A COMPARATIVE ANALYSIS OF MULTI-CRITERIA ROAD NETWORK

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ABSTRACT: Travel Optimization is a method to find the best route to be followed for a journey between two points. This optimization can be based on factors one considers important while traveling. The work presented deals with the design of a multi-criteria based traffic network evaluation technique, using a variety of methods to calculate the cost of the feasible set of solutions, and then to decide which path is most desirable for a given requirement domain. Subsequently, various forms of the desired results e.g. the global depths of all the nodes in a given road network are extracted and presented; and compared by implementing the different methods of cost calculation: a weight based method and a Fuzzy Logic based mechanism. Both methods used are explained and compared through a case study to show that the aims are met in a useful way. Also among the aims of this research is to work out some way of solving the multi objective optimization problem of finding the shortest path using some optimization methods for given multiple objectives. The phenomenon of multiple criterions tells about the nature of the problem where increasing one or more objectives reduce effect of the rest of the criteria or criterions, which effectively, makes the solution more complex but comes out with better results and provides more knowledge of the problem itself.

KEYWORDS: Travel optimization, road network analysis, multi-criteria optimization, fuzzy logic, Pareto optimization

INTRODUCTION

There is enough social evidence to believe that travel optimization, whether systematic or just an individual’s thinking act, has been sought in modern times, and even much before that. It can also be argued that the dominant driving factor behind such optimizations must have been ‘security’, a hundred years ago; today these factors can broadly be divided into two categories, i.e. Economic and Environmental. The factors to optimize can be as economic as the actual hard-dollar cost (as simple as the fare paid for a public travel) and as environmental as the smoke emitted by that vehicle. With more and more costs now being associated with traveling, the optimization of it is getting more and more important.

This work tries to extend the idea by the design of a multi criterion optimization approach which uses five different criterions. Requiring inputs of current location and destination, it will suggest the best route based on factors considered. This approach uses Dijkstra's Algorithm which was developed in 1959. Dijkstra's Algorithm is seen as the most effective method for solving the problem of finding
shortest-path between a given source and destination. Also, weighted graph is often used to solve similar shortest path problems, where the quantities such as distance, crime rate or time are represented using weights. Generally, the cost along a path is calculated is the sum of all such weights of a specific path attached to the actual values. The best path from node \( a \) to \( b \) is the path that has lowest weights for any path from node \( a \) to \( b \). As this approach of travel optimization looks at five different criterions, or simply the factors of optimization, at the same time in order to expand the requirement domain, this makes it usable for a large number of people having a wide spectrum of priorities in their journey.

**RELATED WORK**

Travel Optimization has been identified as a market pull of the near future and thus there has been considerable seminal work on travel optimization and different approaches have been used to search a path that suits best to one’s requirements and preferences.

**Shortest path calculation:**

Finding shortest path from a network is a crucial task when dealing with road networks and transportation related problems. In the recent years finding shortest paths and computing an optimal solution for a number of given criterions has been a hot issue and different research groups have added a lot to the area. Using best suited path finding algorithm is the most crucial step in the whole process of finding shortest paths. The study by Cherkassky *et al.* in 1992 is believed to be one of the most comprehensive evaluations of shortest path algorithms. Cherkassky evaluated a set of 17 shortest path algorithms on a SUN Sparc-10 workstation using C language. Cherkassky *et al.* tested these algorithms on various simulated networks with varying complexity. The results of this study suggest that no one algorithm works consistently well their simulated networks [1]. Hamid Ebadi [2] with a team of researchers evaluated four best shortest paths finding algorithms. Their study showed that the use of Dijkstra’s Algorithm does not always come up with the best solution but still its capabilities make Dijkstra’s Algorithm most commonly used shortest path finding algorithm. Hamid Ebadi with his team performed their analysis using 6 real road networks and tested 4 suggested algorithms.

1) Graph growth algorithm implemented with two queues
2) Dijkstra algorithm
3) Genetic algorithm
4) Dijkstra algorithm implemented with double buckets

Table 1. Results shown by Ebadi after the execution of all four algorithms[2].

<table>
<thead>
<tr>
<th>Condition</th>
<th>nodes</th>
<th>Execution time</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-to-1</td>
<td>( n &lt; 2000 )</td>
<td>Dijkstra &lt; DKD &lt; Graph Growth &lt; Genetic</td>
</tr>
<tr>
<td></td>
<td>( n \geq 2000 )</td>
<td>Dijkstra &lt; Graph Growth &lt; DKD &lt; Genetic</td>
</tr>
<tr>
<td>One-to-all</td>
<td>( n &lt; 500 )</td>
<td>Genetic &lt; Dijkstra &lt; Graph Growth &lt; DKD</td>
</tr>
<tr>
<td></td>
<td>500 &lt; ( n ) &lt; 2000</td>
<td>Dijkstra &lt; Graph Growth &lt; DKD &lt; Genetic</td>
</tr>
<tr>
<td></td>
<td>( n \geq 2000 )</td>
<td>Genetic &lt; Dijkstra &lt; Graph Growth &lt; DKD</td>
</tr>
<tr>
<td>All-to-all</td>
<td>( n &lt; 2000 )</td>
<td>Genetic &lt; Graph Growth &lt; DKD &lt; Dijkstra</td>
</tr>
<tr>
<td></td>
<td>( n \geq 2000 )</td>
<td>Graph Growth &lt; DKD &lt; Dijkstra &lt; Genetic</td>
</tr>
</tbody>
</table>
Ebadi in his study concluded that for small and less complex networks Dijkstra is the best choice, with small but more complex networks metahuristic algorithms perform best where with large and very complex networks heuristic algorithms perform better than any other as shown in Table1.

Another study by Zhan and Noon (1996) tested 15 shortest path algorithms on 21 real road networks. All 15 algorithms were coded in the C programming language. Zhan and Noon based their programs on the set of one-to-all shortest path C programs provided by Cherkassky et al. (1992). They modified one-to-all C programs into all-to-all shortest paths. The summary of all the algorithms implemented in their study are given in Table2.

Table 2: A list of algorithms used by Zhan F.B., and Noon for their evaluation[3]

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Implementation</th>
</tr>
</thead>
<tbody>
<tr>
<td>BFM</td>
<td>Bellman-Ford-Moore</td>
</tr>
<tr>
<td>BFP</td>
<td>Bellman-Ford-Moore with Parent-checking</td>
</tr>
<tr>
<td>DKQ</td>
<td>Dijkstra's Naive Implementation</td>
</tr>
<tr>
<td>DKB</td>
<td>Dijkstra's Buckets - Basic Implementation</td>
</tr>
<tr>
<td>DKA</td>
<td>Dijkstra's Buckets - Approximate</td>
</tr>
<tr>
<td>DKD</td>
<td>Dijkstra's Buckets - Double</td>
</tr>
<tr>
<td>DKH</td>
<td>Dijkstra's Heap - k-array</td>
</tr>
<tr>
<td>DKF</td>
<td>Dijkstra's Heap - Fibonacci</td>
</tr>
<tr>
<td>DKR</td>
<td>Dijkstra's Heap - R-Heap</td>
</tr>
<tr>
<td>PAP</td>
<td>Graph Growth - Pape</td>
</tr>
<tr>
<td>TQQ</td>
<td>Graph Growth with Two Queues - Pallottino</td>
</tr>
<tr>
<td>THR</td>
<td>Threshold Algorithm</td>
</tr>
<tr>
<td>GR1</td>
<td>Topological Ordering - Basic</td>
</tr>
<tr>
<td>GR2</td>
<td>Topological Ordering -- Distance Updates</td>
</tr>
</tbody>
</table>

Although considerable reported research has been done in the area of performance evaluation of shortest path algorithms, no one algorithm or a set of algorithms can be said to run fastest on real road networks in the literature. In our approach of finding multi criterion optimization solution we have Dijkstra algorithm which is most suitable for our case among all algorithms in the literature today.

**Fuzzy Logic and road networks:**

Fuzzy Logic is an analytical control system methodology that is used in a variety of systems ranging from simple and small embedded micro-controllers to large networked multi-channel PC or workstation based control systems [18]. Like all other areas of science and daily life, researchers have implemented a number of road network control systems using Fuzzy Logic. Akiyama T. (2000) in his work provided a travel optimization technique by introducing fuzzy or perceived travel time as shown in Fig1.
The author argued that the uncertainty involved in estimating travel time or other factors like travel cost, safety and comfort factors for travel problems can only be practically implemented using the fuzzy operators [4].

A research team lead by R Narayan (2002) proposed a Fuzzy logic based system for their approach to quantity road congestion. The team used two parameters, Speed and Inter Vehicular Distance (IVD) as the inputs to their suggested fuzzy model. Their approach combined the advantages of both GIS and fuzzy logic; hence, provided strong foundations for researchers to come up with new ideas in the future. Orthophotos (1:2000) were used to assist policy makers and traffic managers in Route Optimization and other network analysis. This work involved digitization of road networks and other characteristics like (Hospitals and Hotels etc.) from the traced map using Arch GIS software and fed in the format acceptable to the system. Additionally, the coverage was translated to Arc/Info format and effort was made to reduce the digitization errors. The tool developed in this work can use a number of different criterions as drive time, street length and traffic conditions etc allowing the user to consider more than just distance based routing facilities but to use additional real world information [5]. Our approach looks at even broader aspects and the criterions used e.g. crime rate in an area, pollution and Tourism attraction allow users to find the best path on a network where the users can actually suit their very needs. In another application of fuzzy logic in GIS and road network problems, Arie Croitoru and Yerach Doytsher (2002) used a combined paradigm of fuzzy logic and Monte Carlo simulation. This work, in particular, addresses the potential impacts of positional accuracy on fuzzy logic decision support systems. In this paper the utilization of a Monte Carlo simulation approach to a fuzzy logic decision making process was described and evaluated as a potential scheme that provides the ability to incorporate data uncertainty. In particular, the authors explored the application of this scheme for modeling uncertainty in a decision making process that is based on vector GIS data. Arie Croitoru and Yerach Doytsher (2002) further argued that the ability to incorporate uncertainties into a spatial decision making process is highly important for reducing the overall risk as failing to incorporate data may result in under or over estimation of the problem and can lead to erroneous decision making [6].

**Road network optimization:**

With the increase of need of considering real world issues in GIS and travel optimization problems, finding an optimal solution with respect to multi criterions has become extremely important. With the need of improved travel optimization techniques different researchers have implemented various methods to solve multi-criterion optimization problems. The complexity of multi criterion optimization solution increases with the increase of total number of criterions considered for a solution technique. Bo Huang and Fery (1995) presented an approach attempts to build a framework for determination of
optimal routes for hazardous material transportation. The proposed study puts emphasis on providing an overview of the possible tradeoffs between routes without generating all of them. A simple characterization of the efficient routes is used to select the best ones with no need for any input from the decision-makers. Huang in his study included a case study with 8 objective functions on a road network in Singapore. Authors used a geographical information system (GIS) to quantify road link attributes, which are assumed linear and deterministic for the sake of simplicity. The proposed algorithm derives four significantly different routes. Huang used Pareto optimality to find the optimal solution by creating a self controlled function. Pareto optimality can be calculated using different methods where each method has its own suitability to various multi-criterion travel optimization problems [7].

Olaf Jahn and Rolf H. Mohring(2000) from University at Berlin in their paper suggested that when traffic flows are routed through a road network, the route guidance system can route them over paths which are perceived as too long. As shown in Fig2, As the route guidance systems can only recommend paths to the drivers, special care has to be taken for such cases. In Jan 2000, Olaf and Rolf, along-with Andreas S. Schulz from Sloan School of Management, MIT, Cambridge presented an algorithm combined with convex combinations algorithm by Frank and Wolfe to solve the problem they stated as a non linear multi commodity flow problem.

![Fig 2The representation of roads; Olaf Jahn and Rolf H. Mohring 's approach(2000) [8].](image)

The results obtained by their algorithm which nests the Frank-Wolfe algorithm from nonlinear optimization show that the running time of an algorithm depends much on the difficulty of the encountered constrained shortest paths problem. They also suggested that the algorithm they implemented is practical for problems with a few thousand nodes, arcs, and commodities [8].

Literature provides sufficient evidence [9], [10],[11] and [12] that provides in detail explanation of optimization problems and methods of handling such problems.

**BACKGROUND THEORY**

Among the basic targets of this research is to develop apposite and agile techniques to reduce the total cost of travel depending on the desired criteria and to ameliorate the technologies used by various researchers and results obtained during their work in past. In order to understand the bases of analysis performed during the research one must understand some different methodologies and tools used in the process.
What is optimization

An optimization problem can be defined as formulating a mathematical method which requires formalization of the design process. In order to understand the process of optimization we must understand the following components involved.

- The Solution space: All solutions we can consider for one given problem. This could be the whole road network given for an optimization problem.
- Criterion: The criterions are used to explicitly describe the feasible solutions for a given problem. It is like a measure of quality for a given solution or design. Five different criterions have been used to calculate the best suited path for a travel depending on the requirements of the user.
- Requirements: Requirements are the constraints that separate the valid or feasible solutions from the ones which are non favorable. The path or solution to be found must the one which combines the source and destination nodes and looks to decrease the total cost of a travel which is calculated using all five criterions involved depending on their respected strength.
- Analysis: testing the solution in order to make prediction of consequences.

A. Multi-criterion optimization

When we talk about more than one criterion, the solution to a multi criterion optimization problem must look at the criterion proportional to their importance. The main aim behind designing this technique for traffic layouts is to find a way of calculating not just one final solution for any traveling problem but the best compromise between various competing solutions, and sometimes conflicting criteria are used to evaluate the quality of solutions. A solution $X$ is believed to be optimized with respect to a solution set $S$ if no other solution in the solution set $S$ is as good as $X$ in all the criteria and no other solution in the solution set $S$ is better than $X$ in at least one criteria.

A general multi-criterion optimization problem can be formally defined as:

\[
\text{Find the vector } X = [X_1, X_2, X_3, \ldots, X_n]^T \text{ which will satisfy the m inequality constraints:}
\]

\[
F_i(X) = 0, \ i = 1, 2, \ldots, m, \text{ and the p equality constraints}
\]

\[
G_i(X) = 0, \ i = 1, 2, \ldots, p, \text{ and will optimize the vector function.}
\]

\[
F(X) = [F_1(X), F_2(X), \ldots, F_k(X)]^T
\]

The existence of multiple objective functions changes the notion of optimum, because, when a best solution to a traveling problem is based on multi criteria, we are really trying to find good compromises between the conflicting criterions rather than a single solution as in global optimization. The most commonly used form of “optimum” is the one originally proposed in 1881 by Francis Ysidro Edgeworth. Vilfredo Pareto later generalized the notion in 1896, which is known as Pareto Optimum.

A.1 Pareto Optimality

The concept of Pareto optimality presented by Vilfredo Pareto(1896) says that a solution $X$ is a Pareto optimal solution if there does not exist a feasible vector of decision variables $x_1 F$ that would decrease some criterion without increasing at least one other criterion. Unfortunately, this concept almost always
gives a set of solutions rather than a single solution. Such a solution set is called the Pareto optimal set and the plot of the objective functions whose non-dominated vectors are in the Pareto optimal set is called the Pareto front.
There are a number of methods which can be used to find Pareto optimal solution or a vector of solutions.

A.2 Linear weighting method

Toomas Lepikult (1999) proposed a famous and widely adopted method to optimize several conflicting criteria simultaneously by combining such criterions linearly and by introducing a weighting factor for each of the criterions. Toomas Lepikult suggested that if these weighting coefficients, denoted by $W_i$, where $i = 0,1,2,3,...,m$, are interpreted as parameters, we obtain a linear weighting method which can be used to generate Pareto optima.

The criterions we have used are crime, pollution level, tourism attraction, distance and time which can be assigned a different weight i.e. $W_1$ for crime weight, $W_2$ for pollution and so on. As all the criterions are conflicting with each other, we can say

$$W_1 + W_2 + ..+ W_m = 1$$

Or

$$\sum_{i=1}^{m} W_i = 1.$$  

The scalar optimization problem, where the objective function consisting of the weighted sum of the criteria is minimized in the original feasible set, takes the form

Minimize such that

$$\sum_{i=1}^{n} W_i f_i(x), f_i(x) \in \Omega$$

The Pareto optimal for our problem can then be calculated by tuning the weights $W_i$ and solving this scalar problem for each fixed parameter combination, separately. Unfortunately, this method can only generate the entire Pareto optimal set where both the criteria and constraint functions are convex. Though, most structural optimization problems are non-convex, no Pareto optima may be lost, even if this optimization method is used but with the no guarantee.

A.3 Norm methods

Norm methods allow the generation of Pareto optima by reducing the distance between the attainable set and some chosen reference point in the whole criteria space. In this method a diagonal matrix $W = \{w_1,w_2,w_3,..,w_m\}$ is introduced that includes weights $W_i \geq 0$, $i = 1,2,3,..,m$.

We can define a function for criteria, where we have a set

$$Z = \{Z_1,Z_2,Z_3,...,Z_n\}$$
representing any fixed point in the criterion space. Usually vector \( Z_i \in \mathbb{R}^m \) is chosen so that it contains the individual minima of the criteria in the feasible set as components, that is \( z_i = \min \min f_i (t_x) \) such that \( t_x \in \Omega \).

In some other efforts Koski used weighted sum method to structural optimization problems [13]. Marglin developed the \( \varepsilon \)-constraint method [14] and Lin presented an equality constraint method for such computational problems [15]. Furthermore, heuristic methods are used for multi-objective optimization whereas Suppapitnarm (2000) made use of simulated annealing to multi-objective optimization problems [16]. Additionally, multi-objective optimization using Genetic Algorithms can be found in Goldberg [17].

**B. Fuzzy Logic**

FL’s approach to solve control problems imitates a human way of decision making, only in a much faster way. FL is an established methodology for finding a definite conclusion while dealing with unclear, vague, distorted, or incomplete inputs. Like all other areas of science and daily life, researchers have implemented a number of road network control systems using fuzzy logic, and FL is a useful method while dealing with operations based on blatant parameters.

**B.1 How to use fuzzy logic**

One must perform the following steps in order to fully implement a FL system.

1. Define the control objectives and criteria
2. Devise the input to output relationships and select minimum input variables for FL engine.
3. Break down the control problem into a series of IF X AND Y THEN Z rules to define the desired system output for given input conditions.
4. Generate FL membership functions that define the meaning of Input/Output terms used in the rules.
5. Test the designed system, evaluate and analyze the results, change the rules and membership functions if required and retest until satisfactory results are obtained.

**B.2 Fuzzy Sets**

The Fuzzy Set theory, presented by Zadeh L. A. [18], can be imagined as an extension of classical sets. Unlike the Boolean algebra, fuzzy sets don’t just have a membership value of in the set and out the set, 1 and 0, but they have partial membership value, between 1 and 0. These values are assigned using a membership function, which tells what should be the value for a given member of the input set.

For example, consider a road network with two routes ‘abd’ and ‘acd’ for a journey between ‘a’ and ‘d’. Depending on the actual values of the factors we have considered in this research, If ‘abd’ reduces the cost of our cost function but ‘acd’ has slightly bigger cost for the same function, it means ‘abd’ is more optimal solution than ‘acd’, but there can be another path, ‘ad’ which reduces the cost even lower than ‘abd’. In order to support such estimations, we need to design the membership functions for all the criterions in a way that they follow the same pattern, and allot high and low membership values to the input variables being transformed from mathematical to fuzzy form accordance to their membership strength.
These sets are generally described through a membership function
\[ m(s) : x \to [0,1] \]
that associates each element \( x_i \) with a degree of membership in \( S \) using the
membership function and 0 means no, 1 means full membership values in between indicate how strongly
an element is affiliated with the set.

### B.3 Fuzzification

Fuzzification is the extension principle which defines how a value, function or set can be represented by
a corresponding fuzzy membership function. It also extends the known membership function of a subset
to a specific value, or a function, or the full set. Let us say we have a function

\[ f: X \to Y \]

Its membership function \( m_A \) for a subset \( A \subseteq X \), extension
\[ mf(A) \ (f(x)) = mA(x) \]

Let’s consider the crime factor to analyze the solution technique developed. Crime is easily the most
important of all the fuzzy functions we have included. Assuming the values of crime between 1 and 10,
the membership function for crime uses the following assumptions.

- Low crime rates don’t affect the travel as much as higher rates.
- For high rates of crime in an area, the cost of a path doesn’t increase to a level when users can’t
  travel at all.

Next step is to define the IF-ELSE-RULES for our control system. One or a series of such
IF-ELSE-RULES can be defined in order to justify the needs of the requirement domain. We can say

If crime is too low then cost is extremely low or
If crime is low then the cost is too low
If crime is not low, cost is not low etc

Such if else rules allow us define the membership functions with complete knowledge of restriction
and requirements.

Next step is to convert such rules into fuzzy functions. The membership function for Crime is given
below.

\[ C=\{(1,0.0),(2,0.2),(3,0.3),(4,0.4),(5,0.4),(6,0.5),(7,0.5),(8,0.6),(9,0.6),(10,0.6)\} \]

Each element closed as one set is called singleton. For instance, let us look at few singletons from the
given membership function.

\[ SC = \{(6,0.5)\}, \text{ it says when the actual value of crime is 6, its membership value is 0.5, n so on.} \]

The membership functions for all criterions are developed based on the background knowledge and natural
pattern affecting the process.

### C. Defuzzification

The process of converting a fuzzy output variable into a single-value variable is called
defuzzification. It can be seen as the reverse of fuzzification. There are different methods used for
defuzzification, but the widely used methods are center of gravity (COG): finds the geometrical center
of the output variable mean of maxima: calculates the mean of the maxima of the membership function.
Ahmad M Ibrahim (2003) revisited all the basic concepts and processes involved, a reference to his
work can be found in the reference list for interested readers [19]. The mathematical functions generated
for every single fuzzy criteria does the job of defuzzification. Now values achieved from theoretical
knowing and converted into fuzzy form, passed through the membership functions give us mathematical
or defuzzyfied results obtained by passing the fuzzy values through the mathematical functions which map exact fuzzy behavior of all the factors.

A list of general rules which help describing how complex sentences can be generated from elementary ones are given below.

**Modification rules:**
We can introduce a linguistic variable into a simple sentence, e.g. “Peter is very old”

**Composition rules:**
Combination of simple sentences through logical operators, e.g. condition (if ... then), conjunction (and), disjunction (or)

**Quantification rules:**
Use of linguistic variables with quantifiers, e.g. most, many, almost all

**Qualification rules:**
Linguistic variables applied to truth, probability, possibility, e.g. very true, very likely, almost impossible

**Fuzzy Inference rules:**
The fuzzy inference rules are the principles that allow the generation of new sentences from existing ones. The general logical inference rules (modus ponens, resolution, etc) are not directly applicable as fuzzy logic has its own inference rules.

**SYSTEM DESCRIPTION**
This section explains the basic methodologies used in implementation of proposed approach for calculating shortest path between any two given nodes. Assumptions have been made to represent real world data as crime rate of an area, tourism attraction and pollution level of a given road network in order to explain the working of the suggested method as shown in Fig 3. As we are dealing with multiple criterions for calculating the total cost of a travel, an approach for calculating the multi criterion optimal solution is provided as shown in Fig 4 and Fig 5.

![Fig 3. System Design](image)

Fig 3. System Design

The main platform of research is based on two criteria of optimization namely Timeline and Environment. The former can be seen as a hard dollar set of criteria, subdivided into two (sub) categories namely Time and Distance. The latter; Environment, has a soft dollar nature to it and is subdivided into three (sub) categories namely; Crime, Pollution and Tourism Attraction. (Figure 4.1)

**Best paths evaluation:**
All the criterions used in our suggested system are given below.

1) Distance to be covered to get to the destination node from a given node.
2) Journey time.
3) Rate of Crime for a given network.
4) Level of pollution for a given road or street.
5) Tourism attraction for a given road or whole path.

Our approach for calculating shortest path between any two given nodes uses all the criterions listed above. The distance needed to cover for a journey from one node to another is calculated from the map directly, whereas the time of travel is calculated using an assumed speed of 30 mph which is a standard speed for most streets in UK.

Dijkstra's Algorithm is used to calculate the shortest-path problem. The minimum distance from node \( a \) to \( b \) is the minimum of the distance of any path from node \( a \) to \( b \). As our method of travel optimization looks at five different criterions, or simply the factors of optimization, at the same time in order to expand the requirement domain, this makes it usable for a large number of people having a wide spectrum of priorities in their journey. Our approach uses two different ways of finding shortest paths between two given nodes in the network of interest.

1) Weighted Sum.
2) Fuzzy Logic based functions.
Figure 5: Structure of multi criterion approach

A. Weighted Sum
In weighted sum method some weight has been given to the entire criterion set, where every weight tells how much concerned we are about every single factor e.g. Tourism attraction, distance that will cover in the journey and the crime level of a given path which could affect the choice of a path.

Separate weights are used for all the criterions respective to their actual values. Given the weight for all the criterions, our implementation of multi criterion optimization comes up with the shortest distance between any two given nodes. The phenomenon of multiple criterions tells about the nature of the problem where increasing one or more objectives reduce effect of the rest of the criteria or criterions, which effectively, makes the solution more complex but comes out with better results and provides more knowledge of the problem itself.

The mathematical expression for Cost calculated by using weighted sum is given below

\[
\text{TotalCost} = \frac{(Wt \times \text{Time} + Wd \times \text{Distance} + Wp \times \text{Pollution} + Wc \times \text{CrimeRate} + Wta \times \text{TourismAttraction})}{(Wt+Wd+Wp+Wc+Wta)}
\]

B. Fuzzy functions
The fuzzy functions used for all three fuzzy criterions i.e. Crime, pollution and Tourism attraction are explained in detail. All the functions for respective criterions are derived according to the assumptions which are based on the real world facts. The rest of the criterions time and distance are directly calculated from the map as said before. Compared to weighted sum method, where we ask the user to enter the weights for all relevant criterions, which is basically a value between 1 and 10, FL uses its own unique way of manipulating the criterion strength. FL, rather than using numbers or exact values, creates a membership function for entire input domain. FL functions developed to calculate the total cost of a travel are based on available evidence and all the membership functions are formed by making some assumptions.

B.1 Fuzzy criterions:
The crime level of an area directly affects the choice of path one wants to select for the journey. As we are dealing with roads where the users are drivers rather than pedestrians, which actually reduces the effect of crime on their travel but still the fact cannot be denied that safer journey is the better journey.

Assuming the values of crime between 1 and 10, the membership function for crime uses the following assumptions.

- Low crime rates don’t affect the travel as much as higher rates.
- For high rates of crime in an area, the cost of a path doesn’t increase to a level when users can’t travel at all.

The singletons for all corresponding values of a crime in real life are given below.

\[ C = \{(1,0),(2,0.2),(3,0.3),(4,0.4),(5,0.5),(6,0.5),(7,0.5),(8,0.6),(9,0.6),(10,0.6)\} \]

As the fuzzy logic only provides the membership functions for the criterions, mathematical function for crime is calculated and implemented as:

\[ C(x) = 0.1445x - 0.00757x^2 - 0.0933 \]

Similarly, for Pollution level, the fuzzy singletons are...
P = {(1,0),(2,0),(3,0.2),(4,0.3),(5,0.4),(6,0.5),(7,0.5),(8,0.6),(9,0.6),(10,0.6)}
and mathematical function is calculated as
P(x) = 0.15x – 0.007x^2 – 0.23
and for Tourism attraction, the fuzzy singletons are
T_A= {(1,0.6),(2,0.6),(3,0.5),(4,0.5),(5,0.4),(6,0.3),(7,0.2),(8,0.2),(9,0.1),(10,0)}
and mathematical function is calculated as
T_A(x) = 0.67 - 0.0023* x^2 - 0.0441*x
All the fuzzy membership functions along with respective mathematical functions for all fuzzy criterions are shown in figures 6, 7 and 8.

C. Pareto Optimality
There are a number of methods which can be used to find Pareto optimal solution or a vector of solutions. The one we have used is explained below.

**Linear weighting method:**
The most famous and most commonly used method to optimize several conflicting criteria simultaneously is to combine them linearly and introduce a weighting factor for each of the criterions. If these weighting coefficients, denoted by $W_i$, where $i = 0,1,2,3,..,m$, are interpreted as parameters we obtain a linear weighting method which can be used for the generation of Pareto optima. Without losing generality the normalization may be applied for convenience.

As all criterions are conflicting with each other, we can say $W_1 + W_2 + ..+W_m = 1$ or $\sum_{i=1}^{n} W_i = 1$

The scalar optimization problem, where the objective function consisting of the weighted sum of the criteria is minimized in the original feasible set, takes the form

Minimize such that

$$\sum_{i=1}^{n} W_i f(x), f \in \Omega$$

By changing the weights $W_i$, and solving this scalar problem separately for each fixed parameter combination we can calculate Pareto optima for our problem. In our case the criterions are; 1) time, 2) distance, 3) crime rate, 4) pollution, and 5) tourism attraction. All these criterions have been separated into two separate groups. As said before the two groups have their own weights to represent their participation in the cost calculation. The equation used to represent cost for Pareto optimality problem is;

$$P(cost) = W_t(\text{Time} + \text{Distance}) + W_e(\text{Crime} + \text{Pollution} + \text{Tourism attraction})$$

Here, as the two given weights are conflicting hence, $W_t + W_e = 1$

By calculating the cost of all possible shortest paths for both criterions, we can easily find the best comprise of given two criterions. Starting with $W_t = 0.5$ and $W_e = 1 - 0.5$, cost for all combinations is calculated, and the areas with lowest costs are combined to form a Pareto front as explained in the case studies.

**CASE STUDY AND RESULTS**
The case study of Liverpool City Center UK is presented to show various techniques developed and used for road network analysis. The analysis is done using a prototype called SmartRNA, a tool developed at Liverpool Hope University College, Liverpool. Edit Network tool provided with the prototype has been used to develop a graph network using the Liverpool city centre map shown in fig.9. The network consists of 42 nodes or junctions and number of edges or roads of it as shown in the Fig9. The table given below provides the names of all nodes against the relative NodeID.
The Local depth of a node or a junction of two or more streets has a local accessibility for the whole network. In the other words we can say, the local depth graph the local depth of the whole network for a given node in which we are interested as shown in Fig 10. To represent a few results, few nodes have been selected as bases for this analysis. To give of a taste of the differences between these approaches, the graph shows the differences between the global depths of the network for both the weighted and fuzzy costs calculation methods.

Fig 10. Red shows the Fuzzy while green shows the weighted sum depth of each node

Fig 11 to Fig 16 demonstrate the results extracted using both Fuzzy functions and the weighted method. The result show that both methods have generated different outputs where fuzzy functions behave more realistically and succeed to find best suitable paths between any two source and destination nodes.
Fig 11. Best path between node 4 and node 30 using weighted sum method

Fig 12. Best path between node 4 and node 30 using fuzzy functions

Shortest distance between node 2 and 21 for the Crime factor

Fig 13. Shortest distance between node 2 and 21 for the pollution factor.

Fig 14. Shortest distance between node 2 and 21 for the tourism factor.

Fig 15. Shortest distance between node 8 and node 53 based on the time and distance factors
Pareto Optimal solution:
In multi criterion optimization, sometimes the criterions are conflicting with each other, which mean the increase in one or more criterions affects other comprising criterions by reducing their contribution. Let’s exploit the idea of two different groups of criterions separating the criterions of similar nature. As discussed before we have all the criterions in 2 groups, let’s assign each group a weight which will represent combined weight for criterions in the group. For time factors we have “Wt” and for environment factors we use “We”. As said earlier both the criterions are conflicting, we can say that
\[ Wt + We = 1 \]
Here cost of a given candidate shortest path is calculated as
\[ \text{Cost} = \text{We} \times (\text{Crime} + \text{Pollution level} + \text{Tourism attraction}) + \text{Wt} \times (\text{Time} + \text{Distance}). \]

Now in order to find a Pareto optimal shortest distance between any two given points using our multi criterion optimization approach we can find the best comprise between given two criterions. For example best path between nodes 12 and 31. By calculating the cost of all possible shortest paths for both criterions, we can easily find the best comprise of given two criterions. Starting with \( Wt = 0.5 \) and \( We = 1 - 0.5 \), cost for all combinations is calculated.

Fig 16 is the graph showing the cost of all the possible shortest paths for both criterions where the values for both criterions is kept between 0 and 1. 

Fig 17 is the graph showing the cost of all the possible shortest paths for both criterions where the values for both criterions is kept between 0 and 1. 

The underlined region in the graph above shows the Pareto front for our solution. And finally, the Pareto optimal path for the said node is shown in the Fig 18.
This result shows that for the given example the environment factor is more favorable to reduce the cost of travel between node 12 and 31.

CONCLUSION
The proposed multi criteria approach for road network evaluation is capable of determining the minimal cost of a journey for given start and destination points, as it can analyze different aspects involved in a multi criteria travel problem.

Our multi criterion traffic layout evaluation technique provides a strong platform for further research in the area. The use of fuzzy based functions to determine the best path in a multi dimensional requirement domain has provided a new technical base for the research field. Working with more than one criterion helps exploring a rich solution space as single criterion based evaluation seems to have become out of date in terms of satisfying today’s complex transportation domain problems.

Our multiple objectives based technique considers most of the criterions one may will to include as a travel’s pre-requisite, but still the idea can further be improved by tuning the fuzzy functions used for all fuzzy criteria to acquire better results. By including a few more criterions to the evaluation can also help in the area of multi criteria based traffic layout analysis to perform in more natural way. A different path finding algorithm e.g. A* or other heuristic approaches can also be embedded, with currently used technologies, in the future.

Use of fuzzy logic is justified using the results and the Pareto optimal solution found is a weak Pareto optimal. Bringing in more and more factors to calculate the goodness of a travel will help reach the root level problem and allow this research to go on achieving high standard results. In addition to the cost factors used here, a few more functions such that; Peak and off/Peak times on the roads, special events, delay on each traffic signal and any other relative criterions can be brought in to allow give the research new directions in the future.

The Pareto platform provided is unfortunately not good enough so it clearly needs to be replaced with a different and more advanced technique. Evolutionary Algorithms, equality constraints in multi-criterion optimization, simulated annealing and Genetic algorithms are some of the methodologies that can be used to improve the Pareto optimal solution representation.
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