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A NEW ALGORITHM USING RANKING FUNCTION TO FIND SOLUTION FOR FUZZY TRANSPORTATION PROBLEM

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ABSTRACT: *In this article a new algorithm is used which depending on proposed ranking function for finding on optimal solution of fully fuzzy transportation problem. In the proposed algorithm, transportation cost, supply and demand are represented by normal or abnormal triangular fuzzy numbers. A numerical example is given to show the efficiency of this algorithm with respect to the Vogel's algorithm with modified distribution algorithm.*

KEYWORDS: Fuzzy numbers, Fuzzy transportation problem, triangular membership function, ranking function.

INTRODUCTION

The transportation problem is a special type of linear programming problem which deals with the distribution of single product from various sours of supply to various destination of demand in such a way that the total transportation cost is minimized.[5]. In real life, there are many problem deals with uncertainty in parameters, then can not applied the traditional method to solve the transportation problem, but we can solved it by using fuzzy methods which depend on ranking function to find the optimal solution for transportation problems. The fuzzy transportation problems connect between fuzzy set theory, ranking function and transportation problems, which means that the supply, demand and total transportation cost are fuzzy numbers.

Many authors studied ranking function to solved the problem of fuzzy transportation.
-Liu and Kao (2004) described a method to solved a fuzzy transportation problem based on extension principle.[5]-Chiang J.(2005) proposed an algorithm for ranking function when the demand and supply are fuzzy numbers only, he found two different triangular membership for demand and supply.[1]
-Liu and Kao(2006) proposed a new method to solved a fuzzy transportation problem based on extension principle.[5]
-Basirzadeh and Abbas (2008)proposed a new method to solved the fuzzy transportation problem using ranking function based on $\alpha-$cut.[2]
- Lin and Tsai (2009)used a two stage genetic algorithm for solving the transportation problem when the demands and supplies were fuzzy numbers.[6]
-Pandian and Nagarajan (2010) proposed a fuzzy zero point method for finding a fuzzy optimal solution for fuzzy transportation problem where all parameters are trapezoidal fuzzy numbers.[5]
-Basirzadeh H.(2011) solved the fuzzy transportation problem depending on the ranking function of Yager (1981) which found ranking function of trapezoidal and triangular memberships.[2]
-poonam S., Abbas S.H, and Gupta V.K.(2012) presented a ranking technique with α optimal solution for solving fuzzy transportation problem, where the demand and supply are triangular fuzzy numbers.[6]

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-Nagoor G. and Abbas S.(2013) used the idea chiang at (2005) and studied that the demand and supply are fuzzy numbers only depending are two different triangular memberships.[4]
-Nareshkumar S. and Kumara G. (2014) proposed method, where the cost, demand and supply are symmetric triangular fuzzy numbers, then they developed fuzzy version of Vogel's algorithm for finding fuzzy optimal solution of fuzzy transportation problem.[5]

The objective of this paper is propose a new algorithm depending on ranking function to solve fully fuzzy transportation problem using triangular fuzzy numbers for the supply, demand and total cost. This paper contain five section:- in section two review some concept of fuzzy theory, in section three define fuzzy transportation problem and it formula, in section four recall ranking function and study some properties, in section five take numerical example and applied a new algorithm depending on ranking function.

Concept of Fuzzy Theory

In this section we will introduce some definitions of fuzzy theory.

Fuzzy set [7]

Let Ωbe a nonempty set. A fuzzy set A in Ω is characterized by its membership function, $\mu_A:\Omega \rightarrow [0, 1]$ and denoted by $\widetilde{A}$ and $\mu_{A(a)}$is interpreted as the degree of membership of element a in fuzzy set A for each a$\in \Omega$, $\widetilde{A} = \{(a, \mu_{A(a)}): a \in \Omega\}$.

Fuzzy number [5]

The fuzzy set A defined on the set of real numbers is said to be a fuzzy number if its membership function $\mu_A: \Omega \rightarrow [0,1]$has the following characteristics

1- A is normal . it means that there exists an a$\in$ R such that $\mu_{A_{(a)}} = 1$.

2- A is convex it means that for every $a_1, a_2 \in R$
$\mu_A(\lambda a_1+(1-\lambda)a_2) \geq \min\{\mu_A(a_1), \mu_A(a_2), \lambda \in [0,1]\}$.

3- $\mu_A$is upper semi-continuous.

4- Supp(A) is bounded in R.

Triangular fuzzy number [5]

A fuzzy number $\widetilde{A}$ in R is said to be a triangular fuzzy number and denoted by $\widetilde{A}$ = (a,b,c) if it membership function $\mu_A : R \rightarrow [0,1]$ has the following characteristics:-

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a} & a \leq x \leq b \\ 1 & x = b \\ \dfrac{c-x}{c-b} & b \leq x \leq c \end{cases}$$

Where a is core(A), b is the left width and c is the right width

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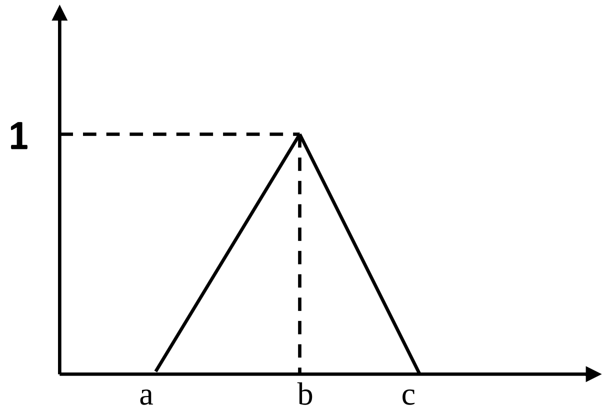

Membership function of triangular fuzzy number

Fuzzy transportation problem

The fuzzy transportation problem is the transportation problem with supply, demand and the total cost are fuzzy quantities.[1]

Now formulate the fully fuzzy transportation problem by

$$\text{Minimize } Z=\sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\, x_{ij}$$
$$\sum_{j=1}^{n} x_{ij} = \tilde{a}_i \qquad , i = 1, \dots, m$$
$$\sum_{i=1}^{m} x_{ij} = \tilde{b}_j \qquad , j = 1, \dots, n$$
$$x_{ij} \geq 0$$

Ranking of triangular fuzzy number

The ranking function defined as, $R{:}F(\mu)\rightarrow R$ which maps each fuzzy number into the real line; $F(\mu)$ represent the set of triangular fuzzy number.[8]

There are many properties for ranking function, anytwo triangular fuzzy number A and B we have the following comparison.**[5]**

1- A<B iff R(A) <R(B).

2- A>B iff R(A)>R(B).

3- A ≈ B iff R(A)≈ R(B).

4- A-B=0 iff R(A)–R(B)=0.

A triangular fuzzy number $\tilde{A}$=(a, b, c) in F(μ) is said to be positive if $R(\tilde{A})\geq 0$ and denoted by $\tilde{A} > 0$.

*** A new algorithm ranking function**

We use the following triangular membership:-

$$\mu_A(x) = \begin{cases} \dfrac{\lambda(x-a)}{(b-a)} & a \leq x \leq b \\ \lambda & x = b \\ \dfrac{\lambda(c-x)}{(c-b)} & b \leq x \leq c \end{cases}$$

By using α-cut, where $\alpha \in \{0,1]$ and $0\leq \alpha \leq \lambda$, then

$$\alpha = \frac{\lambda(x-a)}{(b-a)} \qquad\qquad \alpha = \frac{\lambda(c-x)}{(c-b)}$$
$$x = a + \frac{\alpha}{\lambda}(b-a) \qquad\qquad x = c - \frac{\alpha}{\lambda}(c-b)$$

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$$\widetilde{A}^{l}{}_{(\alpha)} = a + \frac{\alpha}{\lambda}(b-a) \qquad \widetilde{A}^{u}{}_{(\alpha)} = c - \frac{\alpha}{\lambda}(c-b)$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{1}{2}\int_0^{\lambda} \alpha^2\,[\widetilde{A}^{l}{}_{(\alpha)} + \widetilde{A}^{u}{}_{(\alpha)}]d\alpha\right]}{\alpha^2 d\alpha}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{1}{2}\int_0^{\lambda} \alpha^2\,[a+\frac{\alpha}{\lambda}(b-a)+ c-\frac{\alpha}{\lambda}(c-b)]d\alpha\right]}{\alpha^2\, d\alpha}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{1}{2}\int_0^{\lambda}[\alpha^2 a+\frac{\alpha^3}{\lambda}(b-a)+\alpha^2 c-\frac{\alpha^3}{\lambda}(c-b)]d\alpha\right]}{\alpha^2 d\alpha}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{1}{2}\left[\frac{\alpha^3}{3}a+\frac{\alpha^4}{4\lambda}(b-a)+\frac{\alpha^3}{3}c-\frac{\alpha^4}{4\lambda}(c-b)\right]\Big|_0^{\lambda}\right]}{\frac{\alpha^3}{3}\Big|_0^{\lambda}}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{1}{2}\left[\frac{\lambda^3}{3}a+\frac{\lambda^4}{4\lambda}(b-a)+\frac{\lambda^3}{3}c-\frac{\lambda^4}{4\lambda}(c-b)\right]\right]}{\frac{\lambda^3}{3}}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\left[\frac{1}{2}\left[\frac{\lambda^3}{3}a+\frac{\lambda^3}{4}(b-a)+\frac{\lambda^3}{3}c-\frac{\lambda^4}{4}(c-b)\right]\right]}{\frac{\lambda^3}{3}}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\frac{\lambda^3}{2}\left[\frac{a}{3}+\frac{(b-a)}{4}+\frac{c}{3}-\frac{(c-b)}{4}\right]}{\frac{\lambda^3}{3}}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{\frac{\lambda^3}{2}\left[\frac{4a+3b-3a}{12}+\frac{4c-3c+3b}{12}\right]}{\frac{\lambda^3}{3}}$$

$$R(\widetilde{A}_{(\alpha)}) = \frac{[\,a+6b+c\,]}{8}.$$

Numerical Example[3]

This example may be clarify the proposed method .

A company has three sources s_1 , s_2 and s_3 , also three destinations D_1, D_2 and D_3. All the data in this example are triangular fuzzy. We desire to solve this fuzzy transportation problem with proposed algorithm and compare it with traditional algorithm

	D_1	D_2	D_3	Supply
S_1	1	0	2	4
S_2	3	5	4	6
S_3	1	2	3	10
Demand	3	5	12	

First: we solve the problem by using Vogel's approximation algorithm, then apply the modified distribution algorithm to get the optimal solution.

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	D_1	D_2	D_3	supply
S_1	1	0 4	2	4
S_2	3	5	4 6	6
S_3	1 3	2 1	3 6	10
demand	3	5	12	

We find that the total cost is 47
Second: Now, we fuzziness all the parameters in the problem

	D_1	D_2	D_3	Supply
S_1	(0.3,1,2)	(-0.7,0,1)	(1.3,2,3)	(3.3 ,4,5)
S_2	(2.3,3,4)	(4.3,5,6)	(3.3,4,5)	(5.3,6,7)
S_3	(0.3,1,2)	(1.3,2,3)	(2.3,3,4)	(9.3,10,11)
demand	(2.3,3,4)	(4.3,5,6)	(11.3,12,13)	

then apply the proposed algorithm of ranking function, we get the following optimal solution.

	D_1	D_2	D_3	supply
S_1	(1.0375)	(0.0375) 4.0375	(2.0375)	(4.0375)
S_2	(3.0375)	(5.0375)	(4.0375) 6.0375	(6.0375)
S_3	(1.0375) 3.0375	(2.0375) 1	(3.0375) 6	(10.0375)
demand	(3.0375)	(5.0375)	(12.0375)	

we find that the total cost is 35.8667

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CONCLUSION

If the transportation problem is crisp and we use the Vogel's approximation algorithm with modified distribution algorithm the total cost is(47).Now, if we make the supply, demand and cost fuzziness then we solve the fuzzy problem by using the proposed algorithm with ranking function the total cost is (35.8667) which is less than the total cost for traditional algorithm. The proposed algorithm will be helpful for decision maker when they dealing with fuzzy transportation problem.

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