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ADDITIVE SARIMA MODELLING OF MONTHLY NIGERIAN NAIRA - CFA FRANC EXCHANGE RATES

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ABSTRACT: *This paper is on SARIMA modelling of monthly Naira-CFA Franc exchange rates. The time plot of the realisation from January 2004 to June 2013 in Figure 1 shows an overall upward secular trend with no clear seasonality. The time plot of the seasonal (i.e. 12-monthly) differences in Figure 2 shows an overall horizontal trend with no definite seasonality. The time plot of further non-seasonal differences in Figure 3 shows a horizontal trend and still no clear seasonality. The autocorrelation function of the resultant series of Figure 4 has a significant negative spike at lag 12 indicating a 12-monthly seasonality and the involvement of a seasonal moving average component of order one. Its partial autocorrelation function has a significant spike at lag 12 suggesting the inclusion of a seasonal autoregressive component of order one. Using the duality relationship between autoregressive and moving average models, it is argued that this autoregressive component of high order (i.e. of order 12) be replaced by a moving average component of (low) order one. Hence an additive SARIMA model with significant lags 1 and 12 is proposed and fitted. The model is shown to be adequate***.**

KEYWORDS: Naira, CFA Franc, Foreign exchange rates, Additive SARIMA models

INTRODUCTION

Many economic and financial time series are observed to show seasonal as well as volatile behaviour. Foreign exchange rates are inclusive. Attention has been paid by many researchers to the time series modelling of the exchange rates of many currencies. For instance, Etuk and Igbudu(2013) have observed the seasonal nature of the monthly exchange rates of the Naira and the British pounds and fitted to them a (0, 1, 0)x(2, 1, 1)$_{12}$ SARIMA model. Etuk(2013a), after observing a 12-monthly seasonal tendency in the monthly Naira-Euro exchange rates fitted a (0, 1, 1)x(1, 1, 1)$_{12}$ SARIMA model to them. Etuk(2013b) has also fitted a (0, 1, 1)x(0, 1, 1)$_7$ model to daily Naira-Euro exchange rates. A few other researchers who have studied exchange rates are Oyediran and Afieroho (2013), Onasanya and Adeniji (2013) and Appiah and Adetunde (2011). It is the intention of this paper to model the monthly Naira -

CFA Franc exchange rates using SARIMA methods with a view to showing the usefulness of such models.

Box and Jenkins (1976) introduced SARIMA models in order to capture the seasonality of seasonal data. These models have been extensively written on and applied by many authors, a few of whom are Etuk(2012), Surhatono and Lee(2011), Saz(2011) and Madsen(2008).

LITERATURE/ THEORITICAL UNDERPINNING

A stationary time series $\{X_t\}$ is said to follow an *autoregressive moving average model of orders p and q*, designated ARMA(p, q), if it satisfies the following difference equation

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - ... - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + ... + \beta_q \varepsilon_{t-q} \quad (1)$$

where $\{\varepsilon_t\}$ is a white noise process and the coefficients α's and β's are constants such that the model is both stationary and invertible. Model (1) may be put as

$$A(L)X_t = B(L)\varepsilon_t \quad (2)$$

where $A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - ... - \alpha_p L^p$ and $B(L) = 1 + \beta_1 L + \beta_2 L^2 + ... + \beta_q L^q$ and L is the backward shift operator defined by $L^k X_t = X_{t-k}$. For stationarity and invertibility it is necessary that the zeroes of A(L) and B(L) be outside the unit circle respectively.

Suppose that $\{X_t\}$ is non-stationary. Box and Jenkins(1976) proposed that differencing of the series to a sufficient order could make it stationary. Let this minimum order be d and let $\nabla^d X_t$ be the d^{th} difference of X_t. Then $\nabla = 1-L$. If the differences $\{\nabla^d X_t\}$ follow an ARMA(p, q) model, the original series is said to follow an *autoregressive integrated moving average model of orders p, d and q*, designated ARIMA(p, d, q).

Suppose that the time series $\{X_t\}$ is seasonal of period s. Box and Jenkins(1976) proposed further that it be modelled by

$$A(L)\Phi(L^s)\nabla^d \nabla_s^D X_t = B(L)\Theta(L^s)\varepsilon_t \quad (3)$$

where A(L) and B(L) are the non-seasonal autoregressive(AR) and moving average(MA) operators as earlier defined, respectively. $\Phi(L)$ and $\Theta(L)$ are respectively the seasonal AR and MA operators, say of orders P and Q, their coefficients being such that the model is both stationary and invertible. Also $\nabla_s = 1-L^s$. Then the original time series $\{X_t\}$ is said to follow a *multiplicative (p, d, q)x(P, D, Q)$_s$ seasonal autoregressive integrated moving average(SARIMA) model.*

Surhatono(2011) has differentiated between the subset, the multiplicative and the additive SARIMA models. According to him, for a seasonal series of period s, a multiplicative SARIMA model is of the form

$$\nabla^d \nabla_s^D X_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_s \varepsilon_{t-s} + \beta_{s+1} \varepsilon_{t-s-1} \quad (4)$$

where $\beta_{s+1} = \beta_1\beta_s$. Otherwise (4) is a subset model. On the other hand the corresponding additive model is given by

$$\nabla^d\nabla_s^D X_t = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_s\varepsilon_{t-s} \qquad (5)$$

METHODOLOGY

The data for this work are monthly Naira-CFA Franc exchange rates from January 2004 to June 2013 published under the Data and Statistics heading of the Central Bank of Nigeria publication in its website www.cenbank.org. It is to be interpreted as the amount of Naira per CFA Franc.

Time series analysis invariably starts with the time plot and the correlogram. Certain characteristics of the series may become apparent from their visual inspection. For instance seasonality of period s is suggestive by a significant spike at lag s in the autocorrelation function (ACF). Moreover if the spike is negative, the involvement of a seasonal MA component is suggestive; if positive, the involvement of a seasonal AR component is suggestive. Spikes along the partial autocorrelation function (PACF) indicate the presence of AR components; seasonal components for seasonal lags and non-seasonal components for non-seasonal lags. The seasonal orders P and Q could be determined by the highest seasonal lags for which the spikes are significant. Similarly the non-seasonal orders p and q may be estimated by the non-seasonal cut-off lags of the PACF and the ACF, respectively.

Often a time series is non-stationary. If it is seasonal of period s, it is differenced seasonally and then non-seasonally. That is, D = d = 1. This is likely to be sufficient to rid the series of any non-stationarity. That means that the series $\nabla\nabla_s X_t$ is stationary. At each stage, stationarity may be tested by the Augmented Dickey Fuller (ADF) Test.

After the orders p, d, q, P, D, Q and s have been estimated, the parameters of the model (3) are estimated. This is invariably by non-linear optimization techniques since the model involves items of a white noise process. For pure AR or pure MA models, there exist linear optimization techniques (see, Oyetunji, 1985). For the mixed ARMA models efforts have been made to propose linear techniques albeit with limited success (see, for instance, Etuk,1989).

After model fitting, the model is subjected to diagnostic checking to ensure its adequacy. This is done by analysing its residuals. If the model is adequate, the residuals should be uncorrelated as well as possess a normal distribution with zero mean.

For this work use was made of the software Eviews for every aspect of data analysis. This software is based on the least error sum of squares optimization criterion for model estimation.

RESULTS AND DISCUSSION

For the purpose of this work we shall be referring to the realisation of the time series as NXER, to its seasonal (i.e. 12-monthly) differences as SDNXER and to its further non-seasonal differences as DSDNXER. The time-plot of NXER in Figure 1 shows an overall upward trend with no obvious seasonality. That of SDNXER in Figure 2 shows an overall horizontal trend and still with no clear seasonality. The plot of DSDNXER in Figure 3 also shows an overall horizontal trend with no clear seasonality.

With 1%, 5% and 10% critical values of -3.5, -2.9 and -2.6 respectively, the Augmented Dickey Fuller Unit-Root Test shows that with a statistic of -1.7, NXER is non-stationary; with statistics of -3.9 and -4.0 respectively SDNXER and DSDNXER are stationary. However the correlogram of SDNXER gives a contrary impression of non-stationarity.

The correlogram of DSDNXER in Figure 4 has a significant negative spike at lag 12, an indication of a 12-monthly seasonality and the presence of a seasonal MA component of order one. Moreover, the significant spike at lag 12 on the PACF is suggestive of the involvement of a seasonal AR component of order one. However the high AR order of 12 may be replaced by a low MA order following the AR-MA duality relationship. We hereby propose the additive SARIMA model

$$DSDNXER = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_{12}\varepsilon_{t-12}$$

which has been estimated as summarized in Table 1 as

$$DSDNXER = \varepsilon_t + 0.0799\varepsilon_{t-1} - 0.8142\varepsilon_{t-12} \qquad (6)$$
$$(\pm 0.0317) \quad (\pm 0.0002)$$

The model might be said to be adequate on the following grounds:

1. The model closely agrees with the data as evident in Figure 5.
2. The correlogram of the residuals in Figure 6 shows that they are uncorrelated.
3. The Jarque-Bera normality test of Figure 7 shows that the residuals are normally distributed with zero mean.

CONCLUSION

The monthly Naira-CFA Franc exchange rates follow the additive SARIMA model (6). This model has been shown to be adequate in describing the variation in the time series.

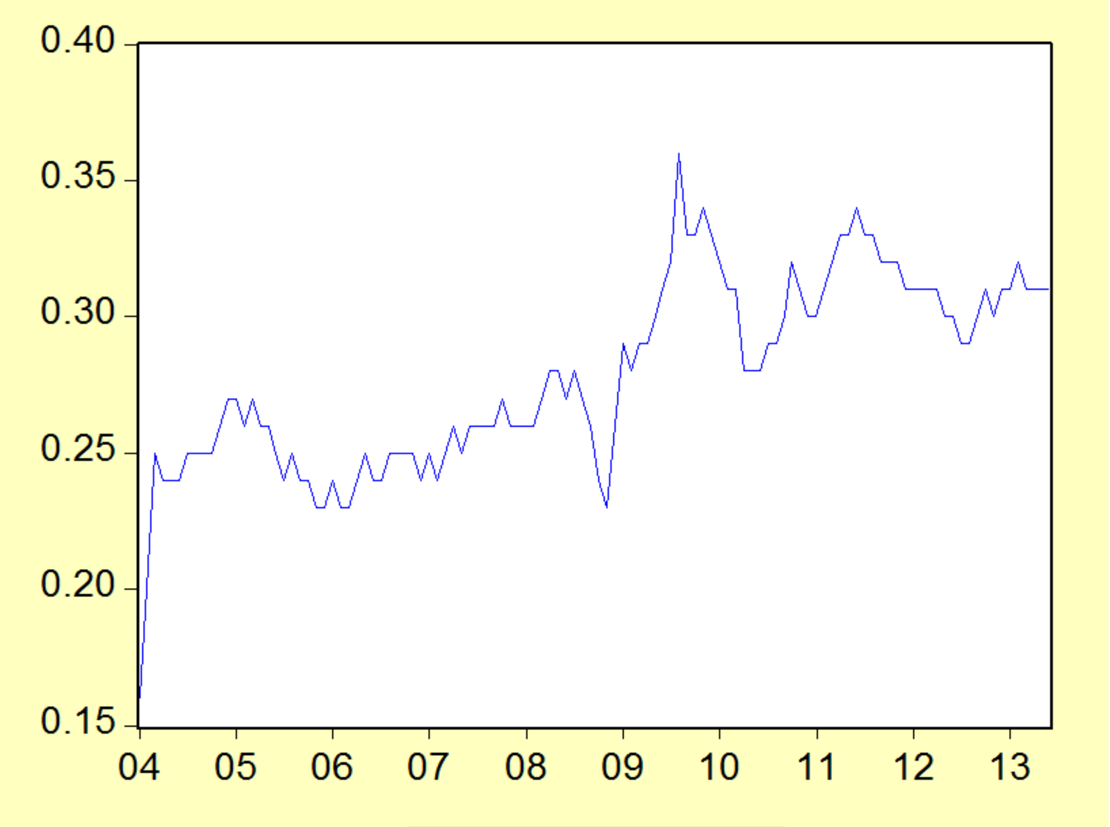


Figure 1: NXER

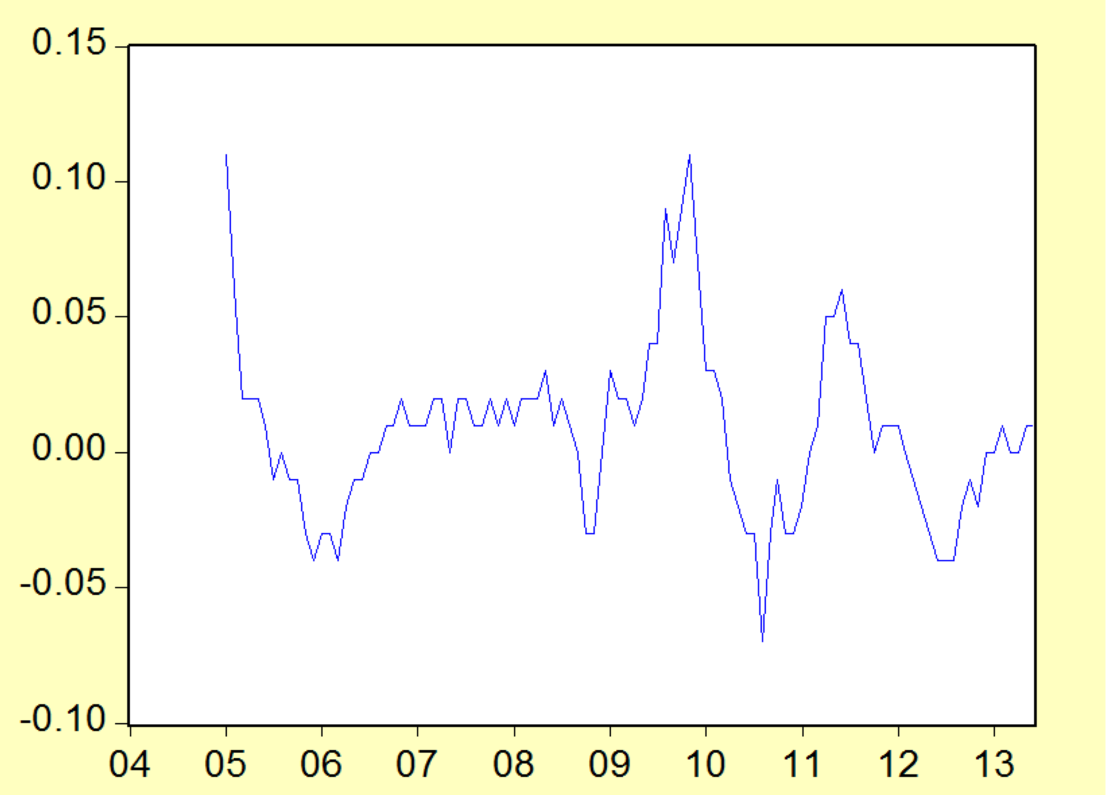


Figure 2: SDNXER

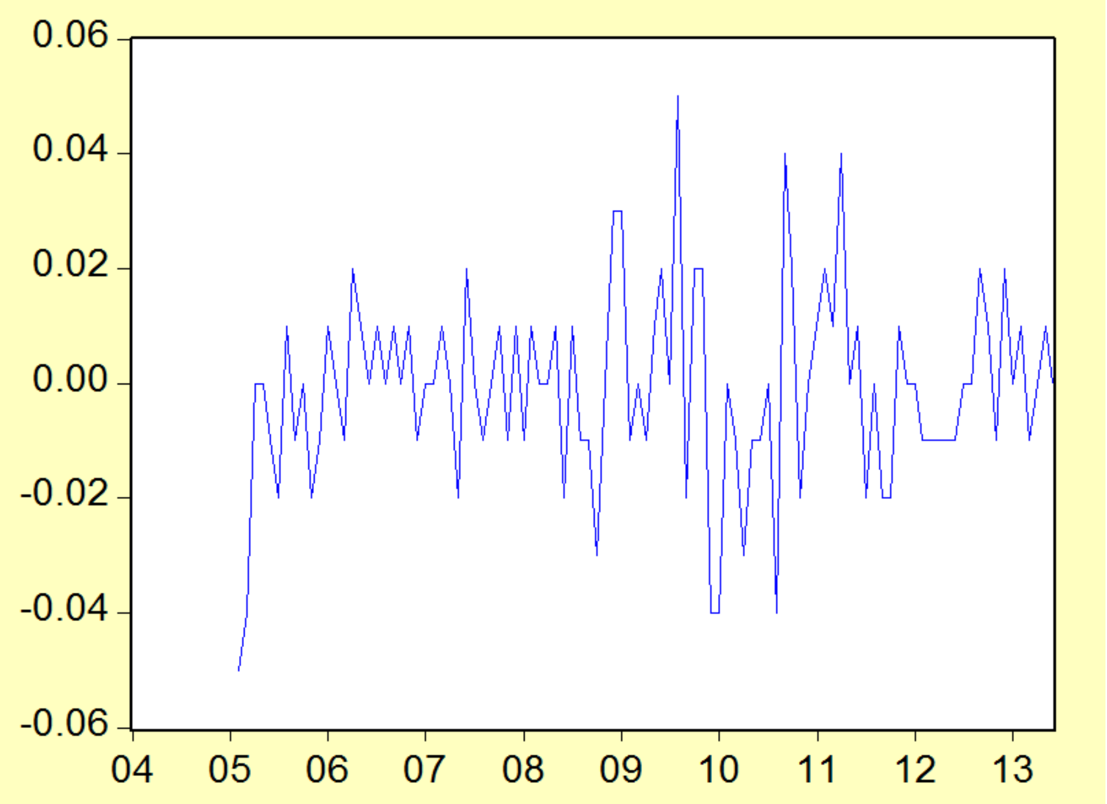


Figure 3: DSDNXER

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.066	0.066	0.4543	0.500
		2	0.016	0.012	0.4819	0.786
		3	0.091	0.089	1.3562	0.716
		4	0.084	0.073	2.1179	0.714
		5	-0.041	-0.054	2.3016	0.806
		6	-0.119	-0.125	3.8394	0.698
		7	0.091	0.095	4.7482	0.691
		8	-0.058	-0.066	5.1231	0.744
		9	-0.099	-0.068	6.2299	0.717
		10	-0.129	-0.121	8.1464	0.615
		11	0.050	0.059	8.4409	0.673
		12	-0.428	-0.445	29.837	0.003
		13	-0.034	0.111	29.976	0.005
		14	-0.088	-0.171	30.901	0.006
		15	-0.034	0.068	31.041	0.009
		16	-0.088	-0.130	31.996	0.010
		17	0.020	0.134	32.047	0.015
		18	0.213	0.030	37.749	0.004
		19	-0.075	0.023	38.461	0.005
		20	0.158	0.111	41.680	0.003
		21	0.088	0.029	42.681	0.003
		22	0.207	0.109	48.308	0.001
		23	-0.118	-0.133	50.178	0.001
		24	0.011	-0.185	50.194	0.001
		25	-0.040	-0.110	50.415	0.002
		26	0.095	0.085	51.668	0.002
		27	0.016	0.023	51.703	0.003
		28	-0.008	0.005	51.712	0.004
		29	0.045	0.011	52.006	0.005
		30	-0.176	-0.040	56.522	0.002
		31	0.008	-0.026	56.531	0.003
		32	-0.083	0.160	57.571	0.004
		33	-0.054	-0.138	58.011	0.005
		34	0.110	0.144	60.212	0.004

FIGURE 4: CORRELOGRAM OF DSDNXER

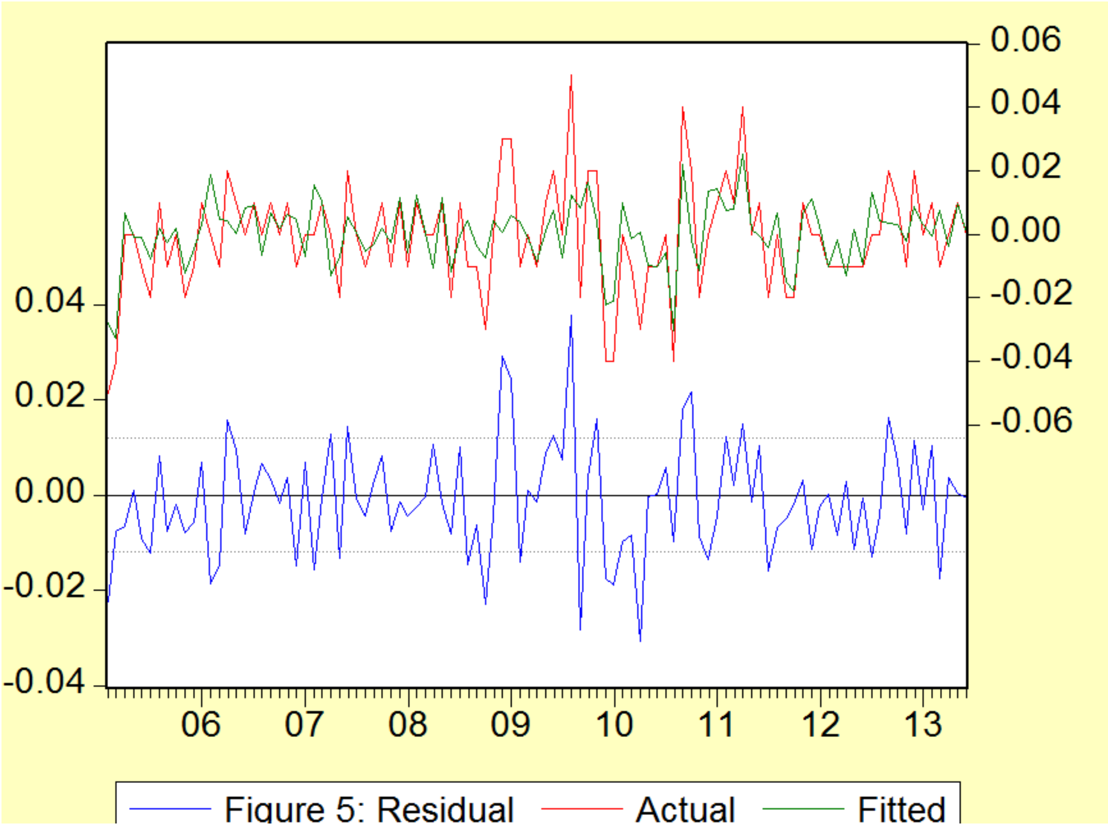
TABLE 1: MODEL ESTIMATION

Dependent Variable: DSDNXER
Method: Least Squares
Date: 08/14/13 Time: 19:49
Sample(adjusted): 2005:02 2013:06
Included observations: 101 after adjusting endpoints
Convergence achieved after 10 iterations
Backcast: 2004:02 2005:01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
MA(1)	0.079909	0.031705	2.520396	0.0133
MA(12)	-0.814177	0.000162	-5025.320	0.0000

R-squared	0.520493	Mean dependent var	-0.000990
Adjusted R-squared	0.515649	S.D. dependent var	0.017176
S.E. of regression	0.011954	Akaike info criterion	-5.995967
Sum squared resid	0.014146	Schwarz criterion	-5.944183
Log likelihood	304.7963	F-statistic	107.4619
Durbin-Watson stat	2.040259	Prob(F-statistic)	0.000000

Inverted MA Roots	.98	.84 -.49i	.84+.49i	.48 -.85i
	.48+.85i	-.01+.98i	-.01 -.98i	-.50+.85i
	-.50 -.85i	-.86+.49i	-.86 -.49i	-.99


0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
0.04
0.02
0.00
-0.02
-0.04
06
07
08
09
10
11
12
13
Figure 5: Residual
Actual
Fitted

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.046	-0.046	0.2248	
		2	-0.075	-0.077	0.8099	
		3	0.050	0.044	1.0805	0.299
		4	0.023	0.022	1.1368	0.566
		5	-0.032	-0.023	1.2492	0.741
		6	-0.147	-0.151	3.6261	0.459
		7	0.094	0.076	4.5996	0.467
		8	0.005	-0.006	4.6021	0.596
		9	-0.154	-0.132	7.2909	0.399
		10	-0.109	-0.133	8.6546	0.372
		11	0.098	0.063	9.7650	0.370
		12	-0.127	-0.149	11.653	0.309
		13	-0.081	-0.054	12.422	0.333
		14	-0.082	-0.140	13.223	0.353
		15	0.059	0.003	13.640	0.400
		16	-0.066	-0.100	14.172	0.437
		17	-0.013	0.022	14.192	0.511
		18	0.152	0.059	17.089	0.380
		19	-0.029	-0.053	17.192	0.441
		20	0.185	0.197	21.574	0.251
		21	-0.019	-0.016	21.621	0.304
		22	0.146	0.120	24.445	0.223
		23	-0.055	-0.080	24.851	0.254
		24	-0.050	-0.017	25.195	0.288
		25	-0.065	-0.149	25.774	0.312
		26	0.075	0.103	26.545	0.326
		27	0.099	0.104	27.912	0.312
		28	0.033	0.114	28.067	0.355
		29	-0.028	-0.033	28.184	0.402
		30	-0.160	-0.115	31.939	0.277
		31	0.028	0.005	32.057	0.317
		32	-0.082	0.011	33.062	0.320
		33	-0.045	-0.048	33.367	0.353
		34	-0.078	-0.050	34.313	0.357
		35	0.100	0.080	35.884	0.335
		36	-0.067	-0.035	36.607	0.349

FIGURE 6: CORRELOGRAM OF THE RESIDUALS

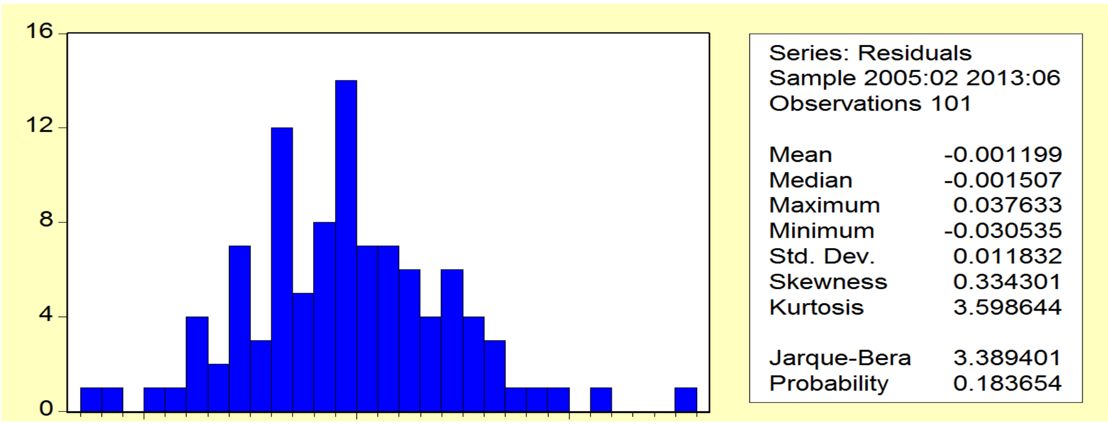


Figure 7: Histogram of the Residuals

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