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BAYESIAN ESTIMATION OF THE PARAMETERS OF THE EXPONENTIATED WEIBULL DISTRIBUTION WITH PROGRESSIVELY TYPE I INTEVAL CENSORED SAMPLE

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ABSTRACT: *In this paper, Bayes, investigation the unknown shape Parameters, of the exponentiated Weibull life time model (EWD) are derived from progressive type I interval censored samples using different loss functions and independent and dependent conjugate priors. Besides, the maximum likelihood estimators have also been attempted. Approximate Credible and Shortest Credible intervals for the parameters of an EWD are obtained. A numerical illustration for these new results is also given.*

KEYWORDS: Exponentiated Weibull distribution; Maximum Likelihood estimator; Bayes estimator; Informative prior; Non-informative prior; Squared error loss function; LINEX loss function; General Entropy loss function; Credible and Shortest Credible intervals; Progressively interval type I censored.

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INTRODUCTION

The exponentiated – Weibull distribution (EWD) was introduced by Mudholkar and Huston (1996) as a simple generalization of well-known Weibull distribution by introducing two shape parameters. Mudholkar and Srivastava (1993) studied the suitability of EWD with bathtub hazard rate life time data.

The probability density function (p.d.f.) and the distribution function of EWD are expressed as

$$f(x) = \alpha\, \theta\, x^{\theta-1}\, e^{-x^{\alpha}}\, (1 - e^{-x^{\alpha}})^{\theta-1}, \quad \alpha, \theta > 0, \qquad x > 0, \tag{1}$$

and

$$F(x) = (1 - e^{-x^{\alpha}})^{\theta}, \tag{2}$$

respectively, where α and θ are the shape parameters of the model. It is important to mention that when $\theta = 1$ the EWD p.d.f. (1) is that of the Weibull distribution. For $\alpha\theta \leq 1$ the EWD is decreasing and unimodal when $\alpha\theta > 1$.

The reliability and failure rate functions of the EWD are given, respectively, by

$$R(t) = 1 - (1 - e^{-t^{\alpha}})^{\theta}, \quad t > 0 \tag{3}$$

$$H(t) = \frac{\alpha\theta\, t^{\theta-1}\, e^{-t^{\alpha}}\, (1 - e^{-t^{\alpha}})^{\theta-1}}{1 - (1 - e^{-t^{\alpha}})^{\theta}}, \quad t > 0 \tag{4}$$

The distinguished feature of EWD from other life time distribution is that it accommodates nearly all types of failure rates both monotone and non-monotone (unimodal and bathtub) and includes a number of distributions as particular cases. The structural properties of EWD have been discussed by Mudholkar and Hutson (1996), Jiang and Murthy (1999) and Nassar and Eissa (2003). Practically, the failure model EWD is more realistic than that of monotone failure rates and play an important role to represent such data. The applications of the EWD in reliability and survival studies were illustrated by Mudholkar et al (1995).

Currently, there are little studies for the use of the EWD in reliability estimation. Singh et al (2002), (2005a), (2005b), obtained the Bayes estimates of the distribution parameters and R(t), H(t) with type II censored sample under squared error as well as under LINEX loss functions . Nassar and Eissa (2004) were obtained the Bayes estimates of the two unknown parameters, the reliability and failure rate function by using Bayes approximation form due to Lindley (1980) under the squared error loss and LINEX loss functions.

This article is concerned with Bayesian estimation using progressive interval type I censored data from the EWD. Maximum likelihood and Bayes estimates of the shape parameters α and θ, reliability, and failure rate functions are derived, using independent and dependent priors for the parameters α and θ are considered. We consider three types of loss functions, the squared error loss function (quadratic loss) which is classified as a symmetric function, the LINEX and general Entropy, (GE), loss functions. As well as Credible and Shortest Credible intervals for the parameters α and θ. A numerical illustration for these new results is also given.

MLE for the unknown parameters.

Progressively type I interval censored sample is a union of type I interval and progressive censoring. A progressively type I interval censored sample is collected as follows: n units are put on life test at time $T_0 = 0$ units are observed at pre – set times $T_1, T_2, \ldots\ldots, T_m$ (therefore, m is also fixed).

At these times, $R_1, R_2, \ldots, R_m$ live units are removed from experimentation, respectively. The values $R_1, R_2, \ldots, R_m$ may be pre – specified as percentages of the remaining live units or , alternatively $R_1, R_2, \ldots, R_m$ units available for removal. In this case, the number of live units removed at time T_i is R_i^{obs} = min (R_i , number of units remaining) , $i = 1, 2, \ldots, m-1$. Again R_m^{obs} = all remaining units at time T_m when experimentation is scheduled to terminate.

Suppose a progressively type I interval censored sample is collected as described above, beginning with a random sample of size n units with a continuous life time distribution F(x) ,(2) , and let $d_1, d_2, \ldots d_m$ denote the number of units known to have failed in the

intervals (0 , T_1] , (T_1 , T_2] , …. , (T_{m-1} , T_m] , respectively. Then based on the observed data, Aggarwala (2001) derived the following joint likelihood function will be proportional to the following expression:

$$\ell(x;\alpha,\theta) \propto \prod_{i=1}^{m} [F(T_i) - F(T_{i-1})]^{d_i} [1-F(T_i)]^{R_i^{obs}}$$

$$= \prod_{i=1}^{m} [(1-e^{-T_i^{\alpha}})^{\theta} - (1-e^{-T_i^{\alpha}})^{\theta}]^{d_i} [1-(1-e^{-T_i^{\alpha}})^{\theta}]^{R_i^{obs}}$$

$$= \prod_{i=1}^{m} [P_i - P_{i-1}]^{d_i} [1-P_i]^{R_i^{obs}} \quad , \qquad (5)$$

where $T_0 = 0$, $P_{i-j} = (1 - e^{-T_{i-j}^{\alpha}})^{\theta}$, $j = 0, 1$ and the constant of proportionality multiplying (5) is independent of parameters α and θ . Clearly if = $R_1 = R_2 ... = R_{m-1} = 0$, we are left with the likelihood function for a conventionally Type I interval censored sample. Additionally, it will be useful to note that $\sum_{i=1}^{m} (d_i + R_i^{obs}) = n$.

The log likelihood function for (5) will be

$$L(x;\alpha,\theta) = \sum_{i=1}^{m} d_i \ln(p_i - p_{i-1}) + \sum_{i=1}^{m} R_i^{obs} \ln(1-p_i) . \qquad (6)$$

The mle $\hat{\alpha}$ and $\hat{\theta}$, of α and θ are obtained by simultaneously solving the equations

$$\frac{1}{\theta}[\sum_{i=1}^{m} d_i\, m_i^{*}/w_i - \sum_{i=1}^{m} R_i^{obs}\, w_i^{\bullet}/w_i^{\Delta}$$

$$\theta \sum_{i=1}^{m} d_i\, \vartheta_{2i}/w_i - \sum_{i=1}^{m} R_i^{obs}\, \varphi_i^{\bullet} P_i^{*}/w_i^{\Delta} \quad , \qquad (7)$$

where $w_i = P_i - P_{i-1}$, $w_i^{\bullet} = P_i \ln P_i$, $w_i^{\Delta} = 1 - P_i$,

$P_i^{*} = (1-e^{-T_i^{\alpha}})^{\theta-1}$, $\varphi_i^{\bullet} = \varphi_i \ln T_i$, $\varphi_i = T_i^{\alpha} e^{-T_i^{\alpha}}$,

$m_i^{*} = w_i^{*} - w_{i-1}^{*}$, $w_j^{*} = P_j(1+\ln P_j)\ln P_j$, $j = i-1, i$,

and

$$\vartheta_{2i} = \sum_{j=0}^{1} (-1)^j \varphi_{i-j}^{\bullet} P_{i-j}^{*} .$$

The mle's of R(t) and H(t) can be obtained by replacing α & θ by $\hat{\alpha}$ and $\hat{\theta}$ in (3) and (4).

The elements of the observed Fisher information are derived as follows:

$$\rho_{11} = \frac{-\partial L(x;\theta,\alpha)}{\partial \theta^2} = -\frac{1}{\theta^2}\Big[\sum_{i=1}^{m} d_i (w_i\, m_i^* - m_i^{\bullet 2}) / w_i^2 - \sum_{i=1}^{m} R_i^{obs}\, \varphi_i^{\bullet} P_i^* / w_i^{\Delta}\Big]$$

$$\rho_{22} = \frac{-\partial L(x;\theta,\alpha)}{\partial \alpha^2}$$

$$= -\theta^2 \Big[\sum_{i=1}^{m} d_i (w_i\, \vartheta_{3i} - \theta\, \vartheta_{2i}) / w_i^2 - \sum_{i=1}^{m} R_i^{obs} (w_i^{\Delta} w_i^* + w_i^{\bullet}) / w_i^{\Delta}\Big]$$

$$\rho_{12} = \frac{-\partial^2 L(x;\theta,\alpha)}{\partial \alpha \partial \theta}$$

$$= -\theta \Big[\sum_{i=1}^{m} d_i (w_i \vartheta_{8i} - m_i^{\bullet} P_i^* m_i^{\Delta}) / w_i^2 - \sum_{i=1}^{m} R_i^{obs}\, \varphi_i^{\bullet} P_i^* (w_i^{\Delta} w_i^{\Delta\Delta} + 1) / w_i^{\Delta 2}\Big] , \qquad (8)$$

where $m_i^{\bullet} = w_i^{\bullet} - w_{i-1}^{\bullet}$, $\vartheta_{3i} = \sum_{j=0}^{1} (-1)^j \varphi_{i-j}^* P_{i-j}^* \psi_{i-j}$,

$\varphi_i^* = \varphi_i \ln^2 T_i$, $\psi_{i-j} = (\theta - 1)\, \varphi_{i-j}\, P_{i-j}^{\bullet} - T_{i-j}^{\alpha} + 1$, $j = i-1, i$,

$P_i^{\bullet} = (1 - e^{-T_i^{\alpha}})^{-1}$, $\vartheta_{8i} = \sum_{j=0}^{1} (-1)^j \varphi_{i-j}^{\bullet} P_{i-j}^* w_{i-j}^{\Delta\Delta}$ $m_i^{\Delta} = \varphi_i^{\bullet} - \varphi_{i-j}^{\bullet}$,

and

$$w_i^{\Delta\Delta} = 1 + \ln P_i .$$

Replacing α , θ by $\hat{\alpha}$, $\hat{\theta}$ respectively , the approximate variance covariance matrix , that is , the inverse of the approximate Fisher information matrix will be

$$I_0^{-1} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\Bigg|_{\substack{\theta=\hat{\theta} \\ \alpha=\hat{\alpha}}} \qquad (9)$$

Where $\sigma_{11} = \dfrac{\rho_{22}}{\rho_{11}\rho_{22} - (\rho_{12})^2}$, $\sigma_{22} = \dfrac{\rho_{11}}{\rho_{11}\rho_{22} - (\rho_{12})^2}$

and

$$\sigma_{12} = \sigma_{21} = \frac{-\rho_{12}}{\rho_{11}\rho_{22} - (\rho_{12})^2}$$

Bayesian estimation with known α

Under the assumption that the parameter α is known ($= \alpha_0$) and the natural family of conjugate prior θ is a gamma distribution with p.d.f. is

$$g_1(\theta) \propto \theta^{a_0-1} e^{-a_1/\theta} \quad , \qquad 0 < \theta < \infty \tag{10}$$

where a_0 and a_1 are positive. When there is no prior information on the parameters, the Bayesian approach his needs to specify a prior distribution the reflects this lack of information. When α is known, then letting $a_0 \to 0$ and $a_1 \to 0$ in equation (10), it would be reasonable to assume a non-information prior for θ as

$$g_1^*(\theta) \propto \frac{1}{\theta} .$$

In this case, the corresponding posterior distribution is given by

$$g_1^*(\theta \mid \alpha_0, t) \propto \frac{1}{\theta} \pi_1(\alpha_0, \theta), \; 0 < \theta < \infty \tag{11}$$

Applying Bayes theorem, we obtain from equations (5) and (10) , the posterior density of θ as

$$g_1(\theta \mid T_i, \alpha_0) = \frac{1}{C_0} \theta^{a_0-1} e^{-a_1/\theta} \pi_1(\theta, \alpha_0) \qquad , \theta > 0 \tag{12}$$

where $\pi_1(\theta, \alpha_0) = \prod_{i=1}^{m} \sum_{k=0}^{d_i} \sum_{j=0}^{R_i^{obs}} (-1)^{k+j} \binom{d_i}{k} \binom{R_i^{obs}}{j} \left(P_i^{\Delta\bullet}\right)^{k+j} \left(P_{i-1}^{\Delta\bullet}\right)^{d_i-k}$,

$P_{i-j}^{\Delta\bullet} = (1 - e^{-T_{i-j1}^{\alpha}})^{\theta}$, $j1 = 0, 1$ and C_0 is the normalizing constant given by

$$C_0 = \int_0^{\infty} \theta^{a_0-1} e^{-a_1/\theta} \pi_1(\alpha_0, \theta) \, d\theta \tag{13}$$

Which could be calculated numerically.

When a squared error loss function is used, the posterior mean of θ, which is the Bayes estimate, is given by

$$\tilde{\theta}_B = \frac{1}{C_0} \int_0^{\infty} \theta^{a_0} e^{-a_1/\theta} \pi_1(\theta, \alpha_0) \, d\theta \tag{14}$$

The corresponding Bayes risk, which is the variance of the posterior p.d.f. (12) is

$$\mathrm{Var}(\tilde{\theta}_B) = \frac{1}{C_0} \int_0^{\infty} (\theta - \tilde{\theta}_B)^2 \theta^{a_0-1} e^{-a_1/\theta} \pi_1(\alpha_0, \theta) \, d\theta. \tag{15}$$

The Bayes estimators, $\tilde{R}_\theta$ and $\tilde{H}_\theta$ of the reliability function $R^\bullet \equiv R^\bullet(t)$ and the hazard rate function $H^\bullet \equiv H^\bullet(t)$, and its variances, respectively are

$$\tilde{R}_B = \frac{1}{C_0} \int_0^{\infty} \theta^{a_0-1} e^{-a_1/\theta} R^\bullet \pi_1(\alpha_0, \theta) \, d\theta.$$

$$\tilde{H}_B = \frac{1}{C_0} \int_0^\infty \theta^{a_0-1} e^{-a_1/\theta} H^\bullet \pi_1(\alpha_0, \theta)\, d\theta. \tag{16}$$

$$\text{var}(\tilde{R}_B) = \frac{1}{C_0} \int_0^\infty \theta^{a_0-1} e^{-a_1/\theta} (R^\bullet - \tilde{R}_B)^2 \pi_1(\alpha_0, \theta)\, d\theta,$$

and

$$\text{var}(\tilde{H}_B) = \frac{1}{C_0} \int_0^\infty \theta^{a_0-1} e^{-a_1/\theta} (H^\bullet - \tilde{H}_B)^2 \pi_1(\alpha_0, \theta)\, d\theta, \tag{17}$$

where $R^\bullet(t)$ and $H^\bullet(t)$ are R(t) and H (t) which defined in equations (3) and (4) with substituting $\alpha = \alpha_0$ and $T_i = t$, $t > 0$.

The means and the variances given equations (14) , (15) , (16) and (17) , respectively, could be evaluated numerically.

Using LINEX loss function Bayes estimate $\tilde{\theta}_L$ of parameter θ relative to the LINEX loss function is

$$\tilde{\theta}_L = \frac{-1}{a} \ln\left[\frac{1}{C_0} \int_0^\infty \theta^{a_0-1} e^{-(a\theta^2 + a_1)/\theta}\right] \pi_1(\alpha_0, \theta)\, d\theta, \tag{18}$$

the Bayes estimate $\tilde{R}_L$ is

$$\tilde{R}_L = \frac{-1}{a} \ln\left[\frac{1}{C_0} \int_0^\infty \theta^{a_0-1} e^{-(a\theta R^\bullet + a_1)/\theta}\right] \pi_1(\alpha_0, \theta)\, d\theta, \tag{19}$$

Also, the Bayes estimate $\tilde{H}_L$ of the hazard rate function of (4) relative to the LINEX loss function is

$$\tilde{H}_L = \frac{-1}{a} \ln\left[\frac{1}{C_0} \int_0^\infty \theta^{a_0-1} e^{-(a_1 + aH^\bullet\theta)/\theta} \pi_1(\alpha_0, \theta)\, d\theta\right], \tag{20}$$

The corresponding Bayes risks of $\tilde{\theta}_L$, $\tilde{R}_L$ and $\tilde{H}_L$ are given by

$$\text{var}(\tilde{\zeta}_L) = \frac{1}{C_0} \int_0^\infty \theta^{a_0-1} e^{-a_1/\theta} (\zeta_L - \tilde{\zeta}_L)^2 \pi_1(\alpha_0, \theta)\, d\theta, \tag{21}$$

where $\zeta = \theta$, $R^\bullet$, $H^\bullet$ and $\tilde{\zeta}_L = \tilde{\theta}_L$, $\tilde{R}_L$, $\tilde{H}_L$.

Using general Entropy loss Bayes estimate $\tilde{\theta}_L$, of parameter θ relative to the general Entropy (GE) loss function is

$$\tilde{\theta}_G = \left[\frac{1}{C_0} \int_0^\infty \theta^{a_0-1-q} e^{-a_1/\theta} \pi_1(\alpha_0, \theta)\right]^{-\frac{1}{q}}, \quad q \neq 0 \tag{22}$$

using GE , the Bayes estimate $\tilde{R}_G$ is

$$\tilde{R}_G = \left[\frac{1}{C_0} \int_0^\infty \theta^{a_0-1} e^{-a_1/\theta} R^{\bullet -q} \pi_1(\alpha_0, \theta)\, d\theta\right]^{-\frac{1}{q}}. \tag{23}$$

Also, using GE loss function, the Bayes estimate $\tilde{H}_G$ is

$$\tilde{H}_G = \left[\frac{1}{C_0} \int_0^\infty \theta^{a_0-1} e^{-a_1/\theta} H^{\bullet -q} \pi_1(\alpha_0, \theta)\, d\theta \right]^{-\frac{1}{q}} . \tag{24}$$

The Bayes Risks of $\tilde{\theta}_G$, $\tilde{R}_G$ and $\tilde{H}_G$ are given by

$$\mathrm{var}(\tilde{\zeta}_G) = \frac{1}{C_0} \int_0^\infty \theta^{a_0-1} e^{-a_1/\theta} (\zeta_L - \tilde{\zeta}_G)^2 \pi_1(\alpha_0, \theta)\, d\theta , \tag{25}$$

where $\zeta = \theta$, $R^\bullet$, $H^\bullet$ and $\tilde{\zeta}_G = \tilde{\theta}_G$, $\tilde{R}_G$, $\tilde{H}_G$.

Bayesian Estimation with Known θ

When θ is known ($\theta = \theta_0$) and α is unknown, a prior p.d.f. for α is

$$g_2(\alpha) \propto \alpha^{a_2-1} e^{-a_3\alpha} , \quad 0 < \alpha < b_1 , \tag{26}$$

where a_2, a_3 and b_1 are Positive. Using likelihood function (5),thus, the posterior p.d.f. for α is

$$g_2(\alpha \mid T_i, \theta_0) = \frac{\alpha^{a_2-1} e^{-a_3\alpha}}{C_1} \pi_2(\alpha, \theta_0) \tag{27}$$

where $\pi_2(\alpha, \theta_0) = \prod_{i=1}^{m} \sum_{k=0}^{d_1} \sum_{j=1}^{R_i^{obs}} (-1)^{k+j} \binom{d_i}{k} \binom{R_i^{obs}}{j} \left(P_i^{\Delta\Delta}\right)^{k+j} \left(P_{i-1}^{\Delta\Delta}\right)^{d_i-k}$,

$P_{i-j_1}^{\Delta\Delta} = (1 - e^{-T_{i-j1}^{\alpha}})^{\theta_0}$, $j1 = 0, 1$ and C_1 is the normalizing constant given by

$$C_1 = \int_0^\infty \alpha^{a_2-1} e^{-a_3\alpha} \pi_2(\alpha, \theta_0)\, d\alpha \tag{28}$$

Under squared error loss function (symmetric), the usual estimator of a parameters (or a given function of the parameters) is the posterior mean. Thus, Bayes estimators of the parameter θ , reliability function $R^* = R^*(t)$ and hazard rate function $H^* = H^*(t)$, given in equations (3) & (4) with $\theta = \theta_0$, respectively, are obtained by using the posterior density (27).

The Bayes estimators $\tilde{\alpha}_B^*$, $\tilde{R}_B^*$ and $\tilde{H}_B^*$ of parameter α , the reliability function R* and the hazard rate function H^* and its variances , respectively are

$$\tilde{\alpha}_B^* = \frac{1}{C_1} \int_0^{b_1} \alpha^{a_2} e^{-a_3\alpha} \pi_2(\alpha, \theta_0)\, d\alpha$$

$$\tilde{R}_B^* = \frac{1}{C_1} \int_0^{b_1} \alpha^{a_2-1} e^{-a_3\alpha} R^* \pi_2(\alpha, \theta_0)\, d\alpha$$

$$\tilde{H}_B^* = \frac{1}{C_1} \int_0^{b_1} \alpha^{a_2-1} e^{-a_3\alpha} H^* \pi_2(\alpha, \theta_0)\, d\alpha$$

$$\text{var}(\tilde{\zeta}^*_B) = \frac{1}{C_0} \int_0^{b_1} \alpha^{a_2-1} e^{-a_3\alpha} (\zeta^* - \tilde{\zeta}^*_B)^2 \pi_2(\alpha, \theta_0)\, d\alpha \tag{29}$$

where $\tilde{\zeta}^*_B = \tilde{\alpha}^*_B, \tilde{R}^*_B, \tilde{H}^*_B$ and $\zeta^* = \alpha, R^*, H^*$.

When there is no prior information on α , an appropriate non informative prior is obtained by letting $a_2 \to 0$ and $a_3 \to 0$ in (26) , as

$$g^*_2(\alpha) \propto \alpha^{-1} .$$

Thus the posterior p.d.f. of α would be

$$g^*_2(\alpha \mid T_i, \theta_0) \propto \alpha^{-1} \pi_2(\alpha, \theta_0) \tag{30}$$

Using LINEX loss, Bayes estimates $\tilde{\theta}^*_L$, $\tilde{R}^*_L$, $\tilde{H}^*_L$ of parameter α , R^* and H^* , respectively, relative to the LINEX loss function are

$$\tilde{\zeta}^*_L = -\frac{1}{a} \ln \left[\frac{1}{C_1} \int_0^{b_1} \alpha^{a_2-1} e^{-(a_3\alpha + a\zeta^*)} \pi_2(\alpha, \theta_0)\, d\theta \right] \tag{31}$$

where $\tilde{\alpha}^*_L = \tilde{\zeta}^*_L$ when $\zeta^* = \alpha$, $\tilde{R}^*_L = \tilde{\zeta}^*_L$ when $\zeta^* = R^*$ and $\tilde{H}^*_L = \tilde{\zeta}^*_L$ when $\zeta^* = H^*$.

By substituting $\tilde{\zeta}^*_B$ with $\tilde{\zeta}^*_L$, in equation (29) The variances of $\tilde{\alpha}^*_L$, $\tilde{R}^*_L$ and $\tilde{H}^*_L$ can be obtained .

Using General Entropy (GE) Loss the Bayes estimates $\tilde{\alpha}^*_G$, $\tilde{R}^*_G$ and $\tilde{H}^*_G$, of parameter α, R^* and H^* relative to the GE loss function are

$$\tilde{\zeta}^*_G = \left[\frac{1}{C_1} \int_0^{b_1} \alpha^{a_2-1} (\zeta^*)^{-q} e^{-a_3\alpha} \pi_2(\alpha, \theta_0)\, d\alpha \right]^{-\frac{1}{q}} , \tag{32}$$

where $\tilde{\alpha}^*_G = \tilde{\zeta}^*_G$ when $\zeta^* = \alpha$, $\tilde{R}^*_G = \tilde{\zeta}^*_G$ when $\zeta^* = R^*$ and $\tilde{H}^*_G = \tilde{\zeta}^*_G$ when $\zeta^* = H^*$.

Also, the variances of $\tilde{\alpha}^*_G$, $\tilde{R}^*_G$ and $\tilde{H}^*_G$ can be obtained by substituting $\tilde{\zeta}^*_G$ in equation (29).

Bayesian Estimation with Unknown α and θ :

In this section, we consider the typical case in which the two shape parameters α and θ of a EWD are unknown. We suppose some information on the shape parameters α and θ are available prior. Formulation of a joint density would be constructed in two cases, independent and dependent priors for α and θ .

Independent priors for α and θ

In this case, we use independent priors for α and θ, therefore the joint prior will be

$$g(\alpha,\theta) \propto \theta^{a_0-1} e^{-a_1/\theta} \alpha^{a_2-1} e^{-a_3\alpha} , \qquad \theta > 0 ,\ 0 < \alpha < b_1 \tag{33}$$

Combining the likelihood equation (5) and the prior density function (33), the joint posterior density of α and θ is

$$g(\theta, \alpha \mid T_i) \propto \theta^{a_0-1} \alpha^{a_2-1} e^{-(a_1/\theta + a_3\alpha)} \pi(\alpha, \theta)$$

$$= \frac{1}{C_2} \theta^{a_0-1} \alpha^{a_2-1} e^{-(a_1/\theta + a_3\alpha)} \pi(\alpha, \theta) \qquad (34)$$

where $\pi(\alpha,\theta) = \prod_{i=1}^{m} \sum_{k=1}^{d_i} \sum_{j=1}^{R_i^{obs}} (-1)^{k+j} \binom{d_i}{k} \binom{R_i^{obs}}{j} (P_i)^{k+j} (P_{i-1})^{d_i-k}$, and C_2 is a normalizing constant given by

$$C_2 = \int_0^{\infty} \int_0^{b_1} \theta^{a_0-1} \alpha^{a_2-1} e^{-(a_1/\theta + a_3\alpha)} \pi(\alpha,\theta)\, d\alpha\, d\theta$$

The joint mode of (34), denoted by $(\tilde{\tilde{\theta}}, \tilde{\tilde{\alpha}})$, may be considered as a Bayes estimate of the unknown parameters and is obtained by simultaneously solving the equations

$$\theta D_1 - \pi(\theta,\alpha)\left[1 - a_0 - \frac{a_1}{\theta}\right] = 0$$

$$\alpha D_2 - \pi(\theta,\alpha)\left[\frac{a_3}{\theta} - a_2 + 1\right] = 0$$

where $D_1 = \dfrac{\partial \pi(\theta,\alpha)}{\partial \theta} = \left(\frac{1}{\theta}\right)^m \prod_{i=1}^{m} \sum_{k=1}^{d_i} \sum_{j=1}^{R_i^{obs}} (-1)^{k+j} \binom{d_i}{k} \binom{R_i^{obs}}{j} (P_i)^{k+j} (P_{i-1})^{d_i-k}$

$$\times [(k+j) \ln P_i + (d_i-k) \ln P_{i-1}],$$

and

$$D_2 = \frac{\partial \pi(\theta,\alpha)}{\partial \alpha} = \theta^m \prod_{i=1}^{m} \sum_{k=1}^{d_i} \sum_{j=1}^{R_i^{obs}} (-1)^{k+j} \binom{d_i}{k} \binom{R_i^{obs}}{j} (P_i)^{k+j-1} (P_{i-1})^{d_i-k-1}$$

$$\times [\varphi_i^{\bullet} P_i^{*} P_{i-1} (k+j) + (d_i - k)\, \varphi_i^{\bullet} P_i P_{i-j}^{*}] \qquad (35)$$

Using the bivariate posterior p.d.f. (34) , we can obtain the univariate marginal densities of α and θ by integrating out one of the two unknown Parameters. Thus the marginal p.d.f. of θ is

$$g_3(\theta \mid T_i) = \frac{\theta^{a_0-1} e^{-a_1/\theta} I_1}{C_2}, \qquad \theta > 0, \qquad (36)$$

where $I_1 = \int_0^{b_1} \alpha^{a_2-1} e^{-a_3\alpha} \pi(\theta,\alpha)\, d\alpha$

The Posterior mean and variance of (36) are

$$\tilde{\tilde{\theta}}_B = \frac{1}{C_2} \int_0^{\infty} \theta^{a_0} e^{-a_1\alpha} I_1\, d\theta,$$

$$\text{var}(\tilde{\tilde{\theta}}_B) = \frac{1}{C_2} \int_0^\infty (\theta - \tilde{\tilde{\theta}}_B)^2 \theta^{a_0-1} e^{-a_1\alpha/\theta} I_2 \, d\theta . \tag{37}$$

From (34) , the marginal density of α is

$$g_4(\alpha | T_i) = \frac{1}{C_2} \alpha^{a_2-1} e^{-a_3\alpha} I_2 \quad , \quad 0 < \alpha < b_1 \quad , \tag{38}$$

where $I_2 = \int_0^\infty \theta^{a_0-1} e^{-a_1/\theta} \pi(\theta, \alpha) \, d\theta$.

The posterior mean and variance of (38) are

$$\tilde{\tilde{\alpha}}_B = \int_0^{b_1} \alpha^{a_2} e^{-a_3\alpha} I_2 \, d\alpha$$

$$\text{var}(\tilde{\tilde{\alpha}}_B) = \int_0^{b_1} (\alpha - \tilde{\tilde{\alpha}}_B)^2 \alpha^{a_2-1} e^{-a_3\alpha} I_2 \, d\alpha. \tag{39}$$

The Bayes estimators, $\tilde{\tilde{R}}_B$ and $\tilde{\tilde{H}}_B$ of the reliability function $R \equiv R(t)$ and the Hazard rate function $H \equiv H(t)$ and its variances, respectively are

$$\tilde{\tilde{R}}_B = \frac{1}{C_2} \int_0^\infty \int_0^{b_1} \theta^{a_0-1} \alpha^{a_2-1} e^{-(a_1/\theta + a_3\alpha)} [1 - P_t^\theta] \pi(\alpha,\theta) \, d\alpha \, d\theta$$

$$\tilde{\tilde{H}}_B = \frac{1}{C_2} \int_0^\infty \int_0^{b_1} \theta^{a_0} \alpha^{a_2} \frac{t^{a-1} e^{-(a_1/\theta + a_3\alpha + t^\alpha)}}{(1 - P_t^\theta)} P_t^{\theta-1} \pi(\alpha,\theta) \, d\alpha \, d\theta \quad , \quad t > 0$$

$$\text{var}(\tilde{\tilde{R}}_B) = \int_0^\infty \int_0^{b_1} (R - \tilde{\tilde{R}}_B)^2 g(\alpha,\theta) \, d\alpha \, d\theta$$

$$\text{var}(\tilde{\tilde{H}}_B) = \int_0^\infty \int_0^{b_1} (H - \tilde{\tilde{H}}_B)^2 g(\alpha/\theta) \, d\alpha \, d\theta \tag{40}$$

where $P_t = (1 - e^{-t^\alpha})$.

The Bayes estimates $\tilde{\tilde{\alpha}}_L$, $\tilde{\tilde{\theta}}_L$, $\tilde{\tilde{R}}_L$ and $\tilde{\tilde{H}}_L$ of parameters α and θ, R (t) and H(t) relative to the LINEX loss function and its variances are

$$\tilde{\tilde{\zeta}}_L = \frac{-1}{a} \ln \left[\int_0^\infty \int_0^{b_1} e^{-a\zeta} g(\theta,\alpha) \, d\alpha \, d\theta \right]$$

$$\text{var}(\tilde{\tilde{\zeta}}_L) = \int_0^{\infty}\int_0^{b_1} (\zeta - \tilde{\tilde{\zeta}}_L)\, g(\theta,\alpha)\, d\alpha\, d\theta \qquad (41)$$

where $\tilde{\tilde{\theta}}_L = \tilde{\tilde{\zeta}}_L$ when $\zeta = \theta$, $\tilde{\tilde{\alpha}}_L = \tilde{\tilde{\zeta}}_L$ when $\zeta = \alpha$, $\tilde{\tilde{R}}_L = \tilde{\tilde{\zeta}}_L$ when $\zeta = R$ and $\tilde{\tilde{H}}_L = \tilde{\tilde{\zeta}}_L$ when $\zeta = H$.

Using general Entropy (GE) loss, Bayes estimates $\tilde{\tilde{\alpha}}_G$, $\tilde{\tilde{\theta}}_G$, $\tilde{\tilde{R}}_G$ and $\tilde{\tilde{H}}_G$ of parameters α and θ, R(t) given by (3) and H (t) given by (4) , relative to GE loss function are

$$\tilde{\tilde{\theta}}_G = \left[\frac{1}{C_2}\int_0^{\infty} \theta^{a_0 - q - 1} e^{-a_1/\theta} I_1 d\theta\right]^{-\frac{1}{q}}$$

$$\tilde{\tilde{\alpha}}_G = \left[\frac{1}{C_2}\int_0^{b_1} \alpha^{a_2 - q - 1} e^{-a_3/\alpha} I_2\, d\alpha\right]^{-\frac{1}{q}}$$

$$\tilde{\tilde{R}}_G = \left[\int_0^{\infty}\int_0^{b_1} [1 - P_t^{\theta}]^{-q}\, g(\theta,\alpha)\, d\alpha\, d\theta\right]^{-\frac{1}{q}}$$

$$\tilde{\tilde{H}}_G = \left[\int_0^{\infty}\int_0^{b_1} \left(\frac{\theta^{\alpha_0}\alpha^{a_2} t^{\alpha-1} P_t^{\theta-1}}{1 - P_t^{\theta}}\right)^{-q} e^{-(a_1/\theta + a_3\alpha - qt^{\alpha})}\, \pi(\alpha,\theta)\, d\alpha d\theta\right]^{-\frac{1}{q}}. \qquad (42)$$

The corresponding Bayes risks, which are the variances of $\tilde{\tilde{\alpha}}_G$, $\tilde{\tilde{\theta}}_G$, $\tilde{\tilde{R}}_G$ and $\tilde{\tilde{H}}_G$ are giver by substituting $\tilde{\tilde{\zeta}}_L$ in equation (41) with $\tilde{\tilde{\zeta}}_G$.

If α and θ are independent and assuming that no prior knowledge about α and θ is available, the appropriate non-informative joint prior of α and k will be

$$g^*(\alpha,\theta) \propto (\alpha\theta)^{-1}, \quad \theta > 0,\ 0 < \alpha < b_1, \qquad (43)$$

using (43) with (5) , the joint posterior density of α and θ is

$$g^*(\alpha,\theta \mid T_i) \propto (\alpha\theta)^{-1}\pi(\alpha,\theta), \quad \theta > 0,\ 0 < \alpha < b_1 \qquad (44)$$

Singh et al (2002 & 2003) used (43) for estimating symmetric and asymmetric LINEX Bayes estimation of EWD parameters in type II censored sample.

Dependent priors for θ and α .

Nassar and Eissa (2004) suggested the following bivariate prior density for α and θ

$$G(\alpha,\theta) = G_1(\theta|\alpha)\, G_2(\alpha), \qquad (45)$$

where

$$G_1(\theta|\alpha) = \frac{\alpha^{-V}\theta^{V-1}}{\Gamma(V)} e^{-\theta/\alpha}, \quad V > 0, \ \theta > 0 \qquad (46)$$

is the gamma conjugate prior, and the scale parameter of this density is α, which is assumed to become known previously with knowledge that may be translated into an exponential distribution with density function

$$G_2(\alpha) = \frac{1}{d} e^{-\alpha/d} \quad , \quad \alpha > 0 \quad , d > 0 \tag{47}$$

Multiplying $G_1(\theta|\alpha)$ by $G_2(\alpha)$, we obtain the bivariate prior density of θ and α, given from (45) by

$$G(\alpha,\theta) = \frac{\alpha^{-V}\theta^{V-1}e^{-(d\theta+\alpha^2)/d\alpha}}{d\Gamma(V)} \quad , \quad \alpha > 0 \quad , \theta > 0 \tag{48}$$

where V and d are positive real numbers .

Combining the likelihood equation (5) and the prior density function (48), the joint posterior density of α and θ, is

$$G(\alpha,\theta \mid T_i) = \frac{1}{C_3}\alpha^{-V}\theta^{V-1} e^{-(d\theta + \alpha^2)/d\alpha} \pi(\theta,\alpha) \quad , \quad \alpha > 0 \quad , \theta > 0 \ , \tag{49}$$

where C_3, is a normalizing constant, given by

$$C_3 = \frac{1}{d\Gamma(V)} \int_0^\infty \int_0^\infty \alpha^{-V}\theta^{V-1} e^{-(d\theta + \alpha^2)/d\alpha} \pi(\theta,\alpha)\, d\theta\, d\alpha$$

The joint mode of (49) , denoted by $\tilde{\tilde{\theta}}^*$ & $\tilde{\tilde{\alpha}}^*$, is the solution of the equations

$$\theta D_1 - [\theta - (V - 1)\alpha]\, \pi(\theta,\alpha) = 0$$

$$\alpha D_2 - [\ V + \frac{\alpha}{d} - \frac{\theta}{\alpha}\ ]\, \pi(\theta,\alpha) = 0 \quad .$$

From (49), the marginal p.d.f. of α is

$$g_5(\alpha \mid T_i) = \frac{1}{C_3}\alpha^{-V} e^{-\alpha/d} I_3 \quad , \quad \alpha > 0 \quad , \tag{50}$$

where $I_3 = \int_0^\infty \alpha^{-V-1} e^{-\theta/\alpha} \pi(\theta,\alpha)\, d\theta$.

The posterior mean and variance of (50) are

$$\tilde{\tilde{\alpha}}^*_B = \int_0^\infty \alpha\, g_5(\alpha \mid T_i)\, d\alpha$$

$$\mathrm{var}(\tilde{\tilde{\alpha}}^*_B) = \int_0^\infty (\alpha - \tilde{\tilde{\alpha}}^*_B)^2 g_5(\alpha \mid T_i)\, d\alpha \quad . \tag{51}$$

Also from (49) , the univariate marginal p.d.f. of θ is

$$g_6(\theta \mid T_i) = \frac{1}{C_3}\theta^{-V-1} I_4 \quad , \quad \alpha > 0 \quad , \tag{52}$$

where $I_4 = \int_0^\infty \alpha^{-V} e^{-(d\theta + \alpha^2)/d\alpha} \pi(\theta,\alpha)\, d\alpha$.

The posterior mean and variance of (52) are

$$\tilde{\tilde{\theta}}_B^* = \int_0^\infty \theta g_6(\theta \mid T_i)\, d\theta$$

$$\text{var}(\tilde{\tilde{\theta}}_B^*) = \int_0^\infty (\theta - \tilde{\tilde{\theta}}_B^*)^2 g_6(\theta \mid T_i)\, d\theta. \tag{53}$$

The Bayes estimators, $\tilde{\tilde{R}}_B^*$ and $\tilde{\tilde{H}}_B^*$ of R(t) and H(t) and its variances, respectively are

$$\tilde{\tilde{R}}_B^* = \frac{1}{C_3} \int_0^\infty \int_0^\infty [1 - P_t^\theta]\, \alpha^{-V} \theta^{-V-1} e^{-(d\theta + \alpha^2)/d\alpha} \pi(\theta,\alpha)\, d\theta\, d\alpha$$

$$\text{var}(\tilde{\tilde{R}}_B^*) = \int_0^\infty \int_0^\infty [R - \tilde{\tilde{R}}_B^*]^2 G(\theta, \alpha \mid T_i)\, d\alpha\, d\theta$$

$$\tilde{\tilde{H}}_B^* = \int_0^\infty \int_0^\infty H(t)\, G(\theta, \alpha \mid T_i)\, d\alpha\, d\theta$$

$$\text{var}(\tilde{\tilde{H}}_B^*) = \int_0^\infty \int_0^\infty [H(t) - \tilde{\tilde{H}}_B^*]^2 G(\theta, \alpha \mid T_i)\, d\alpha\, d\theta. \quad , \quad t > 0 \tag{54}$$

Numerical evaluation, using Computer facilities, are needed to evaluate equations (51, 53 & 54).

The Bayes estimates $\tilde{\tilde{\alpha}}_L^*$, $\tilde{\tilde{\theta}}_L^*$, $\tilde{\tilde{R}}_L^*$ and $\tilde{\tilde{H}}_L^*$ of Parameters α andθ,R (t) and H(t) relative to the LINEX loss function and its variances are

$$\tilde{\tilde{\zeta}}_L^* = -\frac{1}{a} \ln \left[\int_0^\infty \int_0^\infty e^{-a\zeta} G(\theta, \alpha \mid T_i)\, d\alpha\, d\theta \right] \quad , \tag{55}$$

and

$$\text{var}(\tilde{\tilde{\zeta}}_L^*) = \int_0^\infty \int_0^\infty (\zeta - \tilde{\tilde{\zeta}}_L^*)^2 G(\theta, \alpha \mid T_i)\, d\alpha\, d\theta \ , \tag{56}$$

where $\tilde{\tilde{\theta}}_L^* = \tilde{\tilde{\zeta}}_L^*$ when $\zeta = \theta$, where $\tilde{\tilde{\alpha}}_L^* = \tilde{\tilde{\zeta}}_L^*$ when $\zeta = \alpha$, where $\tilde{\tilde{R}}_L^* = \tilde{\tilde{\zeta}}_L^*$ when $\zeta = R(t)$, where $\tilde{\tilde{H}}_L^* = \tilde{\tilde{\zeta}}_L^*$ when $\zeta = H(t)$.

Using general Entropy (GE) loss, the Bayes estimates $\tilde{\tilde{\alpha}}_G$, $\tilde{\tilde{\theta}}_G$, $\tilde{\tilde{R}}_G$ and $\tilde{\tilde{H}}_G$ of parameters α and θ, R (t) given by (3) and H (t), given by (4) , relative to GE loss function are

$$\tilde{\tilde{\zeta}}^{*}_{G} = [\int_{0}^{\infty}\int_{0}^{\infty} \zeta^{-q} G(\theta, \alpha \mid T_i)\, d\alpha\, d\theta]^{-\frac{1}{q}} \tag{57}$$

where $\tilde{\tilde{\alpha}}^{*}_{G} = \tilde{\tilde{\zeta}}^{*}_{G}$ when $\zeta = \alpha$, where $\tilde{\tilde{\theta}}^{*}_{G} = \tilde{\tilde{\zeta}}^{*}_{G}$ when $\zeta = \theta$, where $\tilde{\tilde{R}}^{*}_{G} = \tilde{\tilde{\zeta}}^{*}_{G}$ when $\zeta = R(t)$, where $\tilde{\tilde{H}}^{*}_{G} = \tilde{\tilde{\zeta}}^{*}_{G}$ when $\zeta = H(t)$ .The variances of $\tilde{\tilde{\alpha}}^{*}_{G}$, $\tilde{\tilde{\theta}}^{*}_{G}$, $\tilde{\tilde{R}}^{*}_{G}$ and $\tilde{\tilde{H}}^{*}_{G}$ are given by substituting $\tilde{\tilde{\zeta}}^{*}_{L}$ with $\tilde{\tilde{\zeta}}^{*}_{G}$ in equation (56)

Since EWD have a Unimodal $(\tilde{\tilde{\theta}}^{*}, \tilde{\tilde{\alpha}}^{*})$ if $\alpha\theta > 1$, a l00 (1 –P) two sided shortest credible intervals $\{\tilde{\tilde{\alpha}}_1, \tilde{\tilde{\alpha}}_2\}$ and $\{\tilde{\tilde{\theta}}_1, \tilde{\tilde{\theta}}_2\}$ for α and θ , respectively (or highest posterior Density, HPD, $g_5(\alpha \mid x)$ and $g_6(\theta \mid x)$ are unimodal, Box and Tiao (1972)) are such that

$g_5(\tilde{\tilde{\alpha}}_1 \mid x) = g_5(\tilde{\tilde{\alpha}}_2 \mid x)$, as well as ,

$$\int_{\tilde{\tilde{\alpha}}_1}^{\tilde{\tilde{\alpha}}_2} g_5(\alpha \mid x)\, d\alpha = 1 - P \tag{58}$$

and $g_6(\tilde{\tilde{\theta}}_1 \mid x) = g_6(\tilde{\tilde{\alpha}}_2 \mid x)$, as well as ,

$$\int_{\tilde{\tilde{\theta}}_1}^{\tilde{\tilde{\theta}}_2} g_6(\tilde{\theta} \mid x) = 1 - P \tag{59}$$

Numerical Illustration Situations .

To illustrate the usefullness of the proposed estimators obtained in preceding sections with real situations obtained in preceding sections, we generate a sample of size 50 from the EWD with parameters $\alpha = 3$ and $\theta = 1.5$ (Ahmed et al 2006).

Using MATHCAD (2001) , a sample of size n = 50 was generated from the exponentiated Weibull with parameters $\alpha = 3$ and $\theta = 1.5$ ($\alpha\theta = > 1$ unimodal, see Figure (1)) . The results are :

0.20944	0.60863	0.84716	1.11124	1.28844
0.35593	0.69445	0.88538	1.11346	1.30113
0.41745	0.69717	0.89909	1.11470	1.32253
0.43788	0.72031	0.83488	1.11038	1.33569
0.43814	0.73955	0.93289	1.11875	1.33483
0.52250	0.77944	0.94301	1.12167	1.34592
0.53618	0.78027	0.94607	1.14399	1.41394
0.59273	0.78978	0.95254	1.15737	1.41710
0.60315	0.80577	0.97753	1.16380	1.57964
0.60719	0.81916	1.02507	1.22745	1.78653

Suppose that progressive type I interval censored from EWD with removal occurs at five stages m = 5. Assume that at Time T_1 = 0.41745, none unit selected at random form the survivor, were censored, i.e., $R_1^{obs} = 0$. At T_2 = 0.53618, two additional randomly selected survivors were removed .Three additional randomly selected survivors were removed at T_3= 0.69717. At time T_4 = 0.83488, another one unit selected at random from the survivors, and the test was terminated at T_5 = 0.93289 data, we record :

$T_1 = 0.417$	$T_2 = 0.536$	$T_3 = 0.697$	$T_4 = 0.839$	$T_5 = 0.933$
$d_1 = 3$	$d_2 = 4$	$d_3 = 4$	$d_4 = 5$	$d_5 = 3$
$R_1 = 0$	$R_2 = 2$	$R_3 = 3$	$R_4 = 1$	$R_5 = 25$

The estimates α, θ , R (t) and H (t) at t = 0.75 developed in sections 2 , 3 , 4 and 5 on the basis of above data are obtained and are reported in Table 1 . Also , figures 1and 2 show that the unimodal property for the density function of EWD, f(x)and joint posterior density of the parameters α and θ .

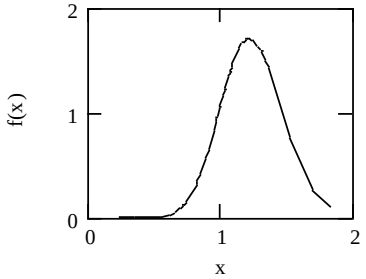
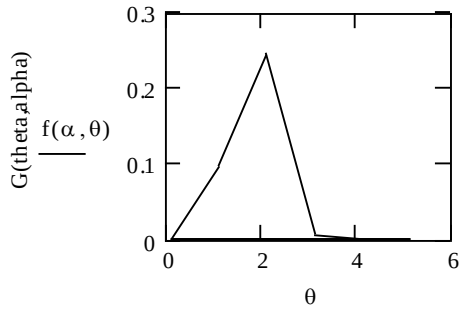


Figure (1)



the joint posterior of parameters

Figure(2)

Table 1 revealed that the Bayes estimators developed with non-informative prior yet the estimated values of Bayes estimators are very enclosed to the estimated values of MLE. Future numerical results using different values for n , m , d_i , R_i , θ and α are needed to establish strong conclusion for comparing mle with Bayes estimates .

Table 1 : Various Estimates For The Progressive Type I Interval Censored Sample Estimators.

	θ	α	Estimate variance of θ	Estimate variance of α	$R(t)_{t=0.75}$	$H(t)_{t=0.75}$
mle	1.55	2.94	0.0043	0.0026	0.802. (0.009) $^{\Delta}$	2.046 (0.074)$^{\Delta}$
α Known=3.0						
Symmetric LOSS	1.581 [1.35,1.82]*		5.38×10^{-4}		0.70 (0.009) $^{\Delta}$	1.757 (0.074) $^{\Delta}$
LINEX	1.545		1.68×10^{-4}		0.872(1.68×10^{-4})$^{\Delta}$	2.07(0.03×10^{-4}) $^{\Delta}$
GE	1.671		0.026		0.696 (8.5×10^{-3})$^{\Delta}$	1.78(4.0×10^{-3}) $^{\Delta}$
Noninformative prior	1.55		0.0042		0.80. (0.0094) $^{\Delta}$	2.036 (0.075)$^{\Delta}$
θ Known=1.5						
Symmetric loss		2.998 [1.59, 3.58]*		0.039	0.784 (0.0031) $^{\Delta}$	2.06(1.7×10^{-4}) $^{\Delta}$
LINEX		3.028		0.052	0.788(2.56×10^{-8}) $^{\Delta}$	2.06(7.7×10^{-9}) $^{\Delta}$
GE		3.018		0.048	0.807 (0.011) $^{\Delta}$	1.83(4.9×10^{-9}) $^{\Delta}$
Noninformative prior		2.95		0.0029	0.803. (0.0096) $^{\Delta}$	2.049 (0.071)$^{\Delta}$
Indep.Unk.α & θ						
Posterior mode	1.848	3.042				
Symmetric loss	1.523 [1.45,1.55]*	3.025 [2.96,3.26]*	6.98×10^{-8}	0.051	0.8 (4.6×10^{-6}) $^{\Delta}$	2.06(1.9×10^{-4}) $^{\Delta}$
LINEX	1.49	3.03	4.86×10^{-3}	0.038	0.8(4.6×10^{-6}) $^{\Delta}$	2.06(1.9×10^{-4}) $^{\Delta}$
GE	1.546	3.058	2.15×10^{-3}	0.667	0.8(4.6×10^{-6}) $^{\Delta}$	2.04(1.92×10^{-4}) $^{\Delta}$
Noninformative prior	1.54	2.94	0.0045	0.0027	0.803. (0.0098) $^{\Delta}$	2.041 (0.070)$^{\Delta}$
Dep.Unk.α & θ						
Posterior mode	1.507	2.914				
Symmetric loss	1.554 [1.43,1.57]*	3.105 [2.4,3.5]*	1.74×10^{-5}	1.49×10^{-5}	0.776(1.25×10^{-3}) $^{\Delta}$	2.11(3.8×10^{-3}) $^{\Delta}$
LINEX	1.508	2.98	1.41×10^{-7}	3.59×10^{-3}	0.762(9.68×10^{-6}) $^{\Delta}$	2.04(9.5×10^{-3}) $^{\Delta}$
GE	1.502	3.016	1.96×10^{-8}	1.63×10^{-6}	0.762(9.68×10^{-6}) $^{\Delta}$	2.05(9.5×10^{-3}) $^{\Delta}$
Noninformative prior	1.53	2.93	0.0041	0.0027	0.801. (0.010) $^{\Delta}$	2.039 (0.072)$^{\Delta}$

* Present the Shortest Credible Intervals for θ & α .
Δ Present the variance estimates for R(t) & H(t).

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