \hat{\rho} - z^{\alpha}.se]$ has probability exactly (1-2α)of containing the true value of ρ . The probability that ρ lies below the lower limit is exactly (α) ,as is the probability that (ρ) exceeds the upper limit ,so that:-

$$prob_\rho\{\rho < \hat{\rho}_{lo}\} = \alpha, \quad prob_\rho\{\rho > \hat{\rho}_{up}\} = \alpha \tag{3.3}$$

The fact that (3.3) hold for every possible value of (ρ) is what we mean when we say that a$(1-2\alpha)$ confidence interval $(\hat{\rho}_{lo}, \hat{\rho}_{up})$ is accurate .It is random variables being $\hat{\rho}_{lo}$ and $\hat{\rho}_{up}$, there for:-

When $\alpha = 0.05 , 1 - 2\alpha = 0.90$ in general so,

$$(\hat{\rho} - z^{(1-\alpha)}.\widehat{se}, \hat{\rho} - z^{\alpha}.\widehat{se})$$

is called the standard confidence interval with coverage probability equal$(1-2\alpha)$, or confidence level 100%, $(1-2\alpha)$% or more simply it is called a$(1-2\alpha)$confidence interval for (ρ) .$z^{\alpha} = z^{(1-\alpha)}$ .but the more accurate form as follows:-

$$\hat{\rho} \pm z^{(1-\alpha)}.\widehat{se}$$

A $(1-2\alpha)$confidence interval $(\hat{\rho}_{lo}, \hat{\rho}_{up})$ with property (3.3) is called equal-tailed, see Efron (1992) .This refers to the fact that the coverage error (2α) is divided up evenly between the lower and upper ends of the interval. Confidence intervals are usually constructed to be equal-tailed and we will restrict attention to equal-tailed interval, in our discussion notice also that property (3.2) implies property (3.1), but not vice-versa. That is, (3.2) requires that the one-sided miscoverage of the interval be α on each side, rather than just an overall coverage of$(1-2\alpha)$.

With supposing that the true (ρ) were equal to ($\hat{\rho}_{lo}$), say.

$$\hat{\rho}^* \sim N(\hat{\rho}_{lo}, se^2).$$

Many types for the bootstrap confidence intervals had been introduced in different statistical areas and either in parametric or non-parametric settings. The simplest bootstrap confidence interval is that one for which the nominal quantiles of the bootstrap distribution of the statistic under study. This is the so-called basic bootstrap confidence interval. Another more important type ,which is proved to be more accurate , is the bootstrap-t (also called the percentile-t)confidence interval introduced first by Efron(1979).This method is based on

using the quantiles of a studentized bootstrap statistic , generalizing thus the student-T method in constructing confidence intervals .

The approximate $(1 - \alpha)$ equitailed confidence interval in this case will be that with confidence limits either $\left(\hat{\rho}_n \pm z_{\frac{\alpha}{2}} \hat{\sigma}_{\hat{\rho}} \ or \ \hat{\rho}_n \pm t_{\frac{\alpha}{2}} \hat{\sigma}_{\hat{\rho}}\right)$, with (z_α) and (t_α) are the α-quantiles of the standard normal and standard (t) distribution, respectively. These approximate methods remain unacceptable in situations where, say, it is known that (G_n) is highly skewed or spare in order to stabilize the variance Efron (1992).

Where, (G_n) be the sampling distribution function of the quantity $\left(\frac{\hat{\rho}_n - \rho}{\sigma_{\hat{\rho}}}\right)$, and G_n^* be its bootstrap approximation with $\sigma_{\hat{\rho}}^2$ the variance or the estimated variance of $\hat{\rho}_n$.

Efron (1987) introduced approximate confidence intervals based on bootstrap compilation like the standard intervals; these can be applied automatically to almost any situation. Though at greater computational expense than (3.1). Unlike (3.1), the bootstrap intervals transform correctly .Each bootstrap sample gives the bootstrap replication of $\left(\hat{\rho} = \frac{\hat{\sigma}_a^2}{\hat{\sigma}_e^2}\right)$.

Methods of Bootstrap Confidence Interval for The Ratio of Variance Component (ρ)

Many types of bootstrap confidence intervals were introduced and have been applicable a wide range of both theoretical and applied field. In this section, the bootstrap methods to construct approximate confidence intervals were described and reviewed. These methods are standard approximate method (SAM) proposed by Lawless (1982).Bootstrap-T (Percentile-T)method ,and Percentile Bootstrap method proposed by Efron (1979) ,The bias -corrected percentile method (BC Method) . Accelerate Bias –corrected method (BCa Method) ,proposed by Efron (1982) and The approximate confidence Interval (ABC Method), proposed by Efron (1987) all these methods will be used for constructing approximate confidence interval for the ratio of variance components ρ.

The Basic Bootstrap Confidence (BB)

Efron (1992) tried to create a α/2 confidence interval for a parameter ρ based on a sample estimate $\hat{\rho}$, we determine the distance that we plausible expect $\hat{\rho}$ to fall from ρ at the α/2% level. As we don`t know the distribution of $\hat{\rho} \ (or \ of \ \hat{\rho}, \rho)$ and their percentiles, we take the corresponding percentiles of the bootstrap distribution. Under the assumption that the percentiles of the sampling distribution and the bootstrap distribution are very close, see Efron (1992).

$$p\left(2\hat{\rho} - \rho_{1-\frac{\alpha}{2}}^* \leq \rho \leq 2\hat{\rho} - \rho_{\frac{\alpha}{2}}^*\right) = (1-\alpha)\% .$$

The approximate $(1 - \alpha)$ % confidence interval of ρ is

$$\rho \in \left(2\hat{\rho} - \hat{\rho}^*_{\left(\frac{\alpha}{2}\right)}, 2\hat{\rho} - \hat{\rho}^*_{\left(1-\frac{\alpha}{2}\right)}\right)$$

$\hat{\rho}$ is the sample which generated from the frist sample ,

$\hat{\rho}^*_\alpha$ is (α) bootstrap percentile confidence interval for (ρ^*) Sample.

The Standard Approximate Interval (S A M)

The exact confidence interval for percentile (ρ) ,have been obtained by Monte Carlo simulations to estimate the percentage points of the distribution of a pivotal quantity that can be used to find confidence limits for percentiles, and S_{ρ^*}.It developed procedures , for complete ,this method which are passed on the asymptotic normality of the unknown parameters. According this method$\Phi(m)$ is the m<u>th</u> quantile of the standard normal distribution function. Using (SAM), the approximate $(1-\alpha)$ % confidence interval of ρ is

$$\rho \in (\hat{\rho} \pm \text{Se}_{\hat{\rho}} \Phi_{\frac{\alpha}{2}}).$$

Where, $\hat{\rho}$ is the sample which generated from the frist sample ,

And $\text{Se}_{\hat{\rho}}$ Is the standard deviation of the $\hat{\rho}$ sample.

,Where Φ_m is the m<u>th</u> quantile of the standard normal distribution.

The Bootstrap-t Interval (BT

Where t_{n-1} represents the student`s –t distribution on (n-1) degrees of freedom. Using the approximation, our interval with $t^{\left(\frac{\alpha}{2}\right)}_{(n-1)}$ denoting the α/2 <u>th</u> percentile of the (t) distribution on (n-1) degrees of freedom the use of t distribution doesn`t adjust the confidence interval to account for sekwness in the underlying population. Idea behind the bootstrap–t method is easier to describe than the percentile-based bootstrap intervals in practice, however, the bootstrap –t can give somewhat erratic results and can be heavily influenced be a few outlying data points. Using (bootstrap-t).

the approximate (1-α) % confidence interval of ρ is

$$\rho \in \left(\hat{\rho} - t^{\frac{\alpha}{2}}_{(n-1)} \widehat{se}_{boot}, \hat{\rho} - t^{\left(1-\frac{\alpha}{2}\right)}_{(n-1)} \widehat{se}_{boot}\right)$$

$\hat{\rho}$ is the sample which generated from the frist sample ,

$\widehat{se}_{boot} = \{\sum_{b=1}^{B}[\rho * - \rho(.)]^2/(B-1)\}^{\frac{1}{2}}$, $\rho(.) = \sum_{b=1}^{B} \rho * /B$.

The Percentile Bootstrap Interval (PB)

Confidence limits can be made more accurate by working on transformed scale. The percentile intervals are those approximate intervals which implicitly exploit the properties of using good transformations of the parameter space without any need to know such transformations explicitly.

Much better is to use the approximate (student-t)distribution for the studentized quantity $\left(\frac{\hat{\rho}_n - \rho}{\hat{\sigma}_{\hat{\rho}_n}}\right)$ with $(\hat{\sigma}_{\hat{\rho}_n}^2)$ is an asymptotic estimate of $\left(\sigma_{\hat{\rho}_n}^2\right)$ .The variance of $(\hat{\rho}_n)$,the approximate $(1-\alpha)$ equitailed confidence interval in this case will be that with confidence limits either $(\hat{\rho}_n \pm Z_{\frac{\alpha}{2}}\hat{\sigma}_{\hat{\rho}}, \hat{\rho}_n \pm t_{\frac{\alpha}{2}}\hat{\sigma}_{\hat{\rho}})$. With $(Z_{\frac{\alpha}{2}})$ and $(t_{\frac{\alpha}{2}})$ are theα-quantiles of the standard normal and student-t distribution, respectively. These approximate methods remain unacceptable in situations. Where, say, it is known that (G_n) is highly skewed or the variance is unstable. In such case, the approximation can be refined by using transformations. Usually, the sekwness is due to the variability of the variance with respect to ρ , which needs the transformation of the parameter space in order to stabilize the variance. Practically, using the resampling techniques , generate a sufficient large number (B) of bootstrap resample of size (n)from the original sample , from each bootstrap resample (b) , b=1,2,…….,B. Compute the value of the estimator $\hat{\rho}_n$,denoted by ρ_b^* .The percentile $\left[G_n^{*-1}(\alpha)\right]$ can be estimated from the histogram of ρ^*. Precisely $G_n^{*-1}(\alpha)$ is the value such that:-

$$\frac{\#\left\{\rho_b^* \le G_n^{*-1}(\alpha)\right\}}{B} = \alpha.$$

The previous discussion suggests how we might use the percentiles of the bootstrap histogram to define confidence limits .This is exactly how the percentile interval works .A bootstrap data set (y^*) is generated according to $\hat{\rho} \to y^*$, and bootstrap replications $[\hat{\rho}^* = \rho(y^*)]$ are computed .Let $(\hat{G})$ be the cumulative distribution function of $\hat{\rho}^*$.

The percentile interval is also "range-preserving", i.e. the interval full within the allowable range of the parameter if there is a restriction on the values the parameter can take. This because $(\hat{\rho}_n)$ obeys the range of the parameter, and both end points of the interval are values of the bootstrap – statistic (ρ^*) .Confidence intervals that are range-preserving tend to be more accurate and reliable.

On the other hand, a whole different class of confidence intervals introduced and refined by Efron(1979,1987), consists of the percentile and the adjusted percentiles methods, namely .The accelerated bias–corrected (BCa) interval. These interval has the advantage of using implicitly all the properties of good transformations without any need to know such transformations, taking thus into consideration the bias and sekwness in the distribution function of the underlying statistic. He (1992) introduced the computational burden in the (BCa) method.

Indeed, like any estimated distributed distribution, the convergence or consistency of (G_n^*) to (G_n) can be measured by the limit of the error:-

$$\|G_n^* - G_n\| = sup_y |G_n^*(y) - G_n(y)| \ .$$

Its magnitude, as indicate by Efron (1979) is due to the different between F_n or ($\hat{F}$) and F, together with nature of functional $\rho(F)$ under study, the study of such error need analytical efforts.

To formulate our problem, Let y= $(y_1, \ldots \ldots \ldots, y_n)$ be a random sample of size (n), drawn from a population with unknown distribution function (F), and let (F_n) be the corresponding empirical distribution function.

Moreover, Let (G_n) be the sampling distribution function of the quantity $\frac{\hat{\rho}_n - \rho}{\sigma_{\hat{\rho}_n}}$, and (G_n^*) be its bootstrap approximation with the variance of the estimated variance of $\hat{\rho}_n$.

Much better is to use the approximate (student-t) distribution for the studentized quantity ($\frac{\hat{\rho}_n - \rho}{\hat{\sigma}_{\hat{\rho}_n}}$) with ($\hat{\sigma}_{\hat{\rho}_n}^2$) is an asymptotic estimate of ($\sigma_{\hat{\rho}_n}^2$) the variance of ($\hat{\rho}_n$).

Thus, if $m_{(.)}$ is a transformation such that $m(\hat{\rho}_n)$ has a symmetric distribution then the bootstrap $\alpha\_quantile$ of the distribution of $[m(\rho^*) - m(\hat{\rho}_n)]$ will be equal to $[m(\hat{\rho}_n) - m_{1-\alpha}(\rho^*)]$ with $[m_{1-\alpha}(\rho^*)]$ the $(1-\alpha)$ quantile of the bootstrap histogram of $[m(\rho^*)]$.

The $(1-\alpha)$ basic bootstrap confidence interval for $[m(\rho)]$ will be:-

$$\left[m_{(\frac{\alpha}{2})}(\rho^*), m_{(1-\frac{\alpha}{2})}(\rho^*) \right].$$

Which by applying the inverse transformations $m_{(.)}^{-1}$ is transformed back to the $(1-\alpha)$ bootstrap percentile confidence interval for (ρ) , that is :-

$$(\hat{\rho}_{(\frac{\alpha}{2})}^* , \hat{\rho}_{(1-\frac{\alpha}{2})}^*)$$

It involves creating many (≤5000) bootstrap samples and calculating a ($\rho_{0.95}^*$) value for each bootstrap sample then a 95% interval for ρ , for example ,would be :-

$$\hat{\rho}_{0.025}^* \leq \rho \leq \hat{\rho}_{0.975}^* \ .$$

$\hat{\rho}_\alpha^*$ Is (α) bootstrap percentile confidence interval for (ρ*) Sample.

The Residual Bootstrap Interval(RM)

For this method, many (≤ 5000) bootstrap samples are created and $e^* = \hat{\rho}^* - \hat{\rho}$ is calculated for each bootstrap sample. A 95% interval for ρ, for example m would then be:-

$$\hat{\rho} - e^*_{0.975} \leq \rho \leq \hat{\rho} - e^*_{0.025} \quad . \quad \text{See Efron (1992)}$$

The approximate ($1 - \alpha$) % confidence interval for ρ.

$$\rho \in (\hat{\rho} - e^*_{\frac{\alpha}{2}}, \hat{\rho} - e^*_{\left(1-\frac{\alpha}{2}\right)})$$

$\hat{\rho}^*_\alpha$ is (α) bootstrap percentile confidence interval for (ρ*) Sample

$\hat{\rho}$ *is the sample which generated from the frist sample* , $e^* = \hat{\rho}^* - \hat{\rho}$.

$\hat{\rho}^*$ Are the bootstrap samples, which generated from $\hat{\rho}$ sample.

The Bias-Corrected And Accelerated Interval (BCa)

Efron (1987) introduced a new approximate bootstrap confidence interval as a generalization of using transformation theory , he showed that of the second order accuracy of percentile bootstrap intervals can be gained by assuming a monotone increasing transformation $m_{(.)}$, for which the asymptotic distribution of the transformed estimator $[\hat{\phi} = m(\hat{\rho}_n)]$ is normal , and that its bias in estimating $[\phi = m(\rho)]$ is (z_0), with some variance depend on ϕ . The idea is to build the interval for the transformed parameter $[\phi = m(\rho)]$, then transform it back using ($m^{-1}_{(.)}$) to the scale of (ρ). Although ($m_{(.)}$) is not always known, the use of the bootstrap distribution makes it possible to build the interval for (ρ) without any knowledge of the form of transformation ($m_{(.)}$) as long as it is only assumed to be existent.

It is more natural to assume that the variance of $\hat{\phi}$ is a function of ϕ. Efron (1987) assumed that the standard deviation ($se_{\hat{\phi}}$) is effected by (ϕ) via an "acceleration" constant (a) that is why the method was called accelerated bias- corrected.

As a modified percentile interval, the (BCa) is still transformation respecting. Moreover, it has the additional property of being second order correct and accurate.

Although the bootstrap-t interval is also second order correct and accurate it failed to be transformation respecting. The matter, which draw the attention to the (BCa) interval as the most promising approximate bootstrap interval.

Assume that for some unknown monotone increasing transformation ($m_{(.)}$), unknown bias correction factor (z_0) and unknown sekwness correction factor (a), the transformed estimator [$\hat{\phi} = m(\rho)$] for [$\phi = m(\rho)$] is asymptotically normally distributed.

$$\hat{\phi} \sim N(\phi - z_0 se_{\phi}, se_{\phi}^2), \quad \text{i.e.} \frac{\hat{\phi}-\phi}{se_{\phi}} + z_0 \sim N(0,1)$$

Then ,

$$\hat{\phi} = \phi + se_{\phi}(z - z_0)z \sim N(0,1). \tag{3.4}$$

With

$$\sigma_{\phi} = 1 + a\phi \tag{3.5}$$

The derivation of the interval in this from relies essentially on the standard transformation form. Note from (3.4) and (3.5) that.

$$1 + a\hat{\phi} = (1 - a\phi)(1 + a(z - z_0))$$

Estimating The Bias Correction Constant (Z_0)

Efron (1987) introduced that:-

$$\Phi(z_0) = \Pr(\hat{\phi} \leq \phi) \text{ and } G_n^*(\hat{\rho}_n) = \Pr(\phi^* \leq \hat{\phi}).$$

Approximating, now,$\Pr(\hat{\phi} \leq \phi) \, by \, \Pr(\hat{\phi}^* \leq \phi)$, it gives; $G_n^*(\hat{\rho}_n) \cong \Phi(z_0)$, i.e.

$$z_0 \cong \Phi^{-1}(G_n^*(\hat{\rho}_n)).$$

Practically, it can be computed the following by the proportion of bootstrap replications less than ρ, i.e.

$$z_0 = \Phi^{-1}(\frac{\#\{\rho_b^* < \rho\}}{B}). \tag{3.6}$$

It measures the median bias ofρ^*, i.e. the discrepancy between the median of ρ^* and ρ in normal units, Efron (1992)

The Acceleration Constant (a)

Efron (1987) showed that equation (3.4)can be written as follows

$$\sigma_{\phi} = \sigma_{\phi_0}[1 + a(\phi - \phi_0)/\sigma_{\phi_0}],$$

Thus, the acceleration constant (a) is may be written as:-

$$a = \frac{d\left(\frac{\sigma_\phi}{\sigma_{\phi_0}}\right)}{d[(\phi-\phi_0)/\sigma_{\phi_0}]} \qquad (3.7)$$

The above three formula are nearly equal for computing the constant (a) in the sense that equal $\left(\frac{1}{6}\right)$ of the coefficient of sekwness for different ways of computing these coefficients in different cases.

For the simple parameter case, let the estimator$\hat{\rho}$ have a p.d.f of the family $[g_\rho(\hat{\rho})]$ is the density $\{\frac{\partial G_\rho(\hat{\rho})}{\partial \hat{\rho}}\}$, and define $(\dot{\ell}_\rho)$ to be the score function of the family $[f_n(\hat{\rho}_n)]$, of the form.

$$\dot{\ell}_\rho(\hat{\rho}) = \frac{\partial}{\partial \rho} log\{g_\rho(\hat{\rho})\}.$$

In this case the formula of (a) is given by:-

$$a = \frac{1}{6} skew_{\rho=\hat{\rho}}(\dot{\ell}_\rho),$$

Where,$skew_{\rho=\rho_n}(y)$ is the sekwness of a random variable of the form $\frac{\mu_3(y)}{(\mu_2(y))^{\frac{3}{2}}}$ for a verification of the (3.7), see Efron (1987).

In the nonparametric case, an easy formula for estimating the constant (a) uses the Jacknife estimation of sekwness coefficient as follows:-

$$\hat{a} = \frac{\sum_{i=1}^{n}(\hat{\rho}_{(.)}-\hat{\rho}_{(i)})^3}{6(\sum_{i=1}^{n}(\hat{\rho}_{(.)}-\hat{\rho}_{(i)})^2)^{\frac{3}{2}}}$$

(3.8)

Where $\hat{\rho}_{(i)}$ is the value of $(\hat{\rho})$ computed from the original sample with (y_i) deleted I.e. $\hat{\rho}_{(i)}$ is the ith delete Jacknife pseudo estimate, and $\hat{\rho}_{(.)} = \frac{\sum_{i=1}^{n} \hat{\rho}_{(i)}}{n}$,

is thus $\frac{1}{6}$ the Jackknife estimation of the sekwness coefficient. $\quad z[\alpha] = z_0 + \frac{z_0+z_{(\alpha)}}{1-a(z_0+z_{(\alpha)})}$

,$\hat{G}_{(.)}$ is the Cumulative distribution function of$\hat{\rho}^*$.

That can be computed from the bootstrap distribution of $\hat{\rho}$ without any knowledge of the form of $m_{(.)}$.This is the (BCa) interval for (ρ).

Let (a=0) the BCa will be:

$$\{\hat{G}_n^{-1}[\Phi(2z_0 + z_{(\alpha)})], \hat{G}_n^{-1}[\Phi(2z_0 + z_{(1-\alpha)})]\}.$$

And let ($a\ and\ z_0 = 0$), we get:-

$$\Phi(z[\alpha]) = \Phi(z_{(\alpha)}) = \alpha, \quad \Phi(z[1-\alpha]) = \Phi(z_{(1-\alpha)}) = 1 - \alpha,$$

So that the interval reduces the percentile interval.

$(\hat{G}_n^{-1}(\frac{\alpha}{2}), \hat{G}_n^{-1}(1 - \frac{\alpha}{2}))$, as given before.

From another hand, we can discuss this point as:-

$$BCa = (\hat{\rho}_{lo}, \hat{\rho}_{up}) = (\hat{\rho}^*_{(\alpha_1)}, \hat{\rho}^*_{(\alpha_2)}).$$

Where,

$$\alpha_1 = \Phi(z_0 + \frac{z_0 + z_{(\frac{\alpha}{2})}}{1 - a(z_0 + z_{(\frac{\alpha}{2})})}), \quad \alpha_2 = \Phi(z_0 + \frac{z_0 + z_{(1-\frac{\alpha}{2})}}{1 - a(z_0 + z_{(1-\frac{\alpha}{2})})}).$$

Depending on $\hat{G}^{-1}(\alpha) = \hat{\rho}^*_{(\alpha)}$.   Then, $\hat{\rho}^*_{(\alpha_1)} = \hat{G}^{-1}(\alpha_1)$.

And $BCa = [\hat{G}_n^{-1}(\alpha_1), \hat{G}_n^{-1}(\alpha_2)]$,

Finally, $\hat{\rho}_{BCa}[\alpha] = \hat{G}^{-1}[\Phi(.z_0 + \frac{z_0 + z_{(\alpha)}}{1 - a(z_0 + z_{(\alpha)})})]$

The Approximate Bootstrap Confidence Interval (ABC)

Efron (1992) introduced the approximate bootstrap confidence interval (ABC) for the case in which $\hat{\rho}$ can be defined smoothly in the sample observations. Their main idea was the possibility to give an automatic tool to be applied for finding the BCa interval. This can be done by approximating the bootstrap random sampling results using Taylor series expansions. The matter, which reduces the computational burden by an enormous factor.

Having observed $y = (y_1, y_2, \ldots \ldots, y_n)$, Efron (1992) assumed a multinomial distribution with support on the observed data. Formally, if we denote the resampling vector by (p^*), we assume that (np^*) has a multinomial distribution success probabilities $p^0 = (\frac{1}{n}, \ldots \ldots, \frac{1}{n})^T$. Our statistic has the form. $\hat{\rho} = T(p)$.

Delta method approximation for the (ρ) is:-

$$\hat{\sigma} = (\sum_{i=1}^n \frac{T_i^2}{n^2})^{\frac{1}{2}},$$

Where, ($\dot{T}$) is the empirical influence component as:-

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$$\dot{T}_i = \lim_{\epsilon \to 0} \frac{T\left((1-\epsilon)P^0 + \epsilon e_i\right) - T(P^0)}{\epsilon},$$

Where (e_i) is the ith cording vector $(0,0,\ldots\ldots,0,1,0,\ldots\ldots,0)^T$.

Let $\hat{\rho}_{(1-\alpha)}$ indicate the end point of an approximate is (100), or $(1 - 2\alpha)\%$ one-side upper confidence interval for ρ then $(\hat{\rho}_{[\alpha]}, \hat{\rho}_{[1-\alpha]})$ is an approximate (100), or $(1 - 2\alpha)\%$ two – side interval.

A quadratic Taylor series expansion of $(\rho = T(p^0))$ gives approximate bias (b):-

$$b = \sum_{i=1}^{n} \ddot{T}_i \,(2n^2)$$

(3.9)

Where, $(\ddot{T}_i)$ is an element of the second order influence function as:-

$$\ddot{T}_i = \lim_{\epsilon \to 0} \frac{T\left((1-\epsilon)p^0 + \epsilon e_i\right) - 2T(p^0) + T((1-\epsilon)p^0 - \epsilon e_i)}{\epsilon^2}.$$

The second quantity needed for (Z_0) is the quadratic coefficient $\hat{c}_q$ as:-

$$\hat{c}_q = \lim_{\epsilon \to 0} \frac{T\left[(1-\epsilon)p^0 + \frac{\epsilon \dot{T}}{n^2 \hat{\sigma}}\right] - 2T(p^0) + T\left[(1-\epsilon)p^0 - \frac{\epsilon \dot{T}}{n^2 \hat{\sigma}}\right]}{\epsilon^2}. \tag{3.10}$$

This coefficient measures the nonlinearity of the function $\rho = T(p)$ as we move in the least favorable direction. Let, $\rho(\lambda) = T(\hat{p} - \frac{\lambda \hat{\delta}}{\hat{\sigma}})$,A quadratic Taylor series expansion gives:-

$$\rho(\lambda) = \hat{\rho} + \hat{\sigma}(\lambda + \hat{c}_q \lambda^2)$$

$\hat{\delta}$ is called the last favorable direction, $\hat{c}_q$ measures the ratio of the quadratic term in $\{\rho(\lambda) + \hat{\rho}\}/\hat{\sigma}$. The size of $(\hat{c}_q)$ does not effect the standard intervals , which treat every function $T(p)$ as if it were linear , but it has an important effect on more accurate confidence intervals.   The bias correction constant $(\hat{z}_0)$ is a function of $(\hat{a}), (\hat{b}) and (\hat{c}_q)$, these three constants are approximated by using a small value of ϵ in formulas (3.6),(3.8),(3.9) and (3.10) . Then we define:-

$$\hat{\gamma} = \frac{\hat{b}}{\hat{\sigma}} - \hat{c}_q. \text{ and estimate } z_0 \text{ by}$$

$$\hat{z}_0 = \Phi^{-1}\{2 - \Phi(\hat{a}) - \Phi(-\hat{\gamma})\} \quad = \hat{a} - \hat{\gamma}.$$

it can be shown that $\hat{\gamma}$ is the total curvature of the level surface $\{p: T(p) = \hat{\rho}\}$. The greater the curvature, the more biased is $\hat{\rho}$. $\quad \hat{z}_0 = \hat{G}^{-1}(\Phi(\hat{\rho}))$.

Where $(\hat{G})$ is the cumulative distribution function of $(\hat{\rho}^*)$. Either form of $(\hat{z}_0)$ approximates (z_0) sufficiently well to preserve the second order accuracy of the (BCa) formulas. The definition of z_0 is more like a median bias than a mean bias, which is why $\hat{z}_0$ in values quantities other than $(\hat{b})$.

The ABC confidence limit for, denoted $\hat{\rho}_{ABC}(1 - \alpha)$ is constructed as follows:-

$$\hat{\rho}_{ABC}(1 - \alpha) = T(p^0 + \tfrac{\lambda \hat{\delta}}{\hat{\sigma}}).$$

Where,

$$\hat{\delta} \equiv \dot{T}(p^0), \qquad \lambda \equiv \frac{\omega}{(1 + \hat{a}\omega)^2} \quad \text{and} \qquad \omega = z_0 + z_{(1-\alpha)}.$$

The direction $\hat{\delta}$ is called the last favorable direction and is discussed the big advantage of the ABC procedure is that the constant $(\hat{z}_0)$ and $(\hat{a})$ can be computed in terms of numerical second derivatives, and hence no resampling.

NUMERICAL RESULTS OF MONTE CARLO SIMULATION

Boardman (1974) compared some different method to obtain the confidence intervals for σ_a^2 in balanced random models using Monte Carlo simulation techniques.

This chapter aim to compare some exact methods denoted previously by section (2.2)with some approximate methods ,see section(2.3) and some bootstrap methods denoted to section(3.2) obtain confidence intervals for the unbalanced random one- way model. For all comparison we will use the Monte Carlo simulation techniques along the lines of Boardman . The criteria of comparisons are the percentage of coverage, average length and The comparison is conducted using five of the unbalanced random one- way designs given by El-Ganzouri (1986)which are chosen to cover a wide range of unbalancedness and more details presented in the following section.

The Designs of Experiment

The following designs were selected from those of Thomas and Hultiquist (1978) to study confidence interval procedures in case of unbalanced random of one- way model.

Table (1): The unbalanced random one- way designs used in our empirical study

Design(i)	Number of groups (t_i).	Values of n_i.
Thomas and Hultiquist design 1	3	10,50,500
2	6	10,10,50,50,500,500
Burnham design 3	5	2,3,5,7,8
4	10	2,2,3,3,6,6,6,7,7,8
5	15	3,3,3,3,3,3,3,3,3,3,3,3,3,3,33

In order to compute the confidence intervals using the above mentioned procedures,

1– Compute the eigenvalues (λ_i) and their multiplicities (r_i).

2 – Let (t_i) be the number of groups having (n_i) observations each.

In fact, LaMotte (1976) showed that λ_i's are the same as the roots of:-

$$h(\lambda) = \sum_{i=1}^{a} \frac{t_i n_i}{(n_i - \lambda)}.$$

Let (a) be the number of distant (n_i`s), say, $n_1 < n_2 < \cdots \ldots \ldots < n_a$. LaMotte (1976) stated that Newton's method, for solving h (λ) =0, has been found to converge very quickly due to the steepness of [h (λ)]. We used this method to compute the λ_i and r_i for the five designs of Thomas and Hultiquist (1978) and Burnham (1975) see El-Ganzouri (1986) .

Table(2):The Eigen Values and their Multiplicities for Our Designs

Design	Values of λ_i	Values of r_i
1	14.13,94.80	1,1
2	10,14.13,50,94.80,500	1,1,1,1,1,1
3	2.23,3.51,5.70,7.52	1,1,1,1,
4	2.0,2.34,3,3.56,6.0,6.54,7.0,7.74	1,1,1,1,2,1,1,1
5	3.0,19.80	13,1

The Simulation

The confidence intervals of exact methods, other approximate procedures, and the bootstrap methods were simulated for the five designs which were proposed by El- Ganzouri (1986).

The values of ρ were (0.001, 0.01, 0.1,0.5,1,2,4and8) the results for α=0.05 appear in tables of Appendix(i). They were computed using the program and subprograms of Appendix (ii). Since the confidence intervals for ρ are based on location and scale invariant pivotal

quantities we took $\mu = 0$ and $\sigma_e^2 = 1$, so that $\rho = \sigma_a^2$. For each design we generated (k) independent chi-square variables with (r_i) degrees of freedom, say $X_1^*, X_2^*, \ldots\ldots, X_k^*$ and an independent chi-square variable with (n-t) degrees of freedom, say, X_0 .Let

$$\chi_i(\rho) = \left(1 + \lambda_i \rho\right)\chi_i^* \qquad i = 1, \ldots\ldots, K$$

It be noted that X_0 plays the role of SSW, see (2.3), while $\chi_i(\rho)$ plays the role of Q_i , so ,

$$\sum_{i=1}^{k} \chi_i(\rho) = \sum_{i=1}^{k} Q_i = SSB(\text{Sum of square between}) = \sum_{i=1}^{t} n_i(\bar{y}_{i.} - \bar{y}_{..})^2$$

See (2.4) .Further

$$\sum_{i=1}^{k} \left(\frac{1}{\lambda_i}\right) \chi_i(\rho) = \sum_{i=1}^{t} (\bar{y}_i - \bar{y})^2. \quad \text{See El-Shahat (1983)}.$$

Monte Carlo and Bootstrap methods are both computer intensive methods used frequently is applied statistics. The bootstrap is a type of Monte Carlo method applied based on observed data (Efron , the bootstrap was described by Efron (1979) and he has written much about the method and its generalizations since then thousands of papers have been written on the bootstrap . However , in practical application , the bootstrap confidence intervals using some form of resampling with replacement from the actual data ,ρ , to generate (B) bootstrap samples ,(ρ^*) . Often, the data sample, consist of (n) independent units and it then suffices to take a simple random sample of size (n), with replacement.

After, we compute SSB, SSW for every design we will generate SSB*, SSW* the bootstrap sample, and generate (B) bootstrap sample from SSB, SSW in order to sample of SSB*, SSW*.

Then we will calculate the value of ρ^* from SSB*, SSW* as:-

$$\rho^* = \frac{SSB^* - SSW^*}{n_0} \quad , \quad n_0 = \frac{n - \frac{\sum_{i=1}^{t} n_i^2}{n}}{B} \quad . \quad \text{Then we will arrange the values of } \rho^*.$$

We next compute the confidence intervals for every method using the formulas in sections (2-2) and (3-3). we scored one if the confidence interval covered ρ, and score zero otherwise the simulated percentage of coverage is 100 times the total score divided by 1000 , the number of simulations the results appear in tables (1-1),……(1-5). We squared the standard normal variables to get chi-square variables with one degree of freedom and used the additive property of independent chi-square random variables with arbitrary degrees of freedom.

However, when (n-t) was found to be larger than 100 we use the fact $\lim_{k\to\infty} \chi_k^2 / k = 1$ to

avoid having to generate chi-square random variables with large degrees of freedom.

Before we compare the methods of obtaining confidence intervals for ρ for unbalanced one-way model ,we first point out that Wald`s method is an exact one but is difficult to compute

,since it involves the solution of two non-linear equations , and we compare it with other approximate methods which are much easier to compute and with bootstrap methods.

The Coverage

The extent or degree to which something is observed, analyzed, and reported. We now examine tables (1.1)….(1.5)of Appendix (i), to study the simulated coverage for different designs and values of ρ . It should be noted that the nominal coverage is 95.0%.

Let us consider group (1), which consists of designs (1) and (2). Design(1) has three groups with 10,50 and 500 observations , respectively ,design(2) has six groups 10,10,50,50,500,and 500 observations , respectively .It is thus seen that more groups and imbalance are introduced as we go from design (1) to design(2), also both of the two designs are extremely unbalanced.

Consider tables (1.1) and (1.2) which present the simulated coverage values for designs (1) and (2). The conservative upper limit method (CON.UL) and conservative lower limit method (CON.LL) have approximately the same coverage value for ρ. These two limits are fairly closely with the Wald`s coverage values for every value of ρ.

On the other hand, the coverage value of the Arithmetic Adjusted Method (A.ADj) coverage is higher than Wald`s for every value of ρ. It is also higher than that of (CON.UL) and (CON.LL). In fact, (A.ADj) method is more conservative for every ρ. While, the Arithmetic Method (A) gives a close coverage to that of Wald for small values of ρ , ρ≤ 0.01 .However ,for larger values of ρ, ρ>0.1, the coverage is much lower than the nominal. The Harmonic Method (H) coverage almost coincides, with Wald's coverage for large and intermediate values of ρ, ρ>0.1, is much Lower than Wald.

Let us now compare the coverage values as we go from design (1) to design (2). The Wald coverage decreases for small values of ρ, ρ≤1.0, however it increases for larger values of ρ, ρ>0.1.

In addition, Bross method is increasing for the all values of ρ in every design. While, Tukey coverage decreases for every ρ .and in Anderson and Bancroft (ANDBAN) coverage decreases for every ρ . These two limits are fairly closely with William's coverage for every ρ.

In Anderson and Bancroft (ANDBAN) coverage value decreases for small values of ρ, ρ≤0.5, However for large values of ρ, ρ>0.5, the coverage value is a constant value, as same as, in William's coverage values. From this we conclude that the value of Anderson and Bancroft (ANDBAN) coverage is the same values of William's coverage value and it's nearly to Tukey coverage values.

The Arithmetic (A) coverage values are the same values of Anderson and Bancroft (ANDBAN) coverage values, this values decrease for small values of ρ, ρ≤0.5. However, for larger values of ρ, ρ> 0.5, the coverage values are constant value.

While, the William and Tukey (WILLTUK) coverage value increases for the small values of ρ, ρ ≤ 1. However, for larger values of ρ, ρ>1, the coverage value decreases.

In addition, Morigiti coverage values are the same values of Bulmer coverage values, these values of coverage decrease for every values of ρ. The Harmonic method (H) coverage value increases for every ρ.

In addition, The Arithmetic Adjusted method (A.ADJ) coverage is higher than the exact and approximate methods for every ρ. In fact, (A.ADJ) method is more conservative for every ρ.

Let us now compare the coverage values as we go from design(1) and (2) .the Wald, the CON.LL, the CON.UL coverage are constant for every ρ. Bross coverage increases for every ρ. But about Tukey, Anderson and Bancroft, William and Arithmetic coverage values of ρ decreases for small values of ρ, ρ≤ 0.5. However ,for larger values of ρ, ρ> 0.5 ,the coverage is a constant value .William and Tukey coverage increases for the small values of ρ, ρ≤ 1 .However ,larger values of ρ ,ρ >1, the coverage decreases . Morigiti, Bulmer and Arithmetic adjusted coverage decrease for all values of ρ. Finally, Harmonic (H) coverage increases for every ρ in general. The (A.ADJ) Method is more conservative than the conservative method.

According to the Bootstrap Methods ,based on design (1) in the standard approximate method(SAM)coverage increases for the small values of ρ, ρ≤ 0.1 .However ,for the larger values of ρ, ρ>0.1the coverage is a constant. While, The Bootstrap-t Method (BT) coverage value increases for the small values of ρ,ρ≤0.5. However, for the larger values of ρ, ρ> 0.5 the coverage is constant. In addition, The Residual Method (RM) coverage decreases for every ρ. The Basic Bootstrap Method (BB) coverage increases for the small values of ρ, ρ≤0.5. However, for the larger values of ρ, ρ> 0.5 the coverage decreases.

However, based on the design (2) in the coverage values of the standard approximate method (SAM) decreases for the small values of ρ. ρ≤ 0.1 .However, for the larger values of ρ, ρ> 0.1 it is a constant. While, the coverage value of Bootstrap-T Method (BT) increases for the small values of ρ. ρ≤ 0.1. However , larger values of ρ, ρ> 0.1it is a constant .The Residual Methods(RM) coverage value decreases for the small values of ρ, ρ ≤1 .However larger values of ρ ,ρ >1 the coverage value is a constant .The Basic Bootstrap Method(BB) coverage value increases for small values of ρ , ρ ≤1 . However, larger values of ρ, ρ>1the coverage value decreases.

Finally, we compute the percentile bootstrap confidence intervals for computing the other intervals not for comparing coverage values. If the bootstrap distribution of $\hat{\rho}^*$ is roughly normal , then the standard normal and percentile intervals will nearly agree .The central limit theorem tells us that as $n \to \infty$, the bootstrap histogram will become normal shaped, but for small samples it may look very non- normal. Then the standard normal and percentile intervals will differ. The argument in favor of the percentile interval should translate into better coverage value performance.

The results of comparison between the coverage values of designs (1) and (2) show that:

The (BT) coverage values increase for small values of ρ .However larger value is a constant, the (BB) coverage values increase for small values of ρ. However, larger values of ρ, the coverage values decrease. The (RM) coverage values decrease for every ρ, but in design (2) the coverage values decrease for small values of ρ. However, larger values of ρ, the coverage values are constant. Finally, (SAM) coverage values and the best method increase in design

(1) but in design (2) coverage values decrease for small values of ρ but in twice design coverage values are constant.

Let us consider group (2), which consists of designs (3) and (4). Design (3) has five groups with 2,3,5,7 and 8 observations ,respectively, while design(4) has 10 groups with 2,2,3,3,6,6,6,7,7 and 10 observations, respectively. It is thus seen that more groups and imbalance are introduced as we go from design (3) to design (4) , also design(4) is a duplicate of design(3). We can consider this group as a typical unbalanced design.

Consider Tables (1-3) and (1-4) which present the simulated coverage values for design (3) and (4).The (CON.LL) and (CON.UL) methods have approximately the same coverage values for ρ, which agrees fairly closely with Wald's coverage values. The Bross method coverage values increase for every ρ. The Tukey method coverage values decrease for every ρ. William method coverage values increase for small values of ρ, ρ≤ 0.1, however, they decrease for larger values of ρ>0.1.

William method coverage values increase for small values of ρ, ρ≤ 0.5 however, they decreases for larger values of ρ, ρ> 0.5. William and Tukey method (WILLTUK) coverage values increase for small values of ρ, ρ≤ 1, however, they decrease for larger values of ρ, ρ> 1.

Anderson and Bancroft (ANDBAN) coverage values decrease then increase for small values of ρ, the for larger values of ρ. As same as ,n (A) method coverage values and the same values of intervals in (ANDBAN) method for the different values of ρ. Morigiti and Bulmer method coverage values decrease of every value of ρ, and they have the same values of intervals for the different values of ρ(for every value of ρ).

The (H) method coverage values increase for every ρ. The (A.ADJ) method coverage values decrease for small values of ρ, ρ≤ 0.5, however, they increase for larger values of ρ, ρ>0.5.

The results of comparison between the cover values of designs (3) and (4) show that: -

The Wald, the CON.LL, the CON.UL coverage values are constant for every value of ρ .Bross and (H) coverage values increase for every values of ρ. But about (ANDBAN),William,(A) and (A.ADJ) coverage values decrease for small values of ρ, ρ≤ 0.5 .However ,for larger values of ρ, ρ> 0.5 ,the coverage values are constant value . William and Tukey coverage values increase for the small values of ρ, ρ≤ 1. However, larger values of ρ, ρ >1, the coverage values decrease .Morigiti and Bulmer coverage values decrease for all values of ρ. Finally, Tukey coverage values decrease for every value of ρ in general. The (A.ADJ) Method is more conservative than the conservative method.

According to the Bootstrap Methods, for designs (3) and (4) the standard approximate method (SAM) increases for every ρ. The Bootstrap –T(BT) coverage values increase for small values of ρ , ρ≤ 0.5 ,however , they decrease for larger values of ρ, ρ> 0.5 .The Residual Method (RM)coverage values increase for small values of ρ, ρ≤ 0.1 , however , they decrease for larger values of ρ ,ρ > 0.1 . The Basic Bootstrap Method (BB) coverage values increase for every ρ.

The results of comparison between the coverage values of designs (3) and (4) show that:-

The (SAM) and (BB) methods coverage values increase for every value of ρ. The (BT) and (RM) methods coverage values increase for small values of ρ, however, this coverage values decrease for larger values of ρ. Finally, (SAM) coverage values and the best method in bootstrap methods.

Let us consider group (3), which consists of design (5). Design (5)has 15 groups with 3,3,3,3,3,3,3,3,3,3,3,3,3and 33observations, respectively, this design present a nearly balanced design with an odd group containing 33 observations . It thus seems that more groups are introduced as we go from groups (1) to group (3).

Consider Tables (1-5) which present the simulated coverage for design (5).

The (CON.LL) and (CON.UL) methods have approximately the same coverage values for all values for ρ, which are too closely with Wald's coverage values. The Bross method coverage values increase for every value of ρ. The Tukey method coverage values decrease for every vale of ρ. William method coverage values increase for small values of ρ, ρ≤ 0.1, however, they decrease for larger values of ρ>0.1.

William and Tukey method (WILLTUK) coverage values increase for small values of ρ, ρ≤ 1, however, it decreases for larger values of ρ, ρ> 1.

Anderson and Bancroft (ANDBAN) coverage values increase for small values of ρ, ρ≤0.1, however, they decreases for larger values of ρ, ρ>0.1. As same as in (A) method coverage values and the same values of intervals in (ANDBAN) method for the different values of ρ.

The coverage values of Morigiti and Bulmer method are constant for small values of ρ, ρ≤ 0.01, however, they decrease for larger values of ρ ,ρ> 0.01, and they have the same values of intervals for the different values of ρ(for every ρ).

While, the (A.ADJ) method coverage values decrease for every ρ. The (H) method coverage values increase for small values of ρ, ρ≤ 0.5, however, they are constant for larger values of ρ, ρ>0.5.

The results of comparison between the coverage values of designs (5) show that:-

the Wald, the CON.LL, the CON.UL coverage values are constant for every value of ρ .Bross coverage values increase for every ρ, but about (ANDBAN) ,William,(A) and (WILLTUK) coverage values increase for small values of ρ, however , for larger values of ρ . Morigiti and Bulmer coverage values decrease for all values of ρ. (H) coverage values increase for small values of ρ .However, they are constant for larger values of ρ. Finally, Tukey and (A.ADJ) coverage values decrease for every value of ρ in general.

The (A.ADJ) Method is more conservative than the conservative method.

On the other hand , the coverage values of Bootstrap Methods, for designs (5) the standard approximate method(SAM)coverage values increase for small values of ρ , ρ≤ 0.5 ,however, they decrease for larger values of ρ, ρ> 0.5. The Bootstrap –T (BT) coverage values increase for small values of ρ, ρ≤ 0.5, however, they decrease for larger values of ρ, ρ> 0.5 .The Residual Method (RM) coverage values increase for small values of ρ, ρ≤ 0.1, however, they

decrease for larger values of ρ, ρ > 0.1. The Basic Bootstrap Method (BB) coverage values increase for small values of ρ, ρ≤ 1, however, they decrease for larger values of ρ, ρ> 1

Let us now compare the coverage values as we go from design (5). The (SAM), (BT), (RM) and (BB) methods coverage values increase for small values of ρ, however, they decrease for larger values of ρ. Finally, (SAM) coverage is the best method in bootstrap methods.

The Average Length

The difference between the upper and lower limits.Let us examine Tables (2.1),…,(2.5) of appendix (i) , to study the simulated average length for different designs and values of ρ.

First, we notice that the average length decreases as we go from group (1) to group (3) also that average length increases as the ρ values increases for each design.

Based on the average length, the results of tables (2.1) and (2.2) show that the results for design (1) and (2) for exact and approximate methods. William and Tukey (WILLTUK) Method has minimum average length for small values of ρ, ρ ≤0.5, however, for large values of ρ,0.5<ρ ≤4 , the average length continues in increasing , but in larger values of ρ , ρ>4 ,the average length is the maximum value. The (A.ADJ) Method maximum average length for every ρ except ρ=8. The (WILLTUK) Method average length is the maximum, but in design (2) .The (A.ADJ) method, average length is the maximum values for every ρ. Wald method average length is larger than (WILLTUK) average length. But it is lower than the other methods for small values of ρ ,ρ ≤1 ,however ,for large values of ρ ,ρ>1 Wald's average length is minimum value.

While ,the (ANDBAN) Method average length equal the (A) Method average length for every ρ and the (Tukey) Method average length equal the (Morigiti) Method average length for large values of ρ, ρ> 0.01. On the other hand, the results of other methods show that:-

Increasing for every ρ, we can summarize the previous results of exact and approximate methods in design (1) as follows:-

Table (3) :-Results of Table (2.1)for exact and approximate methods

P	Comparing between methods
0.001	WILLTUK<BROSS<WALD<TUKEY<MORGITI<BULMER<WIL<ANDBAN=A<H<A.ADJ
0.01	WILLTUK<WALD<BROSS<TUKEY<MORIGITI<BULMER<WIL<ANDBAN=A<H<A.ADJ
0.1	WILLTUK<WALD<TUKEY=MORIGITI<BULMER<WIL<ANDBAN=A<H<BROSS<A.ADJ
0.5	WILLTUK<WALD<TUKEY=MORIGITI<BULMER<WIL<ANDBAN=A<H<BROSS<A.ADJ
1	BROSS=TUKEY=MORIGITI<WILLTUK=BULMER<WIL<ANDBAN=A<H<WALD<A.ADJ
2	WALD<TUKEY=MORIGITI<BULER<WIL<ANDBAN=A<H<BROSS<WILLTUK<A.ADJ
4	WALD<TUKEY=MORIGITI<BULER<WIL<ANDBAN=A<H<BROSS<WILLTUK<A.ADJ

8	WALD<TUKEY=MORIGITI<BULER<WIL<ANDBAN=A<H<BROSS<A.ADJ<WILLTUK

Table (4) :-Results of Table (2.2)for exact and approximate methods

P	Comparing between methods
0.001	WILLTUK<WALD<BROSS<TUKEY<ANDBAN=A=WIL=MORIGITI=BULMER<H<A.ADJ
0.01	WILLTUK<WALD <TUKEY=ANDBAN=A=WIL=MORIGITI=BULMER<BROSS<H<A.ADJ
0.1	WILLTUK<WALD <TUKEY=ANDBAN=A=WIL=MORIGITI=BULMER<BROSS<H<A.ADJ
0.5	WILLTUK<WALD<BROSS<WIL <ANDBAN=A<BULMER<TUKEY=MORIGITI <H<A.ADJ
1	WALD<BROSS<H<WIL<ANDBAN=A<WILTUK=BULMER<TUKEY=MORIGITI<A.ADJ
2	WALD<BROSS<H<WIL<ANDBAN=A< BULMER<TUKEY=MORIGITI< WILTUK<A.ADJ
4	WALD<BROSS<H<WIL<ANDBAN=A< BULMER<TUKEY=MORIGITI< WILTUK<A.ADJ
8	WALD<BROSS<H<WIL<ANDBAN=A< BULMER<TUKEY=MORIGITI< WILTUK<A.ADJ

In addition to Bootstrap Methods comparing ,in group (1) design (1) in small values of ρ, ρ ≤ 0.5 there are very small different between methods , but for large values of ρ, ρ > 0.5 the (BB) method average length has minimum value and the (BT)method average length has maximum value. In design (2), the (RM) method average length has a maximum value for every ρ, fairly, the (SAM) method average length in middle of the (BT) method average length and (BB) method average length. The (BB) method average length has minimum value for small values of ρ, ρ ≤ 0.5. However, in large values of ρ, ρ > 0.5, the (BT) method average length has minimum value.

Table (5) :-Results of Table (2.1)for bootstrap methods

P	Comparing between methods
0.001	BT<RM<BB<SAM
0.01	RM<BT<BT<SAM
0.1	RM<BB<BT<SAM
0.5	SAM<BT<BB<RM
1	BB<RM<SAM<BT
2	RM<BB<SAM<BT
4	BB<RM<SAM<BT
8	BB<RM<SAM<BT

Table (6) :-Results of Table (2.2)for bootstrap methods

P	Comparing between methods
0.001	BB<BT<SAM<RM
0.01	BB=BT<SAM<RM
0.1	BB<BT<SAM<RM
0.5	BB<BT=SAM<RM
ρ≥1	BT<SAM<BB<RM

Let us compare the average length values as we go from group (1) design (1) to design (2). All methods average length decreases for every ρ. However, it decreases by different rates.

Let us consider group (2), Tables (2-3) and (2-4) shows the exact and approximate method results. The William and Tukey (WILLTUK) Method has minimum average length for small values of ρ, ρ ≤0.5, however, for large values of ρ, ρ>0.5, the average length continues in increasing to be the maximum value. The (A.ADJ) Method maximum average length for small values of ρ, ρ ≤ 0.01, and for large value of ρ , 0.01 < ρ ≤ 0.5 , the BROSS method average length has the maximum values , but for larger values of ρ, ρ > 0.5 ,(A.ADJ) average length is lower than the (WILLTUK) method average length , the (WILLTUK) Method average length is the maximum. Wald method average length is larger than (WILLTUK) average length, but it is lower than the other methods for small values of ρ, ρ ≤1, however, for large values of ρ, ρ >1 Wald's average length is minimum value.

The (ANDBAN) Method average length equal the (A) Method average length for every ρ and the (Tukey) Method average length equal the (Morigiti) Method average length for large values of ρ, ρ > 0.01.

The other methods are increasing for every ρ,

Table (7) :-Results of Table (2.3)for exact and approximate methods

P	Comparing between methods
0.001	WILLTUK <WALD<BROSS <WIL <TUKEY<ANDBAN=A<BULMER=MORGITI <H<A.ADJ
0.01	WILLTUK <WALD=BROSS <WIL <TUKEY<ANDBAN=A<BULMER=MORGITI <H<A.ADJ
0.1	WILLTUK<WALD<WIL <TUKEY<ANDBAN=A <BULMER=MORIGITI <H <A.ADJ< BROSS
0.5	WILLTUK<WALD<WIL <TUKEY<BULMER=MORIGITI <ANDBAN=A <H<A.ADJ <BROSS
1	BROSS<WALD <WIL <TUKEY<WILLTUK <MORIGITI =BULMER <ANDBAN=A<H <A.ADJ
2	WALD<WIL <BULMER=MORIGITI <TUKEY<BROSS <ANDBAN=A<H <A.ADJ<WILLTUK
4	WALD<WIL<BROSS <MORIGITI<BULMER <TUKEY <ANDBAN=A<H <A.ADJ<WILLTUK
8	WALD<WIL<BROSS <MORIGITI<BULMER <TUKEY <ANDBAN=A<H <A.ADJ<WILLTUK

Table (8) :-Results of Table (2.4)for exact and approximate methods

P	Comparing between methods
0.001	WILLTUK<BROSS<WALD< TUKEY<ANDBAN=A<BULMER=MORGITI<A.ADJ<H
0.01	WILLTUK<WALD<BROSS< WIL<TUKEY<BULMER=MORGITI<ANDBAN=A <A.ADJ<H
0.1	WILLTUK<WALD<WIL<TUKEY<ANDBAN=A<BULMER=MORIGIT<A.AD<H <BROSS
0.5	WILLTUK <WALD<WIL<BROSS<BULMER=MORIGITI<TUKEY<ANDBAN=A<H<A.ADJ
1	WALD<BROSS<WIL<MORIGITI=BULMER<TUKEY=WILLTUK<ANDBAN=A <H <A.ADJ
2	WALD<WIL<BULMER=MORIGITI<TUKEY<BROSS<ANDBAN=A<H<A.ADJ< WILLTUK
4	WALD<WIL<BROSS<MORIGITI=BULMER<TUKEY<ANDBAN=A<H<A.ADJ< WILLTUK
8	WALD<WIL<BROSS<MORIGITI=BULMER<TUKEY<ANDBAN=A<H<A.ADJ< WILLTUK

About BOOTSTRAP METHODS comparing ,in group(2) design (3) and (4) the (BB) method average length has minimum value for every ρ, and the (BT) method average length has maximum value for large values of ρ, $\rho > 0.1$, and the (SAM) method average length has maximum value for small values of ρ ,$\rho \leq 0.1$(design3) . But in design (4) the (RM) method average length has maximum for $\rho \leq 0.01$, and the (SAM) method average length has maximum value for $\rho = 0.1$, fairly, the(SAM)method average length in middle of the (BT)method average length and (BB) method average length .

Table (9) :-Results of Table (2.3)for bootstrap methods

P	Comparing between methods
0.001	BB<BT<RM=SAM
0.01	BB<BT<RM<SAM
0.1	BB<RM<BT<SAM
$\rho{\geq}0.5$	BB<RM<SAM<BT

Table (10) :-Results of Table (2.4)for bootstrap methods

P	Comparing between methods
0.001	BB<BT<SAM<RM
0.01	BB<BT<SAM<RM
0.1	BB<BT<RM<SAM
0.5	BB<RM<SAM<BT
$P \geq 1$	BB<SAM<RM<BT

Let us compare the average length values as we go from group (2) design (3) to design (4). All methods average length decreases for every ρ. However, it decreases by different rates.

Let us consider group (3), Tables (2-5) shows the exact and approximate method results. The William and Tukey (WILLTUK) Method has minimum average length for small values of ρ, ρ ≤ 0.5, however, for large values of ρ, ρ > 0.5, the average length continues in increasing to be the maximum value for larger values of ρ, ρ ≥ 4. The (A.ADJ) Method maximum average length for small values of ρ, ρ ≤ 0.01, and in 0.1< ρ ≤ 2, the (WILLTUK) Method average length is the maximum for ρ > 2. Wald method average length is larger than (WILLTUK) average length, but it is lower than the other methods for small values of ρ, ρ ≤ 0.5, however, for large values of ρ, ρ >1 Wald's average length is minimum value.

The (ANDBAN) Method average length equal the (A) Method average length for every ρ and the (Tukey) Method average length equal the (Morigiti) Method average length for large values of ρ, ρ > 0.01. The other methods are increasing for every ρ.

Table (11) :-Results of Table (2.5)for exact and approximate methods

P	Comparing between methods
0.001	WILLTUK <WALD<BROSS <WIL <TUKEY<ANDBAN=A<BULMER=MORGITI <H<A.ADJ
0.01	WILLTUK <WALD=BROSS <WIL <TUKEY<ANDBAN=A<BULMER=MORGITI <H<A.ADJ
0.1	WILLTUK<WALD<WIL <TUKEY<BULMER=MORIGITI <ANDBAN=A <H <A.ADJ< BROSS
0.5	WILLTUK<WALD<WIL <TUKEY<BULMER=MORIGITI<BROSS <ANDBAN=A <H<A.ADJ
1	WALD <WIL < BROSS<MORIGITI =BULMER <TUKEY=WILLTUK <ANDBAN=A<H <A.ADJ
2	WALD<WIL<BROSS <BULMER=MORIGITI <TUKEY <ANDBAN=A<H<WILLTUK <A.ADJ
4	WALD<WIL<BROSS <MORIGITI=BULMER <TUKEY <ANDBAN=A<H <A.ADJ<WILLTUK
8	WALD<WIL<BROSS <MORIGITI=BULMER <TUKEY <ANDBAN=A<H <A.ADJ<WILLTUK

About Bootstrap Methods comparing ,in group (3) design(5) the (BB) method average length has minimum value for every ρ, and the (BT) method average length has maximum value for large values of ρ, ρ > 0.1 , and the (RM) method average length has maximum value for small values of ρ ,ρ ≤ 0.1. Fairly, the (SAM) method average length in middle of the (BT) method average length and (BB) method average length.

Table (12) :-Results of Table (2.5)for bootstrap methods

ρ	Comparing between methods
ρ≤0.1	BB<BT<SAM<RM
ρ>0.1	BB<RM<SAM<BT

Let us compare the average length values as we go from group (3) design (5). All methods average length decreases for every ρ. However, it decreases by different rates

Another type for comparing every methods damage all methods in one compensation for each design (exact, approximate and bootstrap)   from this tables

Table (13) :-Results of Table (2.1)for all methods

ρ	Comparing between methods
0.001	WILLTUK<BROSS<WALD<BT<RM<BB<SAM<TUKEY<MORGITI<BULMER<WIL<ANDBAN=A<H<A.ADJ
0.01	WILLTUK<RM<BT<BT<SAM<WALD<BROSS<TUKEY<MORIGITI<BULMER<WIL<ANDBAN=A<H<A.ADJ
0.1	WILLTUK<RM<BB<BT<SAM<WALD<TUKEY=MORIGITI<BULMER<WIL<ANDBAN=A<H<BROSS<A.ADJ
0.5	SAM<BT<BB<RM<WILLTUK<WALD<TUKEY=MORIGITI<BULMER<WIL<ANDBAN=A<H<BROSS<A.ADJ
1	BB<RM<SAM<BT<BROSS=TUKEY=MORIGITI<WILLTUK=BULMER<WIL<ANDBAN=A<H<WALD<A.ADJ
2	RM<BB<SAM<BT<WALD<TUKEY=MORIGITI<BULER<WIL<ANDBAN=A<H<BROSS<WILLTUK<A.ADJ
4	BB<RM<SAM<BT<WALD<TUKEY=MORIGITI<BULER<WIL<ANDBAN=A<H<BROSS<WILLTUK<A.ADJ
8	BB<RM<SAM<BT<WALD<TUKEY=MORIGITI<BULER<WIL<ANDBAN=A<H<BROSS<A.ADJ<WILLTUK

Table (14) :-Results of Table (2.2)for all methods

ρ	Comparing between methods
0.001	WILLTUK<WALD<BROSS<BB<BT<SAM<RM<TUKEY<ANDBAN=A=WIL=MORIGITI=BULMER<H<A.ADJ
0.01	WILLTUK<WALD<BB=BT<SAM<RM<TUKEY=ANDBAN=A=WIL=MORIGITI=BULMER<BROSS<H<A.ADJ
0.1	WILLTUK<WALD<BB<BT<SAM<RM<TUKEY=ANDBAN=A=WIL=MORIGITI=BULMER<BROSS<H<A.ADJ
0.5	WILLTUK<WALD<BB<BT=SAM<RM<BROSS<WIL<ANDBAN=A<BULMER<TUKEY=MORIGITI<H<A.ADJ
1	WALD<BT<SAM<BB<RM<BROSS<H<WIL<ANDBAN=A<WILTUK=BULMER<TUKEY=MORIGITI<A.ADJ
2	WALD<BT<SAM<BB<RM<BROSS<H<WIL<ANDBAN=A<BULMER<TUKEY=MORIGITI<WILTUK<A.ADJ
4	WALD<BT<SAM<BB<RM<BROSS<H<WIL<ANDBAN=A<BULMER<TUKEY=MORIGITI<WILTUK<A.ADJ
8	WALD<BT<SAM<BB<RM<BROSS<H<WIL<ANDBAN=A<BULMER<TUKEY=MORIGITI<WILTUK<A.ADJ

Table (15) :-Results of Table (2.3)for all methods

ρ	Comparing between methods
0.001	WILLTUK<WALD<BROSS<BB<BT<RM=SAM<WIL<TUKEY<ANDBAN=A< BULMER=MORGITI<H<A.ADJ
0.01	WILLTUK<WALD=BROSS<BB<BT<RM<SAM<WIL<TUKEY<ANDBAN=A< BULMER=MORGITI<H<A.ADJ
0.1	WILLTUK<WALD<BB<RM<BT<SAM<WIL<TUKEY<ANDBAN=A<BULME R=MORIGITI<H<A.ADJ< BROSS
0.5	BB<RM<SAM<BT<WILLTUK<WALD<WIL<TUKEY<BULMER=MORIGITI <ANDBAN=A<H<A.ADJ <BROSS
1	BB<RM<SAM<BT<BROSS<WALD<WIL<TUKEY<WILLTUK<MORIGITI=B ULMER<ANDBAN=A<H <A.ADJ
2	BB<RM<SAM<BT<WALD<WIL<BULMER=MORIGITI<TUKEY<BROSS<A NDBAN=A<H<A.ADJ<WILLTUK
4	BB<RM<SAM<BT<WALD<WIL<BROSS<MORIGITI<BULMER<TUKEY<A NDBAN=A<H<A.ADJ<WILLTUK
8	BB<RM<SAM<BT<WALD<WIL<BROSS<MORIGITI<BULMER<TUKEY<A NDBAN=A<H<A.ADJ< WILLTUK

Table (16) :-Results of Table (2.4)for all methods

P	Comparing between methods
0.001	WILLTUK<BROSS<WALD<BB<BT<SAM<RM<WIL<TUKEY<ANDBAN=A< BULMER=MORGITI<A.ADJ<H
0.01	WILLTUK<WALD<BROSS<BB<BT<SAM<RM<WIL<TUKEY<BULMER=M ORGITI<ANDBAN=A <A.ADJ<H
0.1	WILLTUK<WALD<BB<BT<RM<SAM<WIL<TUKEY<ANDBAN=A<BULME R=MORIGIT<A.AD<H<BROSS
0.5	WILLTUK<BB<RM<SAM<BT<WALD<WIL<BROSS<BULMER=MORIGITI< TUKEY<ANDBAN=A<H<A.ADJ
1	BB<RM<SAM<BT<WALD<BROSS<WIL<MORIGITI=BULMER<TUKEY=WI LLTUK<ANDBAN=A<H <A.ADJ
2	BB<RM<SAM<BT<WALD<WIL<BULMER=MORIGITI<TUKEY<BROSS<A NDBAN=A<H<A.ADJ<WILLTUK
4	BB<RM<SAM<BT<WALD<WIL<BROSS<MORIGITI=BULMER<TUKEY<A NDBAN=A<H<A.ADJ< WILLTUK
8	BB<RM<SAM<BT<WALD<WIL<BROSS<MORIGITI=BULMER<TUKEY<A NDBAN=A<H<A.ADJ< WILLTUK

Table (17) :-Results of Table (2.5)for all methods

P	Comparing between methods
0.001	WILLTUK <WALD<BROSS < BB<BT<SAM<RM<WIL <TUKEY<ANDBAN=A<BULMER=MORGITI <H<A.ADJ
0.01	WILLTUK<WALD=BROSS<BB<BT<SAM<RM<WIL<TUKEY<ANDBAN=A<B ULMER=MORGITI<H<A.ADJ
0.1	WILLTUK<WALD< BB<BT<SAM<RM<WIL <TUKEY<BULMER=MORIGITI <ANDBAN=A <H <A.ADJ< BROSS

0.5	WILLTUK<WALD<BB<RM<SAM<BT<WIL<TUKEY<BULMER=MORIGITI<BROSS<ANDBAN=A <H<A.ADJ
1	BB<RM<SAM<BT<WALD<WIL<BROSS<MORIGITI=BULMER<TUKEY=WILLTUK<ANDBAN=A<H <A.ADJ
2	BB<RM<SAM<BT<WALD<WIL<BROSS<BULMER=MORIGITI<TUKEY<ANDBAN=A<H<WILLTUK <A.ADJ
4	BB<RM<SAM<BT<WALD<WIL<BROSS<MORIGITI=BULMER<TUKEY<ANDBAN=A<H<A.ADJ< WILLTUK
8	BB<RM<SAM<BT<WALD<WIL<BROSS<MORIGITI=BULMER<TUKEY<ANDBAN=A<H<A.ADJ<WILLTUK

Looking at different tables, we can state the following remarks:-

1- In coverage of large group (design 1 and 2) the Wald, the CON.LL, the CON. UL coverage are constant for every ρ .Bross coverage increases for every ρ. But about Tukey, Anderson and Bancroft, William and Arithmetic coverage values of ρ decreases for small values of ρ, ρ≤ 0.5 .However, for larger values of ρ, ρ>0.5, the coverage is a constant value. William and Tukey coverage increases for the small values of ρ, ρ≤1. Finally, Harmonic (H) coverage increases for every ρ in general. The (A.ADJ) Method is more conservative than the conservative method. (BT) coverage increases for small values of ρ .However larger value is a constant, (BB) coverage increase for small values of ρ. However, larger values of ρ, the coverage decreases. (RM) coverage decreases for every ρ, but in design (2), the coverage decreases for small values of ρ. However, larger values of ρ, the coverage is constant. Finally, (SAM) coverage is the best method increases in design (1) but in design (2) coverage decreases for small values of ρ but in twice design coverage is a constant.

2- In medium groups (design 3and 4) the Wald, the CON.LL, the CON.UL coverage are constant for every ρ .Bross and (H) coverage increases for every ρ. But about (ANDBAN) ,William,(A) and (A.ADJ) coverage values of ρ decreases for small values of ρ, ρ≤ 0.5. However, for larger values of ρ, ρ> 0.5 ,the coverage is a constant value .William and Tukey coverage increases for the small values of ρ, ρ≤ 1 .However ,larger values of ρ ,   ρ >1, the coverage decreases . Morigiti and Bulmer coverage decrease for all values of ρ. Finally, Tukey coverage decreases for every ρ in general. The (A.ADJ) Method is more conservative than the conservative method. The (SAM) and (BB) methods coverage increase for every ρ. The (BT) and (RM) methods coverage increase for small values of ρ, however, this coverage decrease for larger values of ρ. Finally, (SAM) coverage and the best method in bootstrap methods.

3- In small groups (design 5) the Wald, the CON.LL, the CON.UL coverage are constant for every ρ .Bross coverage increases for every ρ. However, about (ANDBAN), William, (A) and (WILLTUK) coverage values of ρ increases for small values of ρ, however, for larger values of ρ. Morigiti and Bulmer coverage decrease for all values of ρ. (H) Coverage increases for small values of ρ, however, it is constant for larger values of ρ. Finally, Tukey and (A.ADJ) coverage decreases for every ρ in general. The (A.ADJ) Method is more conservative than the conservative method. The (SAM), (BT), (RM) and

(BB) methods coverage increases for small values of ρ, however, it decreases for larger values of ρ. Finally, (SAM) coverage and the best method in bootstrap methods.

4- In the average length for large groups (designs1) WillTuk average length has the smaller values for ρ≤ 0.1, then the Bootstrap methods also has small values ,but for ρ> 0.1 the smaller values of average length for the Bootstrap methods and the A.ADJ method is larger value for all values of ρ except ρ=8 WillTuk has the larger value. But in (design 2) A.ADJ has a larger value for every ρ, however , for ρ≤ 0.5 WillTuk method has the smallest value , and for ρ> 0.5 Wald has the smallest value then for every ρ the Bootstrap methods has small values also but this values are larger than WillTuk and Wald.

5- For medium and small samples WillTuk method average length has smallest values for ρ≤ 0.5 then Bross and Wald then the bootstrap methods but the large value for A.ADJ method , however, for ρ> 0.5 the bootstrap methods is the smaller values but this values are larger than WillTuk method.

Finally, after damaged the results of the coverage and average length the best methods have highest coverage with smallest average length. In design (1) the best method is (SAM) method, because it has appropriate coverage with a small value of average length for all values of ρ this method from bootstrap methods. In design (2) and design (5) the best method is (Wald) method, because it has high coverage with a small value average length with all values of ρ this method from exact methods. In design (3) and design (4) the best method is (Wald) for small values of ρ, ρ≤0.1and it is an exact method. However, for large values of ρ, ρ>0.1 the best method is (SAM) method and it is a bootstrap method.

AN AGRICULTURAL APPLICATION

We apply all methods of constructing confidence intervals for ρ , to two data sets obtained from the central laboratory for design and statistical analysis research, agricultural research center, ministry of agriculture see El-Shahat (1983) and El-Ganzouri (1986). The data set resulted from an experiment in which a sample of 6 varieties of sugar- cane was compared using 28plots (area is 42 square meters). While the second set resulted from another experiment in which a sample of 64 varieties of wheat were compared using 247 plots (area of plot is 2 square meters).

The Sugar-Cane Experiment:-

The data for the sugar-cane experiment is given in table (18). For this data.

Table (18): Data for The Sugar-Cane Experiment.

Varieties(i)	n_i	Yield of plot in kilogram(y_{ij})	Mean yield
1	3	710,665,791	722
2	4	660,626,679,525	622.5
3	5	773,721,561,592,826	694.6
4	5	620,609,609,650,553	608.2

| 5 | 5 | 633,696,626,613,721 | 657.8 |
| 6 | 6 | 574,646,480,514,427,403 | 507.33 |

The group's sizes for the size varieties are as follows:

$n_1 = 3, n_2 = 4, n_3 = n_4 = n_5 = 5, n_6 = 6$. The distinct eigen values were calculated and the results were given in El-Ganzouri (1986): $\lambda_1 = 3.1946, \lambda_2 = 4.2033$, $\lambda_3 = 5.0$, and $\lambda_4 = 5.7449$.

The Eigen values multiplicities are as follows: $r_1 = 1, r_2 = 1, r_3 = 2$ and $r_4 = 1$

The arithmetic mean of $\lambda_i \lambda_A = 4.6286$ and the harmonic mean of $n_i (\tilde{n}) = 4.4444$.

Using equation (2.8), (2.9) and (2.7), respectively, the sum of squares within groups (SSW) = 113013.13,

The sum of squares between groups (SSB) = 1561174.95 and the sample variance of treatment means $(S_{\bar{y}}^2)$ = 6242.856. the calculated $\hat{F}$=MSB/MSW=6.0894, so the null hypothesis that there are no differences among the varieties of sugar –cane, can thus be rejected at the 0.05 level of significance.

The calculated value of the ratio of the variance components

$$\hat{\rho} = \hat{\sigma}_a / \hat{\sigma}_e = \frac{(MSB - MSW)/\lambda_A}{MSW} = 1.0976.$$

The results of comparison between the methods of constructing confidence intervals appear in TABLE (19).

Table (19): Lower and Upper Limits for ρ Sugar cane Design

Methods	Lower limit	Upper limit	Length
Wald	0.2552	9.0033	8.748
Bross	0.531	7.873	7.343
Tukey	0.24	7.724	7.483
And Ban	0.193	8.062	7.869
A	0.2590	8.9917	8.492
Moriguti	0.266	7.715	7.449
Bulmer	0.266	7.715	7.442
H	0.2527	9.0040	8.751
A.ADJ	0.2548	9.3931	9.138
CON.LL	0.3166	-----	
CON.UL	----	28.3419	
SAM	0.00	3.404	3.404
Percentile	0.313	4.089	3.777
Bt	0.00	3.496	3.496
BB	0.00	1.883	1.883

The results of this table shows that the results of limits, the A.ADJ has the biggest upper limit and, (BB) method has the smallest upper limit. While the (A) method has the highest lower limit, where the bootstrap methods (BB), (BT), and (SAM) have the smallest lower limits. But, for average length, the best method that has the smallest average length, and the best method is (BB) .

While the bootstrap methods where appear minus lower limits so we exchange this values with zero this methods have very closely limits the bigger upper limits in it the percentile methods it has also the bigger lower limit and it is only positive limit .

The Wheat Experiment

The data for the wheat experiment was given in table (20) the group sizes for the sixty-four varieties are:

 Values are: $\lambda_1=2$, $\lambda_2=2.0305$, $\lambda_3=3$, $\lambda_4=1$ and $\lambda_5 =56$.

The arithmetic mean of λ_i (λ_A)=3.8586 and the harmonic mean of $n_i(\tilde{n}) =3.7833$.

Using equation (2.3),(2.4)and(2.7) ,respectively, the sum of squares within groups(SSW)= 4.27,the sum of squares between groups (SSB)= 6.94 and the sample variance of treatment means ($S_{\bar{y}}^2$)=0.029.

The calculated $\hat{F}$=MSB/MSW=4.7253, so the null hypothesis that there are no differences among the varieties of sugar –cane ,can thus be rejected at the 0.05 level of significance .The calculated value of the ratio of the variance components

$$\hat{\rho} = \hat{\sigma}_a/\hat{\sigma}_e = \frac{(MSB-MSW)/\lambda_A}{MSW} = 0.9803.$$

The results of comparison between the methods of constructing confidence intervals appear in TABLE (21). It shows that bootstrap methods is larger than the exact and approximate methods, it indicates that Bross method is the shortest then Morigiti=Bulmer then Tukey method but the largest method is (SAM) then Bootstrap-t then Wald then A.ADJ. While, in case of coverage the largest upper limit for Bootstrap-t method and the smallest upper limit for Morigiti and Bulmer. But the largest lower limit for Bross method and smallest lower limit for And Ban .The results of limits, the (Bt) has the biggest upper limit and Morigiti and Bulmer have the smallest upper limit .While ,Bross has the highest lower limit ,Where the And Ban has the smallest lower limit. But, for average length, the best method that has the smallest average length ,and the best method is Bross method.

TABLE (20):Data for The Wheat Experiment.

Varieties	n_i	Yield of plot in kilogram	Mean yield
1	2	1.28, 1.574	1.43
2	2	1.44,1.15	1.295
3	3	1.37,1.45,123	1.35
4	3	1.35,1.16,1.1	1.203

Varieties	n_i	Yield of plot in kilogram	Mean yield
5	3	1.17,1.35,1.2	1.24
6	3	2.1,1.25,1.44,	1.597
7	3	0.91,1.1,0.92	0.977
8	4	1.27,1.2,1.5,1.08	1.255
6	4	1.09,1.56,1.47,1.45	1.393
10	4	1.02,1.38,1.33,1.47	1.3
11	4	1.16,1.3,1.25,1.14	1.213
12	4	1.38,1.34,1.4,1.29	1.353
13	4	1.29,1.05,1.29,1.02	1.163
14	4	1.43,1.45,1.55,1.36	1.448
15	4	1.24,0.9,1.1,1.07	1.078
16	4	1.33,1,1.27,1.3	1.225
17	4	1.14,0.96,1.1,1.05	1.313
18	4	1.32,1.22,1.49,1.3	1.333
19	4	1.3,1.12,1.51,1.57	1.375
20	4	1.36,1.42,1.28,1.13	1.298
21	4	1.45,1.34,1.64,1.62	1.513
22	4	1.36,1.22,1.35,1.25	1.295
23	4	1.06,0.84,1.04,1.04	0.995
24	4	0.83,1.21,1.27,1.27	1.145
25	4	1.28,1.16,1.05,1.1	1.148
26	4	1.15,0.84,1.31,1.2	1.125
27	4	1.05,0.93,1.05,1.06	1.023
28	4	0.88,1.13,1.05,1.32	1.095
29	4	1.29,1.19,1.22,1.16	1.215
Varieties	n_i	Yield of plot in kilogram	Mean yield
30	4	1.32,1.47,1.55,1.12	1.365
31	4	1.23,1.04,1.15,1.41	1.28
32	4	1.36,1.24,1.43,1.25	1.32
33	4	0.9,1.12,0.9,1.26	1.045
34	4	1.08,1.24,1.47,1.37	1.29
35	4	1.26,1.16,1.15,0.92	1.123
36	4	1.23,1.21,1.45,1.16	1.263
37	4	1.23,1,1.16,1.29	1.17
38	4	1.32,1.23,1.22,1.21	1.245
39	4	1.21,1.02,1.14,1.16	1.133
40	4	1.15,1.17,1.3,1.33	1.238
41	4	1.3,1.15,1.24,1.17	1.215
42	4	1.17,0.8,1.33,1.48	1.22
43	4	1.19,1.52,1.25,1.49	1.358
44	4	1.24,1.36,1.06,1.13	1.198
45	4	1.17,1.24,1.32,1.08	1.203
46	4	1.4,1.34,1.2,1.16	1.275
47	4	1.18,1.21,1.24,1.08	1.178
48	4	1.37,1.3,1.32,1.3	1.323
49	4	0.85,0.84,1.07,0.64	0.85

50	4	0.95,1.12,0.99,1.16	1.055
51	4	1.06,1,1.08,1.25	1.098
52	4	1.27,1.01,1.12,1.15	1.138
53	4	1.39,1.2,1.52,1.12	1.308
54	4	1.13,1.28,163,1.23	1.318
55	4	0.99,0.95,1.02,0.88	0.96
56	4	0.89,1.14,1.11,1.19	1.083
57	4	1.02,1.02,1.1,0.62	0.958
58	4	0.98,0.78,1.1,0.89	0.938
59	4	0.82,0.9,1.16,1.27	1.038
60	4	0.94,0.75,0.99,0.75	0.858
61	4	1.1,0.8,1.08,1	0.995
62	4	0.94,0.81,0.92,1.03	0.925
63	4	0.91,0.96,0.92,0.66	0.86
64	4	0.83,0.67,0.72,0.7	0.73

Table (21): Lower and Upper Limits for ρ Wheat Designs.

Methods	Lower limit	Upper limit	Length
Wald	0.5902	1.6659	1.076
Bross	0.904	1.773	0.87
Tukey	0.641	1.547	0.936
And Ban	0.571	1.618	1.047
A	0.5903	1.6613	1.071
Moriguti	0.623	1.537	0.914
Bulmer	0.623	1.537	0.914
H	0.5899	1.6670	1.077
A.ADJ	0.5891	1.6657	1.077
CON.LL	0.5886	-----	
CON.UL	----	1.7899	
SAM	0.804	1.925	1.21
Bt	0.799	1.929	1.13

REFERENCES:

[1] Aitkin, M.(1999)" A General Maximum Likelihood Analysis Of Variance Components In Generalized Linear Model". Biometrics, Vol. 55,117-128.

[2] Anderson, R.L. and Bancroft, T.A. (1952) "Statistical Theory In Research". McGraw. Hill, New York.

[3] Boardman, T.J. (1974) "Confidence Interval for Variance Components A Comparative Monte-Carlo Study". Biometrics, Vol. 30, 251-262.

[4] Bottai, M. and Orsini,N.(2004)" Confidence Intervals For The Variance Component Of Random- Effects Linear Models" .The state Journal ,Vol.4, 429-435.

[5] Breomling, L.D. (1969)."Confidence Regions for Variance Ratios of Random Models" .J.A.S.A, Vol. 64,660-664.

[6]   Bross, I. (1950)" Fiducial Intervals for Variance Components". Biometrics, Vol. 6, 136-44

[7]   Bulmer, M.G. (1957)" Approximate Confidence Limits for Components of Variance". Biometrics, Vol. 44, 159-67.

[8]   Burdick, R.K. and Sielken, R.L. (1978)" Exact Confidence Intervals for Linear Combinations of Variance Components in Nested Classifications" .J.A.S.A., Vol. 73, 632-635.

[9]   Carpenter.J. and Bithell .J. (2000) "Bootstrap Confidence Intervals When, Which, What? A Practical Guide for Medical Statisticians" .Statistics in medicine, Vol.19, 1141-1164.

[10]   Cran, G.W., Martin, K.J. And Thomas, G.E. (1977)" Remark As R19and Algorithm As 109."A Remark on Algorithms as 63. The Incomplete Beta Integral and As 64. Inverse of the Incomplete Beta Function Ratio ".Appl. Statist.   Vol. 26, 111-114.

[11]   Efron, B. (1979)" Bootstrap Methods for Errors Confidence Intervals, And Other Measures of Statistical Accuracy". The Annals of Mathematical Statistics. Vol. 1, 54-77.

[12]   Efron, B. (1987)" Better Bootstrap Confidence Intervals". J.A.S.A., Vol. 82, 171-185.

[13]   Efron, B. and Tibshirani,R.J, R.J (1992)"An Introduction To The Bootstrap" Chapman &Hall.

[14]   El-Bassiouni, M.Y. and El-Shahat, M.A.T. (1985) "On Testing Treatment Effects In Unbalanced Random One-Way Model The Egyptian Statistical Journal, Vol. 30, 65-91.

[15]   El-Bassiouni, M.Y.(1978) "Interval Estimation Of The Parameters Of The Covariance Matrix Of Unbalanced Normal Models" .   Technical. E.Port.

[16]   El-Shahat, M.A.T. (1983) "Power Considerations for Testing Variance Components In The Unbalanced Random One-Way Model". ISSR. Cairo university .Egypt.

[17]   El-Ganzouri, T.M. (1986)"Approximate Confidence Intervals for   the Variance Components In The Unbalanced Random One-Way Model". Unpublished ISSR .Cairo University. Egypt.

[18]   Engel.J. (2010)" ON Teaching Bootstrap Confidence Intervals" Bulletin Of International Statistical Institute, lisoon.

[19]   Graybill, A. and Chih-Ming Wang (1979)" Confidence Intervals For Proportions Of Variability In Two- Factor Nested Variance Component Models". J.A.S.A., Vol. 74, 368-374.

[20]   Graybill, A. and Ming Wang, C. (1980)"Confidence Intervals on Nonnegative Linear Combinations of Variance" .J.A.S.A., Vol. 75. 869-873.

[21]   Khuri, A.I. (1981)" Simultaneous Confidence Intervals for   Functions of Variance Components in Random Model". J.A.S.A., Vol. 76,878-885.

[22]   Kimball, A.W. (1951)" On Dependent Tests Of Significance In The Analysis Of Variance" The Annals Of Mathematical Statistics, Vol.22, 600-602.

[23]   LaMotte, L.R. (1973)" Quadratic Estimation of Variance Components". Biometrics, Vol. 29, 311-330.

[24]   LaMotte, L.R. (1976)"Invariant Quadratic Estimators in the Random One-Way ANOVA Model ".Biometrics, Vol.32, 793-804.

[25]   Moulton, L.H. & zeger,S.L. (1991)"Bootstrapping Generalized Linear Models" .Computational Statistic and Data Analysis, Vol.11, 53-63.

[26] Majumder, K.L. and Bhattacharjee, G.P. (1973)"Algorithm As 63. "The Incomplete Beta Integral". Appl. Statist, VOL. 22, 409-411.

[27] Ming Wang, C. (1990)" On The Lower Bound Of Confidence Coefficients For A Confidence Interval On Variance Components ".Biometrics, Vol.47,187-192.

[28] Morigiti, Sigeiti (1954)"Confidence Limits For A Variance Component" .Rep.Stat.Appl.Res. JUSE, Vol.3,7-19.

[29] Naylor, T.H., Balintfy,J.L., Burdick, D.S. and Chu, V. (1966) "Computer simulation techniques". Wiley, New York p.95.

[30] Olsen, A. Seely, J. And Birkes, D. (1976)"Invariant Quadratic Unbiased Estimation For Two Variance Components". The Annals Of Mathematical Statistics, Vol. 4, 878-890.

[31] Sahai, H. And Anderson, R.L. (1973)" Confidence Regions For Variance Ratios Of Random Models For Balanced Data". J.A.S.A., Vol. 68, 351-359.

[32] Sanogo, K. (2008)" Tolerance Intervals In Random –Effect Models". Virinia Commonwealth University, Richmond.

[33] Satterthwait, F.E. (1941)"Synthesis of Variance". Psychometrika, Vol. 6, 309-316.

[34] Satterthwait, F.E. (1946)" An Approximate Distribution of Estimates of Variance Components". Biometrics, VOL. 2, 110-114.

[35] Seely, J. (1972)" Completeness for A Family of Multivariate Normal Distributions". The Annals of Mathematical Statistics, Vol. 42, 1644-1647.

[36] Thomas, J.D. and Hultiquist, R.A. (1978) "Interval Estimation for the Unbalanced Case Of The One –Way Random Effect Model". The Annals of Mathematical Statistics, Vol. 6, 582-587.

[37] Tukey, J.W. (1951)"Components in Regression". Biometrics, Vol. 7, 33-69.

[38] Wald, A. (1947)"A Note on Regression Analysis" .The Annals Of Mathematical Statistics, Vol. 18, 585-589.

[39] Weerahandi, S. (1993) "Generalized Confidence Intervals". J.A.S.A., Vol. 88, 899-905.

[40] Welch, B.L. (1956) "On linear combinations of several variances". J.A.S.A., Vol. 51,132-148.

[41] Williams,J.S. (1962) "A Confidence Interval For Variance Components". Biometrics, Vol. 49, 278-281.

[42] Zhang, T. and Wood Roof, M. (2002)"Credible and Confidence Sets for the Ratio of Variance Components in the Balance one –way model." The Indian Journal of Statistics, Vol. 64,545-560.