A NEW PROPOSED RANKING FUNCTION FOR SOLVING FUZZY GAMES PROBLEMS

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ABSTRACT: *In this paper, we deal with games of fuzzy payoffs problems while there is uncertainty in data. We use the trapezoidal membership function which make the data fuzziness and utilize the new proposed ranking function algorithm by using trapezoidal fuzzy numbers for the decision maker to get the best gains*.

KEYWORDS: Fuzzy numbers, Fuzzy game problem, Trapezoidal membership function, New proposed ranking function.

INTRODUCTION

The theory of games is a mathematical theory that deals with general features of competitive situations [4]. It is usually used when two or more individuals or organizations with conflicting objectives try to make decisions, it's based on the minimax-principle [8]. The set of objective functions in the game may have uncertain values where the way to deal with uncertainty is to use the concept of fuzzy games [6]. A ranking function is used which helps us not just to find the solution but also to find the best gain for the fully fuzzy game problem.

Many authors studied fuzzy games some of them with ranking functions to solve the game problem. Aristuidou. M, Sarangi.S at (2005) represented a non-cooperative model of a normal form game using tools from fuzzy set theory [1]. Gao. J at (2007) represented a strategic game with fuzzy payoffs [3]. Medinechiene. M, Zavadskas. E. K, Turskis. Z at (2011) described a model of dwelling selection using fuzzy games theory on buildings [10]. Jawad. M. A at (2012) represented fuzzy sets and fuzzy processes with game theory to address the uncertainty in data for mobile phone companies in Iraq [6]. Kumar. R. S, Kumaraghura. S at (2015) represented a solution of fuzzy game problem with triangular fuzzy numbers using a ranking function to compare the fuzzy numbers [7]. Selvakumari. K, Lavanya. S at (2015) considered a two zero sum game with imprecise (triangular or trapezoidal) fuzzy numbers using ranking function for an approach to solve the problem [12]. Kumar. R. S, Gnanaprakash. K at (2016) represented a (3×3) two zero sum game with octagonal fuzzy payoffs using ranking function to solve the fuzzy game [8].

The objective of this paper is to propose a new algorithm depending on a ranking function to solve the fuzzy game problem using trapezoidal fuzzy numbers and trying to get a desirable gain. This paper contains five sections: in section two a review of some fuzzy theory concepts, section three defines the fuzzy games problem and its formula, section four represents the ranking function and some properties and anew proposed algorithm, finally in section five a numerical example is represented depending on the new proposed algorithm.

Concept of Fuzzy Set Theory

In this section we will introduce some definitions of fuzzy set theory.

Fuzzy Set [8]

Let X be a non empty set, a fuzzy set in X is characterized by its membership function A → $[0, 1]$ and $A_{(x)}$ is interpreted as the degree of membership of element X in fuzzy A for each $x \in X$.

α-Cut Set (α-Level Set) [13]

The crisp set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α-level set

$$A_\alpha = \{ x \in X / \mu_{\tilde{A}} \geq \alpha \}$$

$$\acute{A}_\alpha = \{ x \in X / \mu_{\tilde{A}} > \alpha \} \text{ is called strong } \alpha\text{-level set or strong } \alpha\text{-cut.}$$

Convex Fuzzy Set [13]

A fuzzy set $\tilde{A}$ is convex if

$$\mu_{\tilde{A}} [\, \alpha x_1 + (1-\alpha) x_2] \geq \min\{ \mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)$$

$$x_1, x_2 \in X, \alpha \in [0, 1].$$

Normality [2]

A fuzzy set is called normal if h (A) = 1. A nonempty fuzzy set can always be normalized by dividing $A_{(x)}$ by hgt (A).

Fuzzy Number [12]

A fuzzy number $\tilde{A}$ is a fuzzy set on the real line R, must satisfy the following conditions:

1- There exist at least one $x_0 \in R$ with $\mu_{\tilde{A}} = 1$.

2- $\mu_{\tilde{A}_{(x)}}$ is piecewise continuous.

3- $\tilde{A}$ must be normal and convex.

Trapezoidal Fuzzy Number [11]

A fuzzy number $\tilde{A} = (a, b, c, d; 1)$ is said to be a fuzzy trapezoidal fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}_{(x)}} = \begin{cases} \frac{(x-a)}{(b-a)} & , \quad a \leq x \leq b \\ 1 & , \quad b \leq x \leq c \\ \frac{(d-x)}{(d-c)} & , \quad c \leq x \leq d \\ 0 & , \quad \text{otherwise} \end{cases}$$

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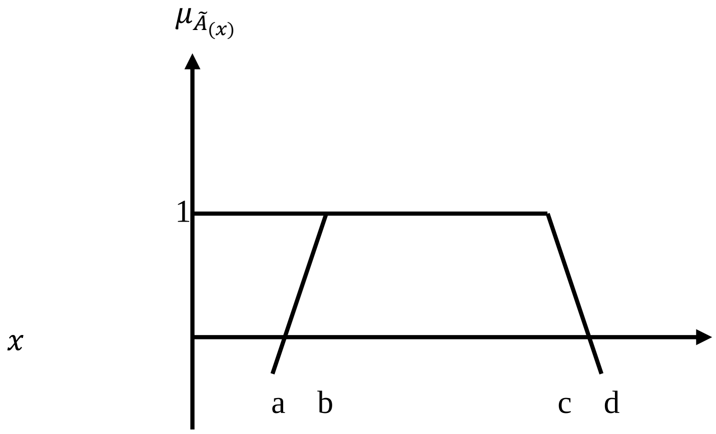

Membership of trapezoidal fuzzy number

Fuzzy Game Problem

The fuzzy game problem is where all the payoffs of the game matrix are fuzzy quantities. Now the formula of the fully fuzzy game problem is as follows:

Player B

$$\text{Player A}\quad \begin{array}{c} \\ 1\\2\\3\\ \vdots\\ i\\ \vdots\\ m \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3\ \dots & j\ \dots & n \end{array} \\ \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13}\ \dots & \tilde{a}_{1j}\ \dots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23}\ \dots & \tilde{a}_{2j}\ \dots & \tilde{a}_{2n} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33}\ \dots & \tilde{a}_{3j}\ \dots & \tilde{a}_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{i1} & \tilde{a}_{i2} & \tilde{a}_{i3}\ \dots & \tilde{a}_{ij}\ \dots & \tilde{a}_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \tilde{a}_{m3}\ \dots & \tilde{a}_{mj}\ \dots & \tilde{a}_{mn} \end{bmatrix} \end{array}$$

Ranking Function

If F(R) is the set of all fuzzy numbers defined on R the set of real numbers then a ranking function

R: F(R) → R maps each fuzzy number into a real ordinary number where there is a natural order, the order rules are as follows [5]:

- $\tilde{A} > \tilde{B}$ if and only if R ($\tilde{A}$) > R ($\tilde{B}$).
- $\tilde{A} = \tilde{B}$ if and only if R ($\tilde{A}$) = R ($\tilde{B}$).
- $\tilde{A} < \tilde{B}$ if and only if R ($\tilde{A}$) < R ($\tilde{B}$).

Where A and B are two fuzzy numbers belong to F(R).

A New proposed Algorithm

We use the previous trapezoidal membership function and by the α- cut, $\alpha \in [0, 1]$ then

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$\frac{x-a}{b-a} = \alpha \Rightarrow x = \alpha(b-a) + a = \inf \tilde{A}_{(\alpha)}$

$\frac{d-x}{d-c} = \alpha \Rightarrow x = d - \alpha(d-c) = \sup \tilde{A}_{(\alpha)}$

Applying the following ranking function R $(\tilde{A})$ = $\int_0^1 [\, k \inf \tilde{A}_{(\alpha)} + (1-k) \sup \tilde{A}_{(\alpha)}]\, d_\alpha, k \in [0,1]$ then

R $(\tilde{A})$ = $\int_0^1 k\,[\,\alpha(b-a) + a\,] d_\alpha + \int_0^1 (1-k)[\, d - \alpha(d-c)] d_\alpha$

R $(\tilde{A})$ = $k\,[\frac{\alpha^2}{2}\,(b-a) + \alpha a\,]_0^1 + (1-k)[\, d\alpha - \frac{\alpha^2}{2}\,(d-c)]_0^1.$

R $(\tilde{A})$ = $k\,\left[\frac{1}{2}\,b - \frac{1}{2}\,a + a\right] + (1-k)\,\left[d - \frac{1}{2}\,d + \frac{1}{2}\,c\right].$

R $(\tilde{A})$ = $k\,\left[\frac{1}{2}\,a + \frac{1}{2}\,b\right] + (1-k)\left[\frac{1}{2}\,c + \frac{1}{2}\,d\right].$

R $(\tilde{A})$ = $\frac{k}{2}\,a + \frac{k}{2}\,b + \frac{c}{2} + \frac{d}{2} - \frac{k}{2}\,c - \frac{k}{2}\,d.$

R $(\tilde{A})$ = $\frac{1}{2}\,(c+d) + \frac{k}{2}\,(a+b-c-d).$

Numerical Examples

This example of two game matrices may clarify the new proposed ranking algorithm.

Example (1) [4]:

Find the value of the game for the following payoff matrix.

$$
\begin{array}{ccc}
 & & \text{Player B} \\
 & & \begin{array}{cc} 1 & \quad 2 \end{array} \\
\text{Player A} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 18 & -12 \\ 6 & 3 \\ -12 & 15 \\ 5 & 2 \end{bmatrix}
\end{array}
$$

Which represents the crisp payoff game matrix.

We transform the crisp game problem for the payoff matrix to fuzzy game problem and then solve it by applying the new proposed algorithm. Consider the payoff matrix as trapezoidal fuzzy numbers.

Let Δ_1= 0.5 and Δ_2= 1.5 for R_1 *and* R_4 , Δ_1= 0.4 and Δ_2= 1.6 for R_2, Δ_1= 0.25 and Δ_2= 1.75 for R_3 where

$R_i = (a_{ij} - 1, a_{ij} + 1, \Delta_1, \Delta_2)$ for $i = 1,2,3,4$ *and* $j = 1, 2.$

$$\begin{array}{cc} & \text{Player B} \\ & \begin{array}{cc} 1 & 2 \end{array} \\ \text{Player A} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} (17,19,0.5,1.5) & (-13,-11,0.5,1.5) \\ (5,7,0.4,1.6) & (2,4,0.4,1.6) \\ (-13,-11,0.25,1.75) & (14,16,0.25,1.75) \\ (4,6,0.5,1.5) & (1,3,0.5,1.5) \end{bmatrix} \end{array}$$

Applying the new proposed ranking algorithm R $(\tilde{A}) = \frac{1}{2}(c+d) + \frac{k}{2}(a+b-c-d), k \in [0,1]$ for the fuzzy payoff matrix. Stating all the values of k neglecting (0) using principle of dominance and method of sub game where each single sub game is solved with probabilities represented in table (1-1) as follows:

Table (1-1) represents the solution of the new proposed algorithm for example (1)

(k)	Values of the game
0.1	1.32
0.2	1.6
0.3	1.96
0.4	2.28
0.5	2.2.6
0.6	2.92
0.7	3.24
0.8	3.56
0.9	3.88
1	4.2

The best shrinkage constant (k) that gives the best gains is when $k \geq 0.8$, where player A when $k \in 0.8$ with an optimal strategy for A (0, 0.9, 0.1, 0) and for player B (0.4, 0.6).

Example (2) [4]:

Find the value of the game for the following payoff matrix.

$$\begin{array}{cc} & \text{Player D} \\ & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{Player C} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} -4 & -2 & -2 & 3 & 1 \\ 1 & 0 & -1 & 0 & 0 \\ -6 & -5 & -2 & -4 & 4 \\ 3 & 1 & -6 & 0 & -8 \end{bmatrix} \end{array}$$

Which represents the crisp payoff game matrix.

We transform the crisp game problem for the payoff matrix to fuzzy game problem and then solve it by applying the new proposed algorithm. Consider the payoff matrix as trapezoidal fuzzy numbers.

Let $\Delta_1 = 0.5$ and $\Delta_2 = 1.5$ for R_1 *and* R_4 , $\Delta_1 = 0.4$ and $\Delta_2 = 1.6$ for R_2, $\Delta_1 = 0.25$ and $\Delta_2 = 1.75$ for R_3 where

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$R_i = (a_{ij} - 1, a_{ij} + 1, \Delta_1, \Delta_2)$ for $i = 1,2,3,4$ *and* $j = 1, 2, 3,4,5.$

$$
\begin{array}{cc}
 & \text{Player D} \\
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\text{Player C} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} &
\begin{bmatrix}
(-5,-3,0.5,1.5) & (-3,-1,0.5,1.5) & (-3,-1,0.5,1.5) & (2,4,0.5,1.5) & (0,2,0.5,1.5) \\
(0,2,0.5,1.5) & (-1,1,0.4,1.6) & (-2,0,0.4,1.6) & (-1,1,0.4,1.6) & (-1,1,0.4,1.6) \\
(-7,-5,0.25,1.75) & (-6,-4,0.25,1.75) & (-3,-1,0.25,1.75) & (-5,-3,0.25,1.75) & (3,5,0.25,1.75) \\
(2,4,0.5,1.5) & (0,2,0.5,1.5) & (-7,-5,0.5,1.5) & (-1,1,0.5,1.5) & (-9,-5,0.5,1.5)
\end{bmatrix}
\end{array}
$$

Stating all the values of k neglecting (0) where the matrices are solved with pure strategy represented in table (1-2) as follows:

Table (1-2) represents the solution for the new proposed algorithm for example (2)

(k)	Values of the game
0.1	0.8
0.2	0.6
0.3	0.4
0.4	0.2
0.5	0
0.6	-0.2
0.7	-0.4
0.8	-0.6
0.9	-0.8
1	-1

The best shrinkage constant (k) that gives the best gains is when $k \leq 0.4$, where player C is losing for player D with an optimal strategy for C (0, 1, 0, 0) and for D (0, 0, 1, 0, 0) and a saddle point (2, 3).

CONCLUSION

The value of the game for the first example is v = 4.2 when $k = 1$, the value of the game for the second example is v = 0.8 when $k = 0.1$.

Then the new proposed algorithm will be helpful for the decision maker when he deals with a fuzzy game problem to get the best gain.

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