A SIMPLE RULE TO DISTINGUISH PRIME FROM COMPOSITE NUMBERS

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ABSTRACT: *In this paper, it was tried to investigate the characteristics of prime and composite numbers. In theorem (1), it was proved that using the natural numbers a, m and n, prime and composite numbers will be distinguished from each other. In theorem (2), it was proved that if x and y are odd natural numbers, then, x and y can be defined using m and n.*

KEYWORDS: Prime Numbers, Composite Numbers

INTRODUCTION

For more than two millennia, the topic of prime numbers and the related issues is one of the most important topics in mathematics and has caught the attention of many great mathematicians. Finding a solution for calculating P_{m+1} from P_m, in which P_m is the mth prime number, is still unresolved. The distribution of prime numbers is so disorderly that has led to many errors on the part of mathematicians. According to [1]:

"A natural question seems illogical after investigation. For example: Is there a general rule for calculating the mth prime number? The answer to this question is as follows: Although this is a natural and acceptable question, it is quite illogical."Though many theorems about prime numbers have been proven, the most well-known of which is Wilson theorem and its inverse, but it can be said that the findings about prime numbers are insignicant compared to the issues that have not been resolved. The present study tries to throw some light on the theory of numbers.

Theorem (1):
Suppose that a is an odd natural number and $a > m > n > 2$. If m and n are found to be natural

numbers, so that $a^2 - (a-n)^2 = (a-1)^2 - (a-1-m)^2$.

Then, $2a - 1$ is a composite number; otherwise, it is a prime.

Proof:

After simplification, we will have:

$$a^2 - (a-n)^2 = (a-1)^2 - (a-1-m)^2$$

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$$a^2 - (a-1)^2 = (a-n)^2 - (a-1-m)^2$$

$$2a - 1 = (2a - 1 - m - n)(m - n + 1)$$

As it was observed, $2a - 1$ is a product of the two terms. Thus, it is a composite number. It can be concluded that if the equation is not satisfied, $2a - 1$ is a prime and the theorem is proved.

Numerical example:

1) Is 91 a prime or composite? $2a - 1 = 91$. Therefore, $a = 46$.

   There are numbers like $n = 36$ and $m = 42$ which satisfy the following equation:

$$a^2 - (a-n)^2 = (a-1)^2 - (a-1-m)^2$$
$$(46)^2 - (46-36)^2 = (46-1)^2 - (46-1-42)^2$$
$$2016 = 2016$$

   Based on theorem (1), it can be concluded that 91 is a composite number. If we assume that $xy = 91$, then x and y can be found. In theorem (2), in the following section, it will be proved that $x = \dfrac{m+n}{m-n}$ and $y = m - n + 1$.

   Therefore, $x = \dfrac{42+36}{42-36} = 13$ and $y = 42-36+1 = 7$. Thus, $7 \times 13 = 91$.

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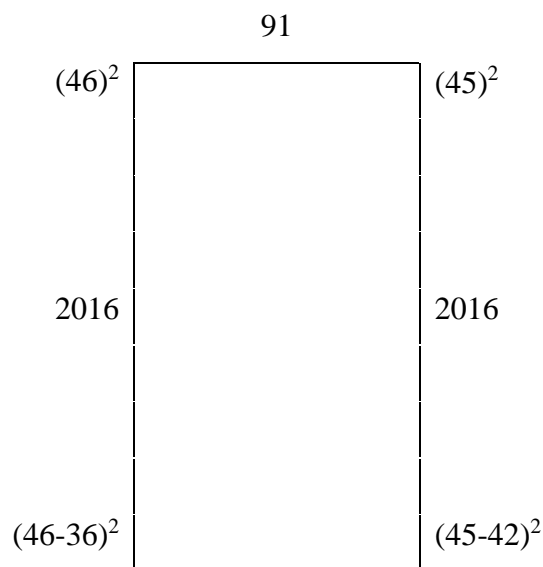
2) Is 31 a prime or composite?

   Since natural numbers such as m and n are not found to satisfy the equation, thus we'll have the following inequality:

   $$a^2 - (a-n)^2 \neq (a-1)^2 - (a-1-m)^2$$

   Therefore, 31 is a prime.

In the next part, some interesting results drawn from theorem (1) will be presented.

Theorem (2)

Suppose that x and y are odd natural numbers, m and n are also natural numbers and x, y, m, $n > 1$, $m > n$. Then, x and y have a common structure of m and n, so that:

$$x = \frac{m+n}{m-n} \text{ and } y = m-n+1$$

Proof:

In theorem (1), it was proved that

$$2a - 1 = (2a - 1 - m - n)(m - n + 1)$$

Now, we replace $2a - 1$ with xy which is an odd composite number. Then,

$$xy = (xy - m - n)(m - n + 1)$$

In that case, $x = (xy - m - n)$ and $y = (m - n + 1)$.

Then, $xy - x = m + n$ and $x = \dfrac{m+n}{y-1}$.

Now, we replace y with $m - n + 1$. Therefore, $x = \dfrac{m+n}{m-n}$. Thus, theorem (2) is proved.

Other findings of theorem (2):

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The other equations drawn from theorem (2) are as follows:

$$xy = m + n + x$$

$$2m = xy - (x - y) - 1$$

$$2m = xy - (x - y) - 1$$

$$2n = xy - (x + y) + 1$$

$$\frac{m}{n} = \frac{x+1}{x-1}$$

$$\frac{m}{n} = \frac{xy - n + 1}{xy - m - 1}$$

$$2m = (x+1)(y-1)$$

$$2n = (x-1)(y-1)$$

$$m^2 - n^2 = (xy - 1)\,\mathrm{m} - (\mathrm{xy}+1)n$$

$$m^2 - n^2 \neq (P - 1)\,\mathrm{m} - (P + 1)n$$

The following numerical examples are based on the following equations which were proved in theorem (2).

$$x = \frac{m+n}{m-n} \quad \text{and} \quad y = m - n + 1.$$

If $x = 13$ and $y = 7$, then $m - n = 6$ and $m + n = 13 \times 6$, thus $m = 42$ and $n = 36$.

If $x = 31$ and $y = 17$, then $m - n = 16$ and $m + n = 31 \times 16$, thus $m = 256$ and $n = 240$.

If $x = 103$ and $y = 11$, then $m - n = 10$ and $m + n = 103 \times 10$, thus $m = 520$ and $n = 510$.

By investigating the abovementioned equations, it can be concluded that m is divisible by $m-n$ and n is also divisible by $m-n$.

CONCLUSION

Based on what was mentioned in the previous parts, a computerized model to distinguish prime from composite numbers can be presented. If $x, y, a, b > 1$ and natural numbers, therefore, according to theorem (1), for odd composite numbers, it can be concluded that:

$$\underline{xy} + (xy-2) + (xy-4) + ... + (xy-2a) = (xy-2) + (xy-4) + ... + (xy-2a) + \underline{(xy-2a-2)} + \underline{+(xy-2a-4) + ... + (xy-2a-2b)}$$

In this relation, $\underline{21}$ is taken as the numerical example:

$\underline{21} + 19 + 17 + 15 + 13 + 11 = 19 + 17 + 15 + 13 + 11 + \underline{9+5+7}$

The following inequality can be drawn from theorem (1). We assume that P is a prime:

$$P + (P-2) + (P-4) + ... \neq (P-2) + (P-4) + (P-6) + ...$$

In this relation, the prime number 23 is taken as numerical example:

$23 + 21 + 19 + 17 + ... \neq 21 + 19 + 17 + 15 + ...$

Now, using computer and a program appropriate for the abovementioned model, prime and composite numbers can be distinguished.

2) Regarding theorem (2), the conclusion is that for any odd composite number like xy, there will be natural numbers like m and n which will satisfy the following equations:

$$x = \frac{m+n}{m-n} \quad \text{and} \quad y = m - m + 1.$$

In other words, it can be claimed that x and y have a common structure like m and n.

References

[1] Hardy, G.H. and Wright, E.M. An Introduction to the Theory of Numbers. Oxford: Claremond Press, 1960